WW Scattering & Strongly Interacting Electroweak Sector

Tao Han
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(Lattice Meets Experiment, FNAL, Oct. 15, 2011)

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A Lesson from QCD

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Effective Lagrangian Approach

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Signals at Colliders

A Lesson from QCD: $\pi\pi$ Scattering

(1). Characteristics of $\pi\pi$ Amplitudes

In low energy QCD, the pions are the Nanbu-Goldstone bosons from the spontaneous chiral symmetry breaking

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

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The amplitudes for $\pi^i + \pi^j \to \pi^k + \pi^\ell \quad (i, j, k, \ell = 1, 2, 3)$ has the form

$$\mathcal{M}_{ijk\ell}(s,t) = A(s,t,u)\delta_{ij}\delta_{k\ell} + B(s,t,u)\delta_{ik}\delta_{j\ell} + C(s,t,u)\delta_{i\ell}\delta_{jk}$$

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Satisfying:

- Massless approximation $s + t + u = 0$.
- Crossing symmetry among $B(s,t,u) = A(t,s,u),\ C(s,t,u) = A(u,t,s)$.
- Bose symmetry $A(s,u,t) = A(s,t,u)$,

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Thus only one analytical function. Low energy theorem when $s = t \to 0$:

$$A(s,t,u) = \frac{s}{f_\pi^2} + \mathcal{O}(s^2, st, t^2) + ..., \quad f_\pi \approx 93 \text{ MeV}.$$

Partial wave expansion:

$$a_{I\ell}(s) = \frac{1}{64\pi}\int_{-1}^{1} d\cos\theta \; P_\ell(\cos\theta) \; \mathcal{M}^I(s,t),$$

$$\mathcal{M}^{I=0} = 3A + B + C,$$

$$\mathcal{M}^{I=1} = B - C,$$

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Elastic dominance ($\pi\pi \to 4\pi, \; KK$ small):

$$a_{I\ell}(s) = e^{i\delta_\ell} \; \sin\delta_\ell$$

$$Im\,(a_{I\ell}) = |a_{I\ell}|^2, \quad Im\left(a_{I\ell}^{-1}\right) = -1, \quad Re\,(a_{I\ell}) \propto l_{decay} + slope \; k^2 + ...$$

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How high in energies?

$$a_{00} = \frac{s}{16\pi f_\pi^2}, \; a_{11} = \frac{s}{96\pi f_\pi^2}, \; a_{20} = -\frac{s}{32\pi f_\pi^2}$$

$$\Rightarrow E < 4\pi^{1/2} f_\pi < 1 \text{ GeV}.$$

in accordance with the elasticity condition.

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Different schemes:

- K-matrix unitarization (simple, starting from the leading term)

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- N/D unitarization (correct analytical behavior)

$$a_{IJ} = \frac{N_{IJ}}{D_{IJ}},$$

$$N_{IJ} = a_{IJ}, \quad D_{IJ}(s) = -\frac{1}{\pi}\int_0^\infty \frac{ds'}{s' - s} N_{IJ}(s').$$

Padé and N/D could lead to resonant structure: any dynamics?

(C). Order $\frac{k^4}{f_\pi^4}$

$$\mathcal{L}_4 = \alpha_1 [\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U)]^2 + \alpha_2 \, \mathrm{Tr}(\partial_\mu U^\dagger \partial_\nu U) \, \mathrm{Tr}(\partial^\mu U^\dagger \partial^\nu U),$$

corresponding to

$$A(s,t) = \frac{s}{f_\pi^2} + \frac{8s^2}{f_\pi^4}\alpha_1 + \frac{4(t^2+u^2)}{f_\pi^4}\alpha_2.$$

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The chiral coefficients α's parameterize the heavy states that have been integrated out, and will delay the unitarity violation.

(D). Scalar Dominance

Assuming unitarity saturated by a lowing-lying state with $I = J = 0$

$$A(s,t,u) = \frac{g_\sigma s}{m_\sigma^2 - s},$$

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(E). Vector Dominance

Assuming a lowing-lying vector state with $I = J = 1$

$$A(s,t,u) = g_\rho \left(\frac{s-u}{m_\rho^2 - t} + \frac{s-t}{m_\rho^2 - u} \right)$$

Electroweak Symmetry Breaking

(1). Enhanced Electroweak Interactions

Yang-Mills gauge self-interactions:

$$
\begin{aligned}
\mathcal{L}_{W3} &= -ig(\partial_\rho W^3_\nu) W^+_\mu W^-_\sigma [g^{\rho\mu} g^{\nu\sigma} - g^{\rho\sigma} g^{\nu\mu}] \\
&\quad -ig(\partial_\rho W^+_\mu) W^3_\nu W^-_\sigma [g^{\rho\sigma} g^{\mu\nu} - g^{\rho\nu} g^{\mu\sigma}] \\
&\quad -ig(\partial_\rho W^-_\sigma) W^3_\nu W^+_\mu [g^{\rho\nu} g^{\mu\sigma} - g^{\rho\mu} g^{\nu\sigma}], \\
\mathcal{L}_{W4} &= \frac{g^2}{4} \left[W^+_\mu W^+_\nu W^-_\sigma W^-_\rho \mathcal{Q}^{\mu\nu\rho\sigma} - 2 W^+_\mu W^3_\nu W^3_\sigma W^-_\rho \mathcal{Q}^{\mu\rho\nu\sigma} \right], \\
&\quad \mathcal{Q}_{\mu\nu\rho\sigma} \equiv 2 g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}.
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Transversely polarized gauge bosons

$$\epsilon^\mu_T \sim (0, \cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta) \quad \Rightarrow \quad A(W_T W_T \to W_T W_T) \sim \mathcal{O}(g^2).$$

Scattering amplitudes well behaved at high energies.

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Scattering amplitudes well behaved at high energies.

Longitudinally polarized gauge bosons

$$\epsilon^\mu_L \sim (E/M_W, ...) \;\Rightarrow\; p^\mu/M_W$$

How would this go?

Longitudinally polarized gauge bosons scattering

$$A(W_L W_L \to W_L W_L) \sim \epsilon_{1L} \cdot \epsilon_{2L} \; \epsilon_{3L} \cdot \epsilon_{4L}$$

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Goldstone-boson Equivalence Theorem:

"At high energies, $\sqrt{s} \gg M_W$, the longitudinally polarized gauge bosons (W_L^i) behave like a scalar, the Nambu-Goldstone bosons (ω^i)."

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Nambu-Goldstone boson (thus $W_L W_L$) scattering:
Most direct probe to Electroweak Symmetry Breaking!
(though may not be the luckiest/easiest)

All $\pi\pi$ scattering techniques can be borrowed.

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- Inconsistent with precision EW measurements.
- Partial wave unitarity violation at $\sqrt{s} < 4\pi^{1/2} v$.
- Toward a UV completion model.

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Add model-independent effective Lagrangian terms.

(B). Scalar Resonance:

$$+ \quad \frac{1}{2}\partial^\mu H \partial_\mu H - \frac{1}{2} M_H^2 H^2$$
$$+ \quad \frac{1}{2} g_h (vH + H^2/2) Tr \partial^\mu U \partial_\mu U - \kappa_t \frac{m_t}{v} H \bar{t} t.$$

Parameters: $M_H,\ g_h,\ \kappa_t$ or $M_H,\ \Gamma_{WW},\ \Gamma_{tt}$

Note: $g_h = \kappa_t = 1$ for SM (Linear σ-model).

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(C). Vector Resonance:

$$- \frac{1}{4}V^{\mu\nu}V^{\mu\nu} - \frac{v^2}{4}Tr J^{a\mu} J^a_\mu$$
$$- \frac{av^2}{4}Tr J^{v\mu} J^v_\mu + \frac{b}{2}\bar{\psi}\gamma^\mu \tau^a \psi V^a_\mu + ...$$

Parameters: $M_V,\ a,\ b$ or $M_V,\ \Gamma_{WW},\ \Gamma_{tt}$.

Note: $A_1,\ \omega_T, ...$ can be incorporated.

(D). Next-to-Leading Order Chiral $\mathcal{L}$:

In the "Goldstone Bosons Only" scenario, there are 15 $SU_L(2) \otimes U_Y(1)$ gauge invariant operators, parameterized by $\ell_i \sim \mathcal{O}(1)$.

$$\left\{ \begin{array}{lcl} \mathcal{L}_4 & = & \ell_4 \left(\frac{v}{\Lambda}\right)^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}_\nu)]^2 \ , \\ \mathcal{L}_5 & = & \ell_5 \left(\frac{v}{\Lambda}\right)^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}^\mu)]^2 \ ; \end{array} \right\} \qquad (SU(2)_c\text{–conserving})$$

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where $V^\mu \equiv \partial^\mu U$.

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where $V^\mu \equiv \partial^\mu U$.

They characterize the underlying physics:

For a heavy scalar $M_H = 2$ TeV:

$\ell_4 \approx 0.0, \quad \ell_5 \approx 0.33$

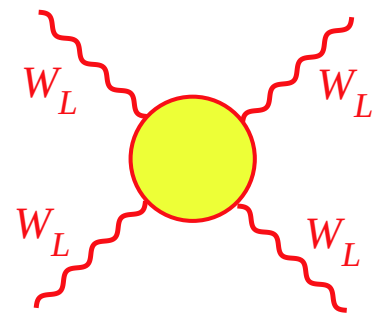
For a heavy vector $M_V = 2$ TeV:

$\ell_4 \approx 0.38 \quad \ell_5 \approx -0.31$.

Signals at Colliders

(1). What Do We Hope to See?

$$
\begin{aligned}
\mathcal{M}(w^+w^- \to w^+w^-) &= \frac{1}{3}\mathcal{M}^{I=0} + \frac{1}{2}\mathcal{M}^{I=1} + \frac{1}{6}\mathcal{M}^{I=2} && (H,\ \rho) \\
\mathcal{M}(w^+w^- \to zz) &= \frac{1}{3}\mathcal{M}^{I=0} - \frac{1}{3}\mathcal{M}^{I=2} && (H) \\
\mathcal{M}(zz \to zz) &= \frac{1}{3}\mathcal{M}^{I=0} + \frac{2}{3}\mathcal{M}^{I=2} && (H) \\
\mathcal{M}(w^\pm z \to w^\pm z) &= \frac{1}{2}\mathcal{M}^{I=1} + \frac{1}{2}\mathcal{M}^{I=2} && (\rho) \\
\mathcal{M}(w^\pm w^\pm \to w^\pm w^\pm) &= \mathcal{M}^{I=2} && (no\ resonance,\ \sim s/v^2)
\end{aligned}
$$



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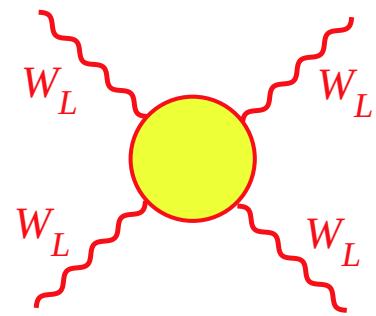
\mathcal{M}(zz \to zz) &= \frac{1}{3}\mathcal{M}^{I=0} + \frac{2}{3}\mathcal{M}^{I=2} && (H) \\
\mathcal{M}(w^\pm z \to w^\pm z) &= \frac{1}{2}\mathcal{M}^{I=1} + \frac{1}{2}\mathcal{M}^{I=2} && (\rho) \\
\mathcal{M}(w^\pm w^\pm \to w^\pm w^\pm) &= \mathcal{M}^{I=2} && (no\ resonance,\ \sim s/v^2)
\end{aligned}
$$



For model connections:

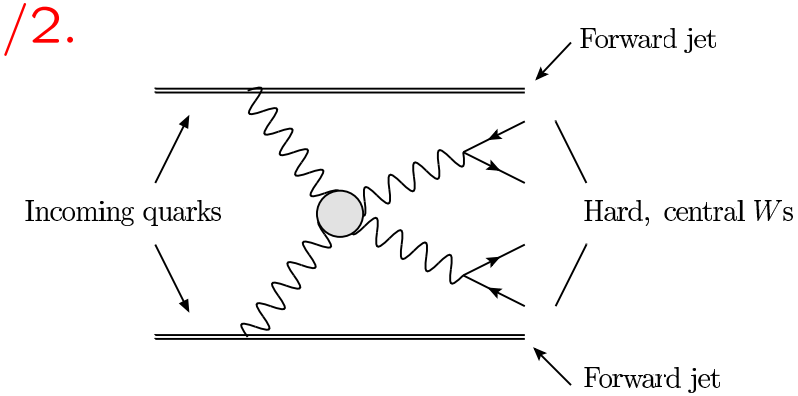
$$
\frac{\sigma(W_L^+W_L^- \to W_L^+W_L^-)}{\sigma(W_L^+W_L^- \to Z_LZ_L)} \begin{cases} \sim 2 & \text{scalar } H^0, \\ \gg 1 & \text{vector } \rho_{TC}^0, \\ \sim 2/3 & \text{LET } \sqrt{s} \ll M. \end{cases}
$$

(2). How Do We See the Signal?

Signal features:

- high-energy gauge boson pairs $E_W \sim 0.5$ TeV.
- forward jets $E_j \sim \mathcal{O}(1)$ TeV, $p_{Tj} \sim M_W/2$.

$$pp \to W_1 W_2 \; j_1 j_2 \; X.$$



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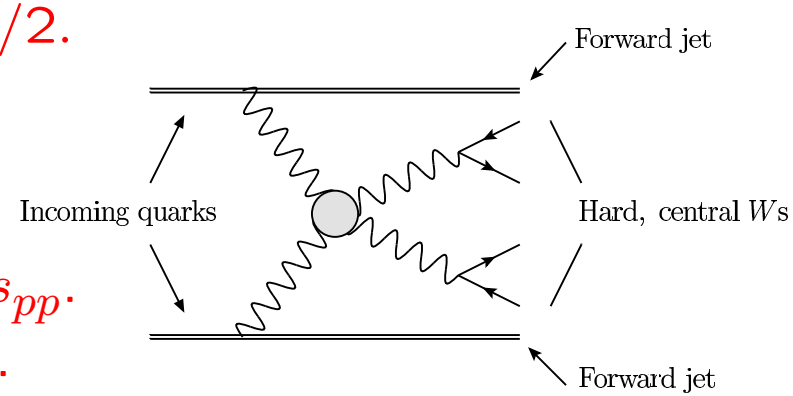

Challenges:

- need high-energy $s_{WW} \sim 0.5^2 s_{qq} \sim \frac{0.3^2}{4} s_{pp}$.
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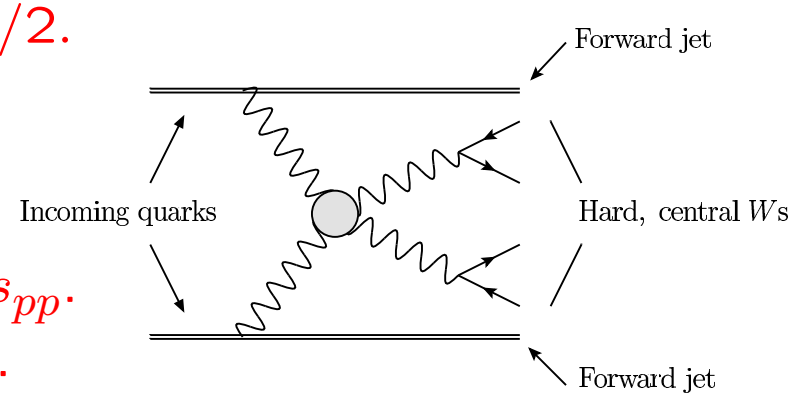

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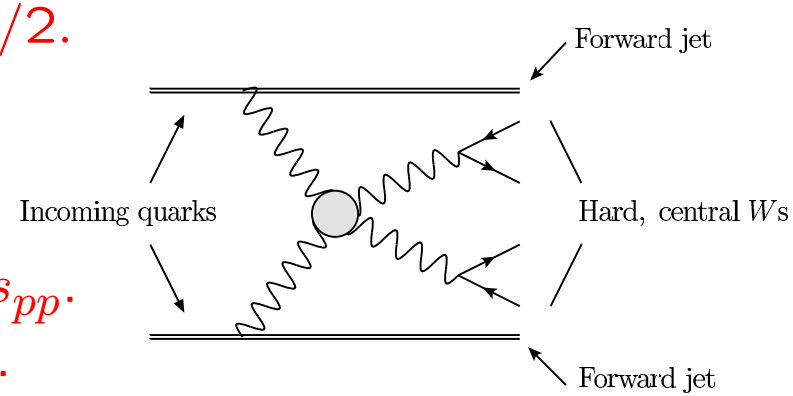

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  branching fractions, detection efficiency ...
- background, background, background!

$pp \to W_1 W_2$ QCD jets X (large)

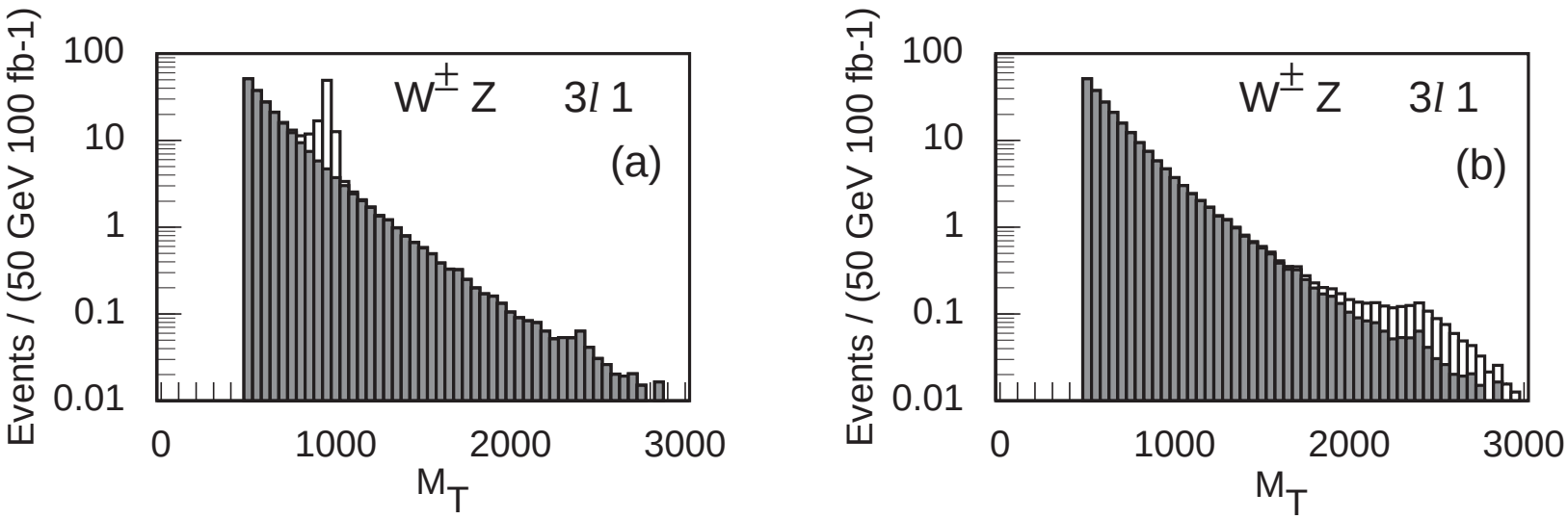
$pp \to W_{T1} W_{T2}$ EW jets X (mimick signal)

$pp \to t\bar{t} \; X \to W^+ W^- \; b\bar{b} \; X$ (very large)

(3). Searches at the LHC

LHC Analysis of the Strongly Interacting WW System: Gold-Plated Modes*

"Gold-plated: pure leptonic final states.
By far, the best channel is the DY $\rho^{\pm}$ production:



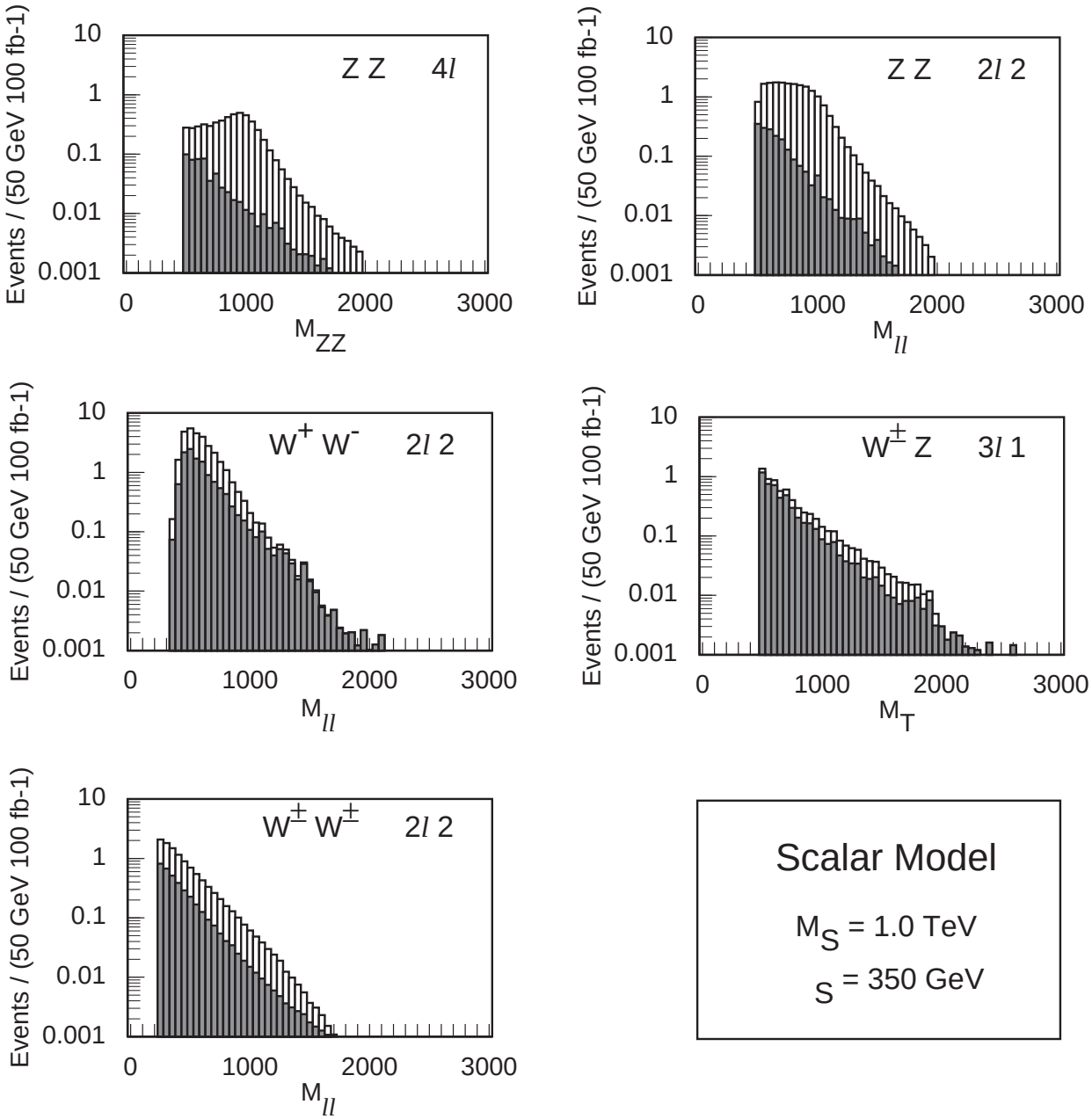
*Bagger, Barger, Cheung, Gunion, Han, Ladinsky, Rosenfeld, Yuan, Phys.Rev.D52:3878,1995.

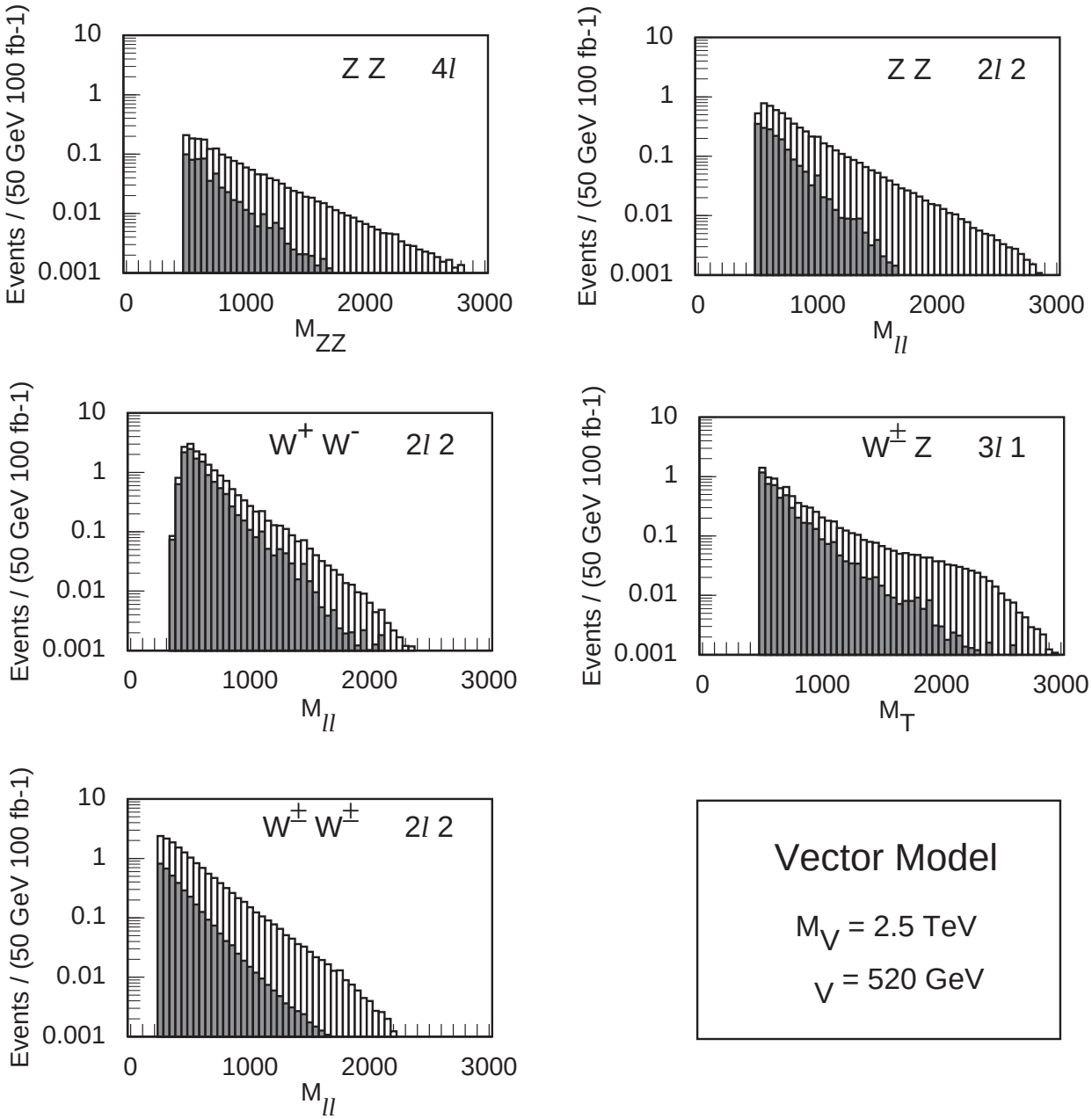
For the WW fusion processes:

Brief version of the cuts	
$\lvert y(l)\rvert < 2.0$	$E(j_{\text{tag}}) > 0.8$ TeV
$p_T(l) > 100$ GeV	$3.0 < \lvert j_{\text{tag}}\rvert < 5.0$
$\Delta p_T(ll) > 440$ GeV	$p_T(j_{\text{tag}}) > 40$ GeV
$\cos\phi_{ll} < -0.8$	$p_T(j_{\text{veto}}) > 30$ GeV
$M(ll) > 250$ GeV	$\lvert y(j_{\text{veto}})\rvert < 3.0$

	Bkgd.	SM	Scalar	O($2N$)	Vec 1.0	Vec 2.5	LET-CG	LET-K	Delay
$ZZ(4\ell)$	0.7	9	4.6	4.0	1.4	1.3	1.5	1.4	1.1
$ZZ(2\ell 2\nu)$	1.8	29	17	14	4.7	4.4	5.0	4.5	3.6
W^+W^-	12	27	18	13	6.2	5.5	5.8	4.6	3.9
$W^\pm Z$	4.9	1.2	1.5	1.2	4.5	3.3	3.2	3.0	2.9
$W^\pm Z(M_T^{cut})$	0.82				2.3				
$W^\pm W^\pm$	3.7	5.6	7.0	5.8	12	11	13	13	8.4

$ZZ(4\ell)$	Leptonic Cuts	Jet Cuts
	$\lvert y(\ell)\rvert < 2.5$	$E(j_{tag}) > 0.8$ TeV
	$p_T(\ell) > 40$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
	$p_T(Z) > \frac{1}{4}\sqrt{M^2(ZZ) - 4M_Z^2}$	$p_T(j_{tag}) > 40$ GeV
	$M(ZZ) > 500$ GeV	No Veto
$ZZ(\ell\ell\nu\nu)$	Leptonic Cuts	Jet Cuts
	$\lvert y(\ell)\rvert < 2.5$	$E(j_{tag}) > 0.8$ TeV
	$p_T(\ell) > 40$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
	$p_T^{\text{miss}} > 250$ GeV	$p_T(j_{tag}) > 40$ GeV
	$M_T(ZZ) > 500$ GeV	$p_T(j_{veto}) > 60$ GeV
	$p_T(\ell\ell) > M_T(ZZ)/4$	$\lvert y(j_{veto})\rvert < 3.0$
W^+W^-	Leptonic Cuts	Jet Cuts
	$\lvert y(\ell)\rvert < 2.0$	$E(j_{tag}) > 0.8$ TeV
	$p_T(\ell) > 100$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
	$\Delta p_T(\ell\ell) > 440$ GeV	$p_T(j_{tag}) > 40$ GeV
	$\cos\phi_{\ell\ell} < -0.8$	$p_T(j_{veto}) > 30$ GeV
	$M(\ell\ell) > 250$ GeV	$\lvert y(j_{veto})\rvert < 3.0$
$W^\pm Z$	Leptonic Cuts	Jet Cuts
	$\lvert y(\ell)\rvert < 2.5$	$E(j_{tag}) > 0.8$ TeV
	$p_T(\ell) > 40$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
	$p_T^{\text{miss}} > 50$ GeV	$p_T(j_{tag}) > 40$ GeV
	$p_T(Z) > \frac{1}{4}M_T(WZ)$	$p_T(j_{veto}) > 60$ GeV
	$M_T(WZ) > 500$ GeV	$\lvert y(j_{veto})\rvert < 3.0$
$W^\pm W^\pm$	Leptonic Cuts	Jet Cuts
	$\lvert y(\ell)\rvert < 2.0$	
	$p_T(\ell) > 70$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
	$\Delta p_T(\ell\ell) > 200$ GeV	$p_T(j_{tag}) > 40$ GeV
	$\cos\phi_{\ell\ell} < -0.8$	$p_T(j_{veto}) > 60$ GeV
	$M(\ell\ell) > 250$ GeV	$\lvert y(j_{veto})\rvert < 3.0$


Events / (50 GeV 100 fb-1)
10
1
0.1
0.01
0.001
0
1000
2000
3000
Z Z 4l
M_ZZ
Z Z 2l 2
M_ll
W+ W- 2l 2
M_ll
W± Z 3l 1
M_T
W± W± 2l 2
M_ll
Scalar Model
M_S = 1.0 TeV
S = 350 GeV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10
1
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Z Z 4l
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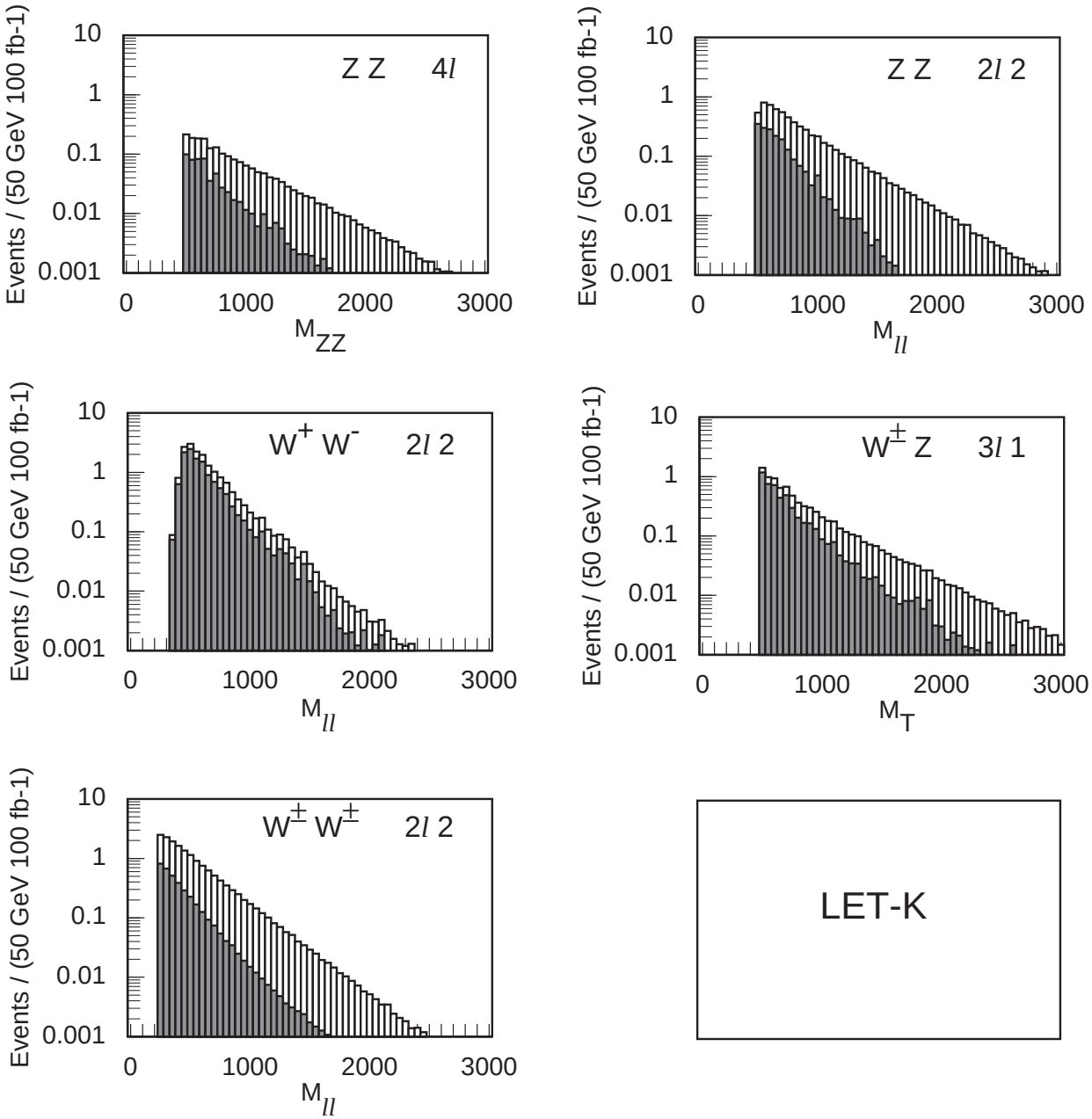
Z Z 2l 2
M_ll
W+ W- 2l 2
M_ll
W± Z 3l 1
M_T
W± W± 2l 2
M_ll
Vector Model
M_V = 2.5 TeV
_V = 520 GeV


Events / (50 GeV 100 fb-1)
10
1
0.1
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0
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Z Z 2l 2
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M_T
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LET-K

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$W^+W^- \to \ell\nu \; + 2$ jets

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Resonance accessible[†] $M_{H,V} \sim 1.5$ TeV with 100 fb^{-1}.

Chiral coefficients accessible[‡] With 100 fb^{-1}

$$-0.39 < \ell_4 < 0.75, \quad -0.6 < \ell_5 < 0.5.$$

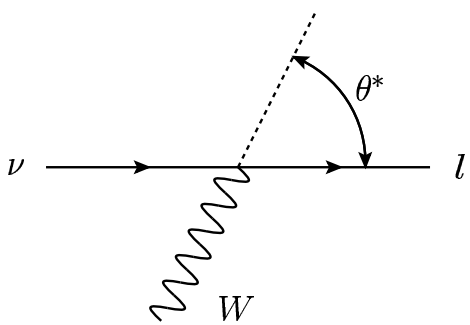
[†]Butterworth, Cox and Forshaw, hep-ph/0201098

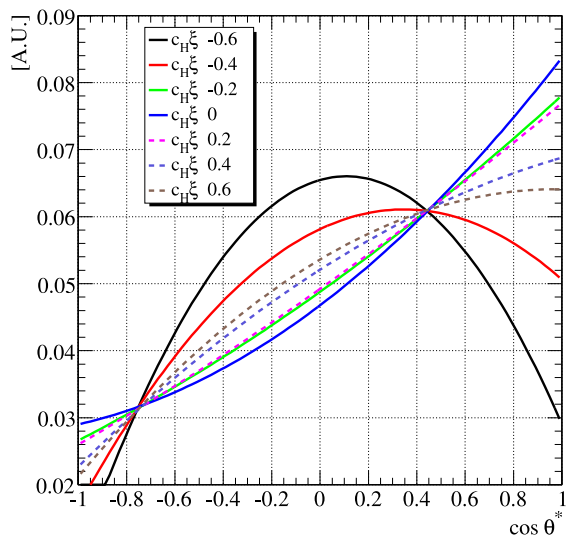
[‡]Eboli, Gonzalez-Garcia, Mizukoshi, hep-ph/0606118

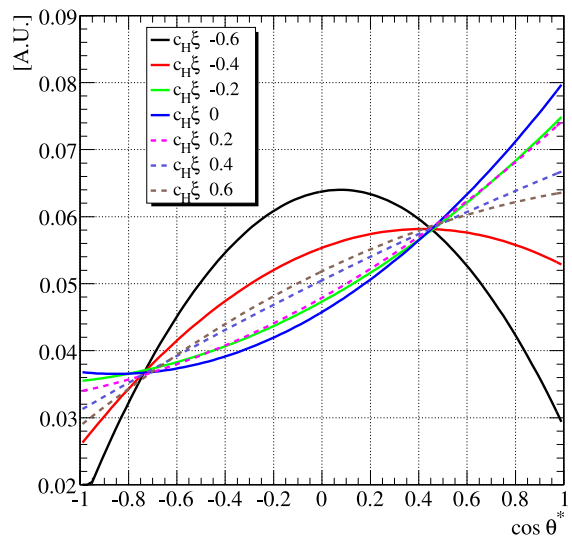
More recent improvement:[§]

- Semi-leptonic decay mode: VV system to fully reconstructed
- Distributions decomposed into longitudinal and transverse components
- Improve fit to the E^2 growth

Angular distributions: $W_\pm \sim (1 \pm \cos\theta)^2,\ W_L \sim \cos^2\theta$







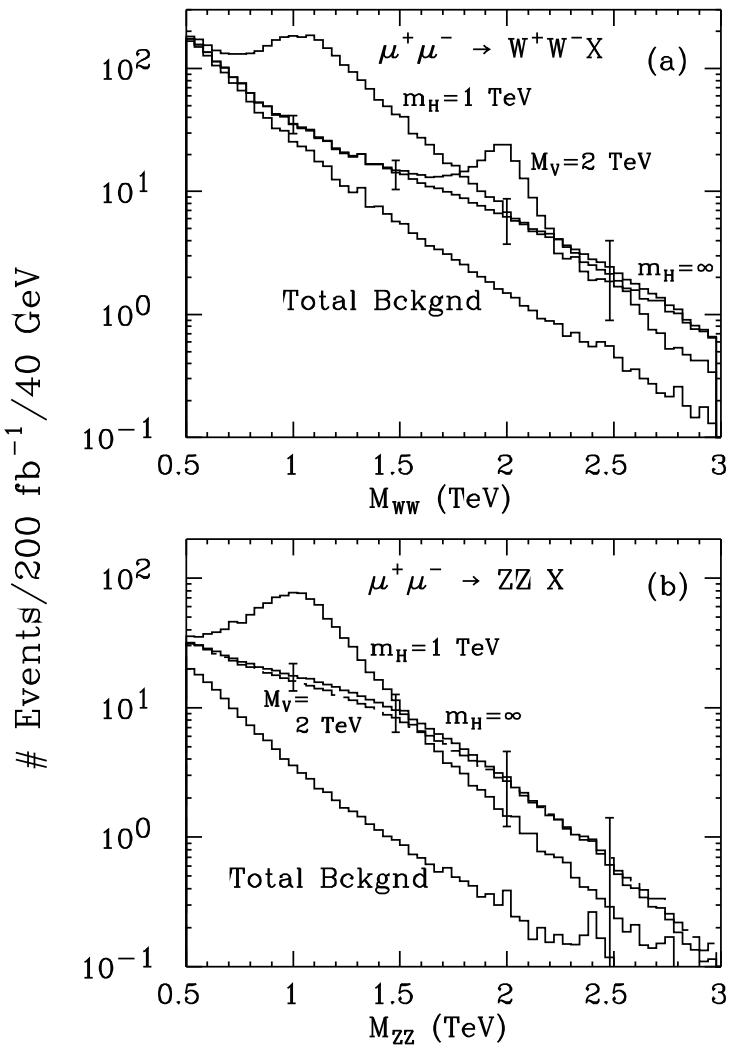
The distributions of $\cos\theta$ for different anomalous couplings at parton level (left) and for fully showered, hadronized, clustered, and reconstructed events (right).

[§]TH, Krohn, Wang, Zhu, e-Print: arXiv:0911.3656

(4). Searches at a Lepton Collider

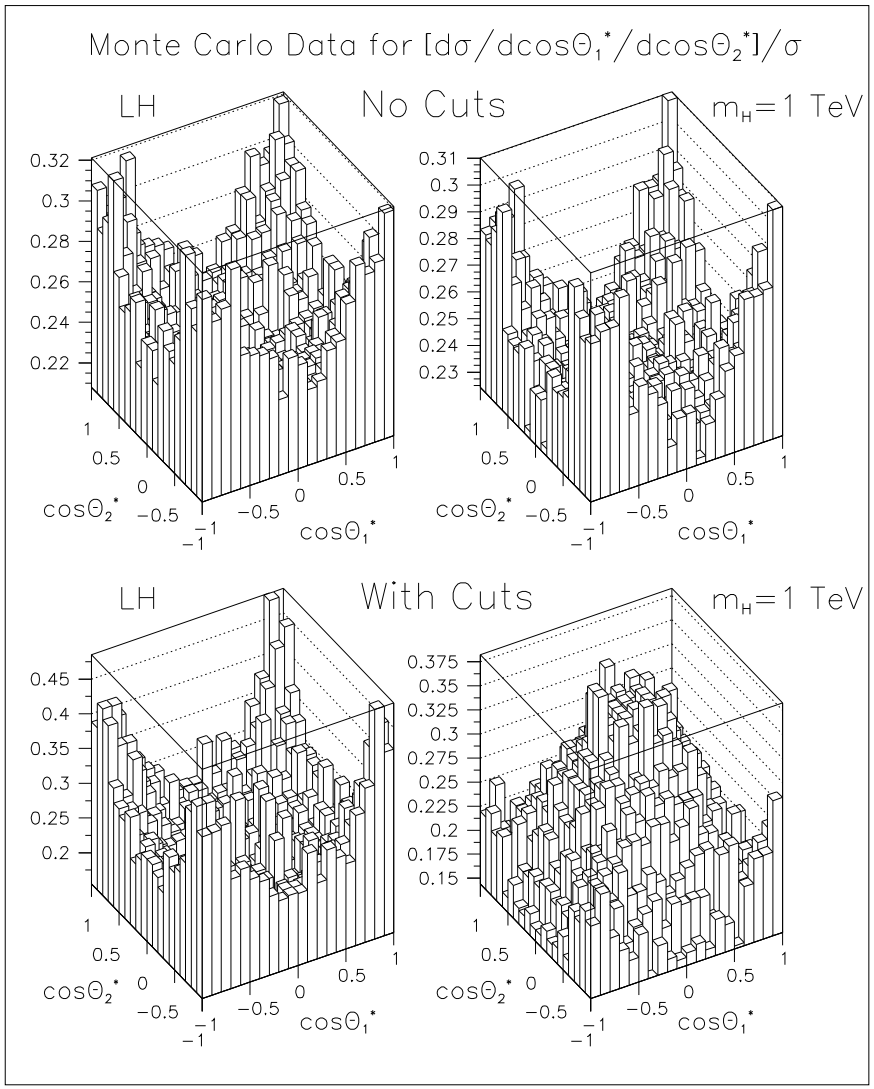
Consider $\mu^+\mu^- \to \nu\nu W^+W^-$ and $\nu\nu ZZ$ at $\sqrt{s} = 4$ TeV

compare H, V or non-resonance.



$W_L W_L$ polarization:

Angular distributions: $W_\pm \sim (1 \pm \cos\theta)^2, \; W_L \sim \cos^2\theta$



Comment: Could not reach this level at the LHC.

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Work on going.