WW Scattering Parameters from BSM Pseudoscalar Phase Shifts

Presented by
Michael I. Buchoff

For the
LSD Collaboration

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Lattice Strong Dynamics Collab.

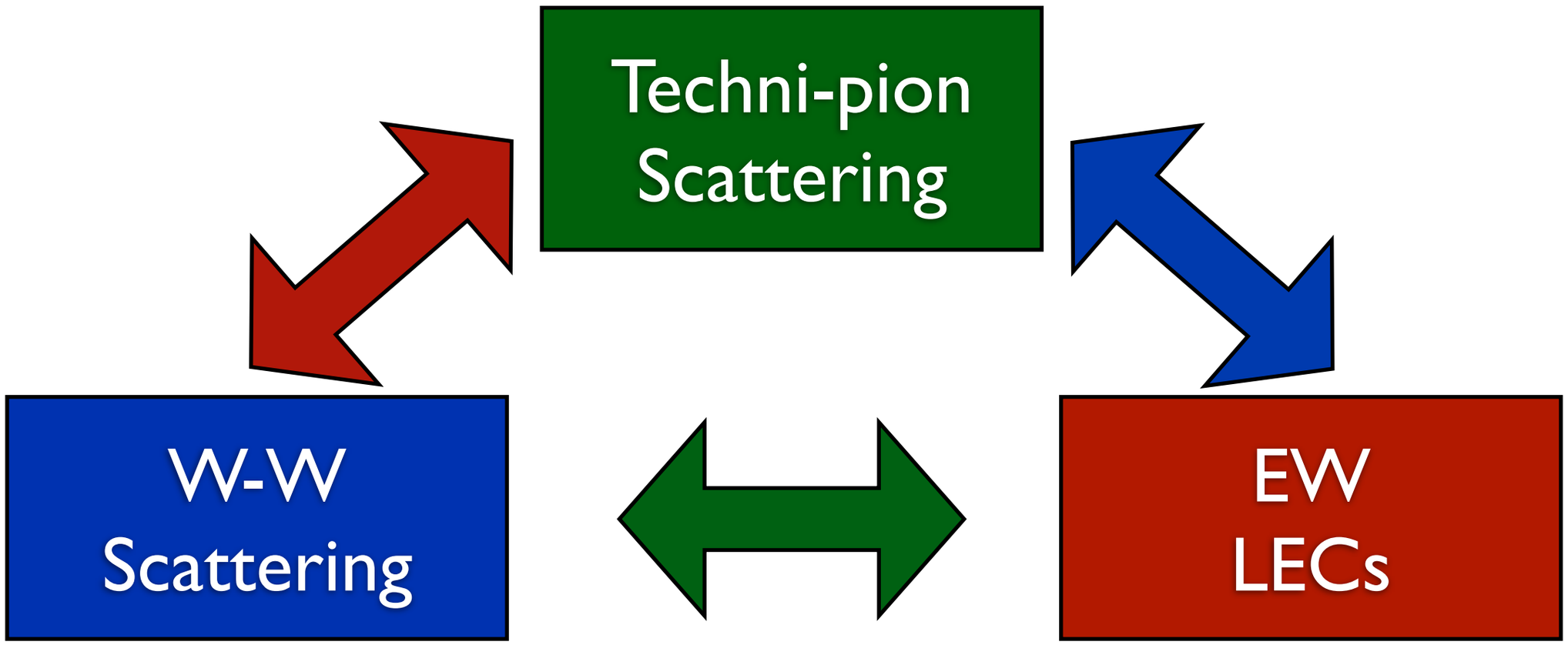
(as of Oct. 4, 2011)

Argonne	Heechang Na, James Osborn
Berkeley	Sergey Syritsyn
Boston	Ron Babich, Richard Brower, Michael Cheng, Claudio Rebbi
Colorado	David Schaich
Fermilab	Ethan Neil
Harvard	Mike Clark
Livermore	MIB, Pavlos Vranas, Joe Wasem
UC Davis	Joe Kiskis
Washington	Saul Cohen
Yale	Tom Appelquist, George Fleming, Meifeng Lin, Gennady Voronov

Classic Question

"So...if the lattice can answer BSM questions, can it make any phenomenological predictions?"

Goal: Address this question using pion scattering as probe



WW Scattering... Why?

It would be GREAT if LHC landscape looked like this:

...but what if it ends up looking like this:

Depending on models, techni-rho can be >2 TeV

WW Scattering... Why?

Bottom Line: Precisely understanding SM processes gives window into BSM physics

Scale:

~1 - 10 MeV: Nuclear Beta Decay (Huey-Wen talk)

~100 MeV: Muon g-2

~100 GeV: S-parameter (David Schaich talk)

~0.1 - 2+ TeV: W-W Scattering

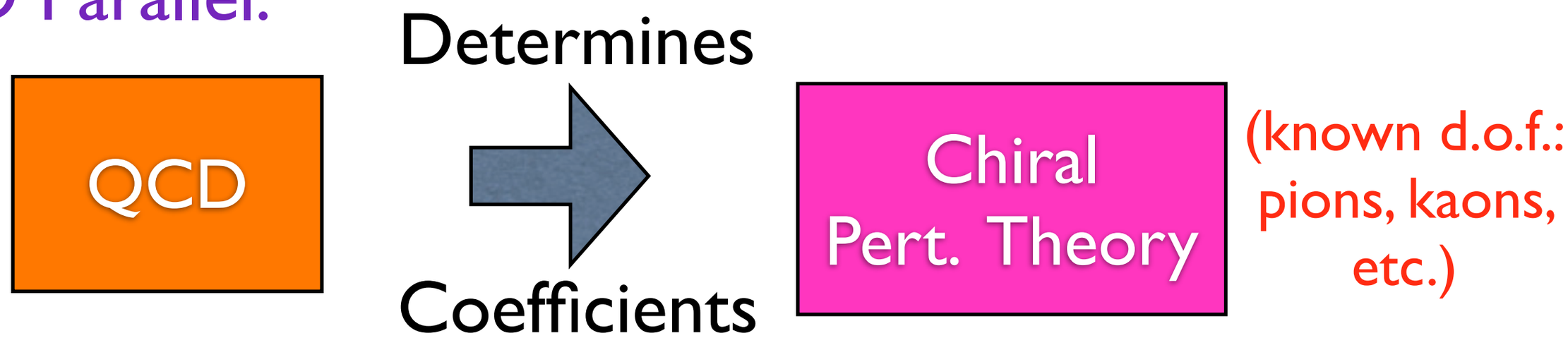
Heavier Scale → More BSM Effects

Our approach to WW

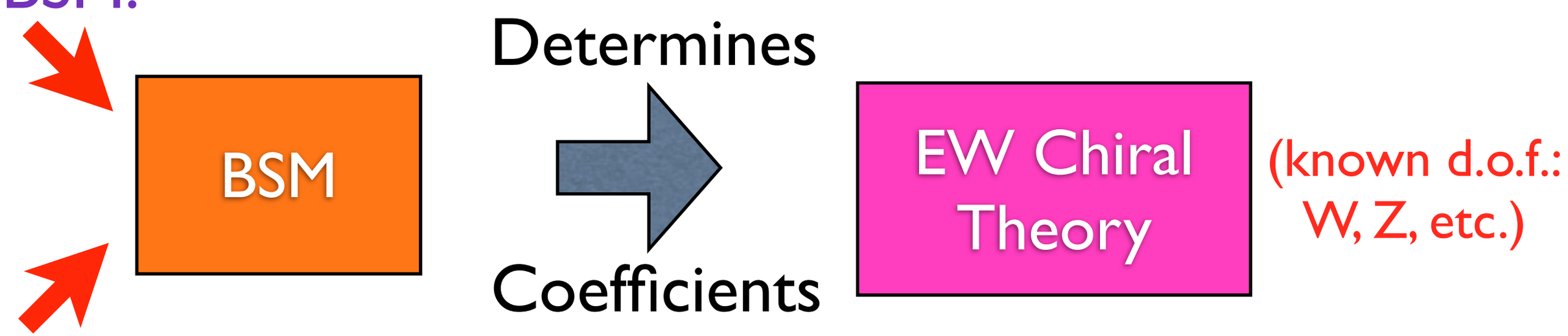
Effective Field Theory

(other approaches include Equivalence Theorem, etc.)

QCD Parallel:



BSM:



Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

Include all terms that respect: $SU(2)_L \otimes U(1)_Y$

At leading order: (Quite Simple)

$$\mathcal{L}_{LO} = \frac{f^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\mathrm{tr}W_{\mu\nu}W^{\mu\nu}$$
$$+ \frac{1}{4}\beta_1 g^2 f^2 [\mathrm{tr}(TV_\mu)]^2$$

$$D_\mu U = \partial_\mu U + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu U - ig' U\frac{\tau_3}{2}B_\mu$$

$$T = U\tau_3 U^\dagger \qquad V_\mu = (D_\mu U)U^\dagger$$

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$SU(2)_C$

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At leading order: f β_1

At NLO: α_1-α_5 α_6-α_{11}

$$\frac{1}{2}\alpha_1 g g' B_{\mu\nu}\mathrm{tr}(TW^{\mu\nu}) \qquad \frac{1}{4}\beta_1 g^2 f^2[\mathrm{tr}(TV_\mu)) \qquad \frac{1}{4}\alpha_8 g^2[\mathrm{tr}(TW_{\mu\nu})]^2$$

$$S \sim \alpha_1 \qquad T \sim \beta_1 \qquad U \sim \alpha_8$$

Dominant terms in WW: (other coefficients experimentally bound/small)

$$\alpha_4[\mathrm{tr}(V_\mu V_\nu)]^2 \qquad \alpha_5[\mathrm{tr}(V_\mu V^\mu)]^2$$

Electroweak Chiral Lagrangian

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Include all terms that respect: $SU(2)_L \otimes \cancel{U(1)_Y}$ $SU(2)_C$

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At NLO: α_1-α_5 $\cancel{\alpha_6\text{-}\alpha_{11}}$

$$\frac{1}{2}\alpha_1 g g' B_{\mu\nu}\mathrm{tr}(TW^{\mu\nu})$$

$$S \sim \alpha_1$$

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Electroweak Chiral Lagrangian

$$V_\mu = (D_\mu U)U^\dagger$$

$$g, g' \to 0$$

$$\mathcal{L}_{LO} = \frac{f^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] + \alpha_4[\mathrm{tr}(V_\mu V_\nu)]^2 + \alpha_5[\mathrm{tr}(V_\mu V^\mu)]^2$$

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$m_f \to 0$

Hadronic Chiral Lagrangian

$\ell_1 = 4\alpha_5$ $\qquad$ $\ell_2 = 4\alpha_4$

Parameters contribute to tree level W-W scattering!

Previous Literature

Two flavor picture is not new:

Distler, Grinstein, Porto, Rothstein 2006
Vecchi 2007

Finds bounds using the
Equivalence Theorem with unitarity

$$\alpha_4 + \alpha_5 \geq 1.14 \times 10^{-3}$$
$$\alpha_4 \geq 0.65 \times 10^{-3}$$

$$\mu \sim 246 \text{ GeV}$$

What is new is generalizing to general number of flavors

Multi-Flavor Extension

Fundamental Complication: Only 3 GBs eaten!

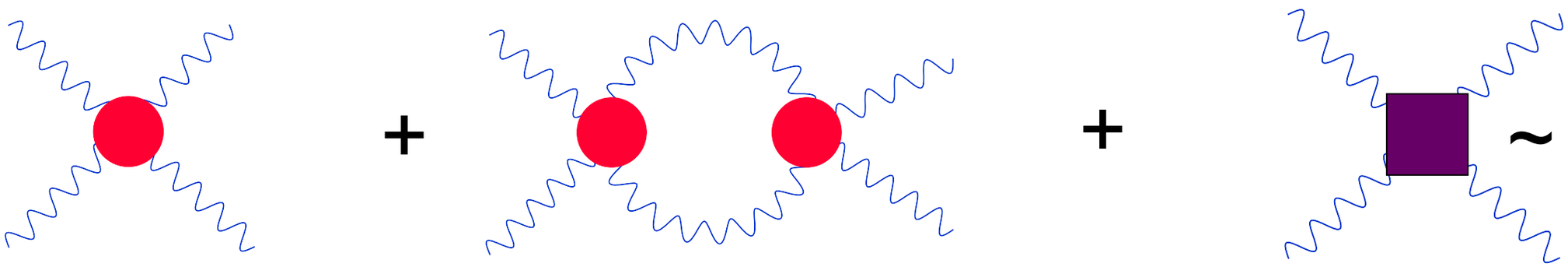
➡EW Chiral Lagrangian has only two flavors

Include additional massive flavors: (mapping LECs)

- Two massless charged
- Rest massive (degenerate) uncharged

$$\alpha_4 = \frac{\ell_2}{4} = L_0 + L_2$$

$$\alpha_5 = \frac{\ell_1}{4} = -\frac{1}{2}L_0 + L_1 + \frac{1}{2}L_3$$

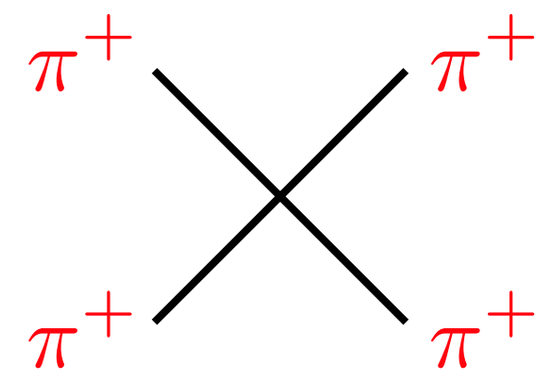


$\mathcal{O}(g^2)$ + $\mathcal{O}(g^4)$ + $\mathcal{O}(g^4)$ ~ α_4, α_5

Pi-Pi Scattering

- Cleanest and most understood hadronic scattering process: theoretically, experimentally, and numerically

I = 2



Weinberg 1966: LO Prediction

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}$$

Gasser and Leutwyler 1985: NLO Prediction

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2}\left[3\log\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - \ell_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

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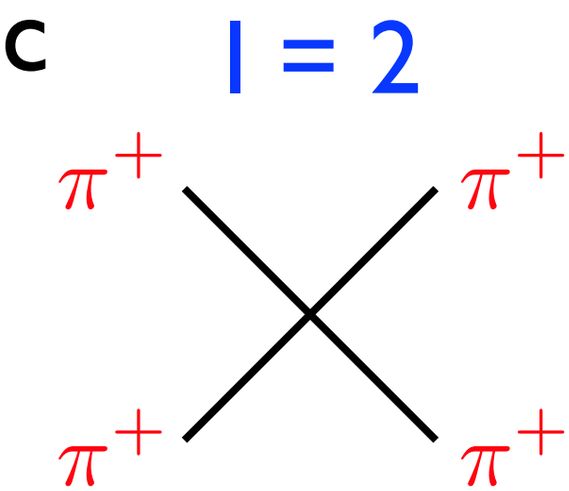

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Many Flavor Scattering

Bijnens, Lu 2011: "$I = 2$"

$$M_P a_{PP} = -\frac{M_P^2}{8\pi F_P^2}\left\{1 + \frac{M_P^2}{16\pi F_P^2}\left[\frac{4(1-N_F+N_F^2)}{N_F^2}\ln\frac{M_P^2}{\mu^2} - \frac{4(N_F-1)}{N_F^2} - L_{PP}(\mu)\right]\right\}$$

$$L_{PP} = 512\pi^2(L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8)$$

Key Points:

1) No explicit flavor dependence in L_{PP}

2) Reduces to two flavor result via matching conditions

$$\alpha_4 = \frac{\ell_2}{4} = L_0 + L_2 \qquad \alpha_5 = \frac{\ell_1}{4} = -\frac{1}{2}L_0 + L_1 + \frac{1}{2}L_3$$

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Key Points:

1) No explicit flavor dependence in L_{PP}

- Can compare different flavor theories directly!!

2) Reduces to two flavor result via matching conditions

$$\alpha_4 = \frac{\ell_2}{4} = L_0 + L_2 \qquad \alpha_5 = \frac{\ell_1}{4} = -\frac{1}{2}L_0 + L_1 + \frac{1}{2}L_3$$

Scattering on Lattice

- Usual LSZ Formalism does not hold for Euclidean:
  Maiani, Testa, 1990

- Measure energy of interaction of multiple hadrons in finite volume
  Luscher, 1986

  Energy of interaction (two hadrons):

$$m_1 + m_2 + \Delta E$$

$$\hat{\pi}(\mathbf{p}, t) \equiv \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, S^\dagger S$$

$$C_\pi(t) = \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)\Big)$$

$$C_{\pi\pi}(t) = \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)\Big)^2 - \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)^2\Big)$$

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Energy of interaction (two hadrons):

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Related to phase shift

$$\hat{\pi}(\mathbf{p}, t) \equiv \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, S^\dagger S$$

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Lattice Scattering Subtleties

1) Scattering on lattice inherently "low-energy"

- Invalid above inelastic threshold $\Delta E > 2M_P$

$$|\vec{k}| \cot\delta = \frac{1}{a} + \frac{M_P^2 r}{2}\left(\frac{|\vec{k}|^2}{M_P^2}\right) + \mathcal{O}\left(\frac{|\vec{k}|^4}{M_P^4}\right)$$

2) Only discrete scattering states at finite volume

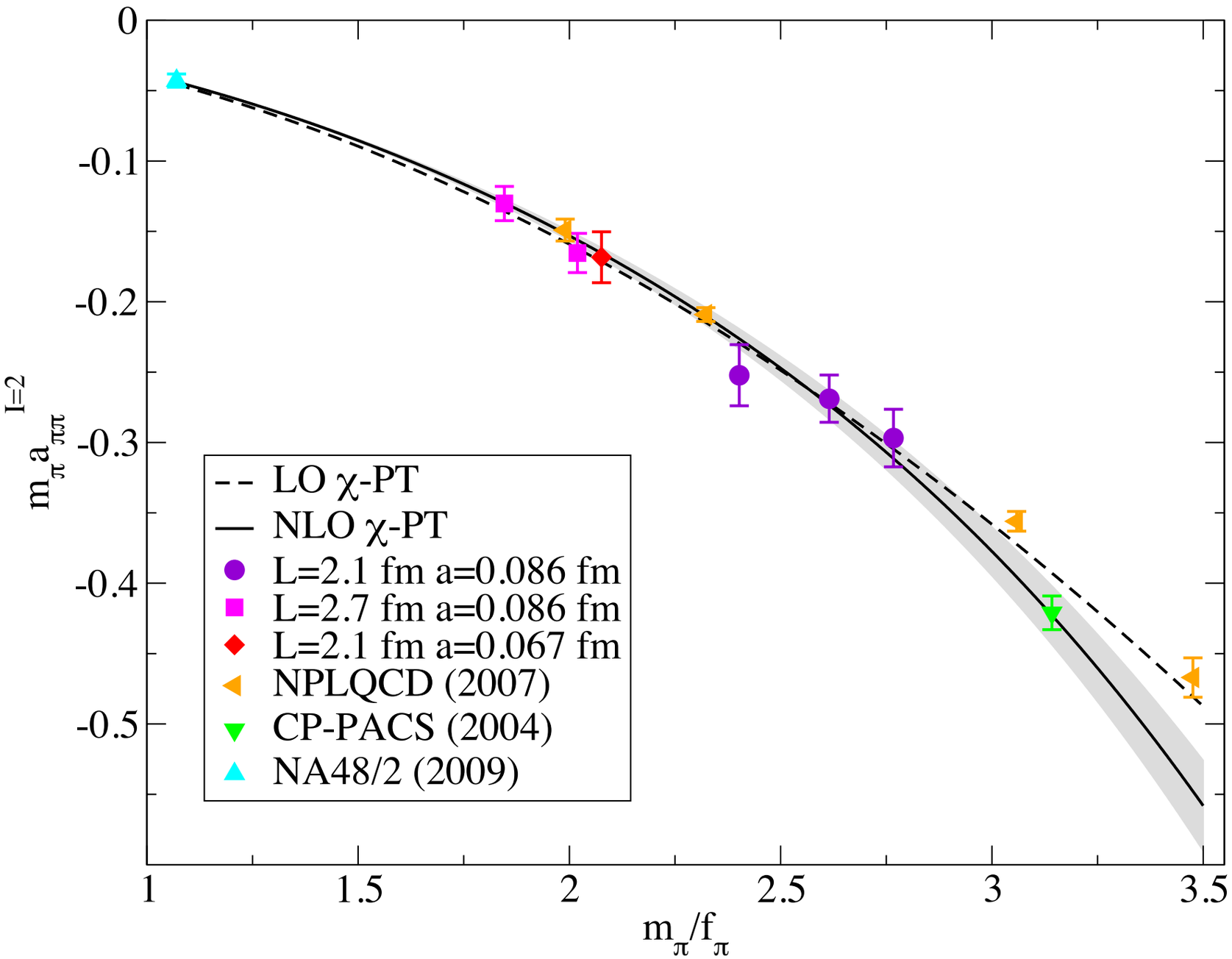
For a<<L:

$$\Delta E = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \cdots\right]$$

3) Relates better to LECs than the limit $\frac{s}{M^2} \to \infty$

Too bad: $M(W^+W^+ \to W^+W^+) \to M(\pi^+\pi^+ \to \pi^+\pi^+)$

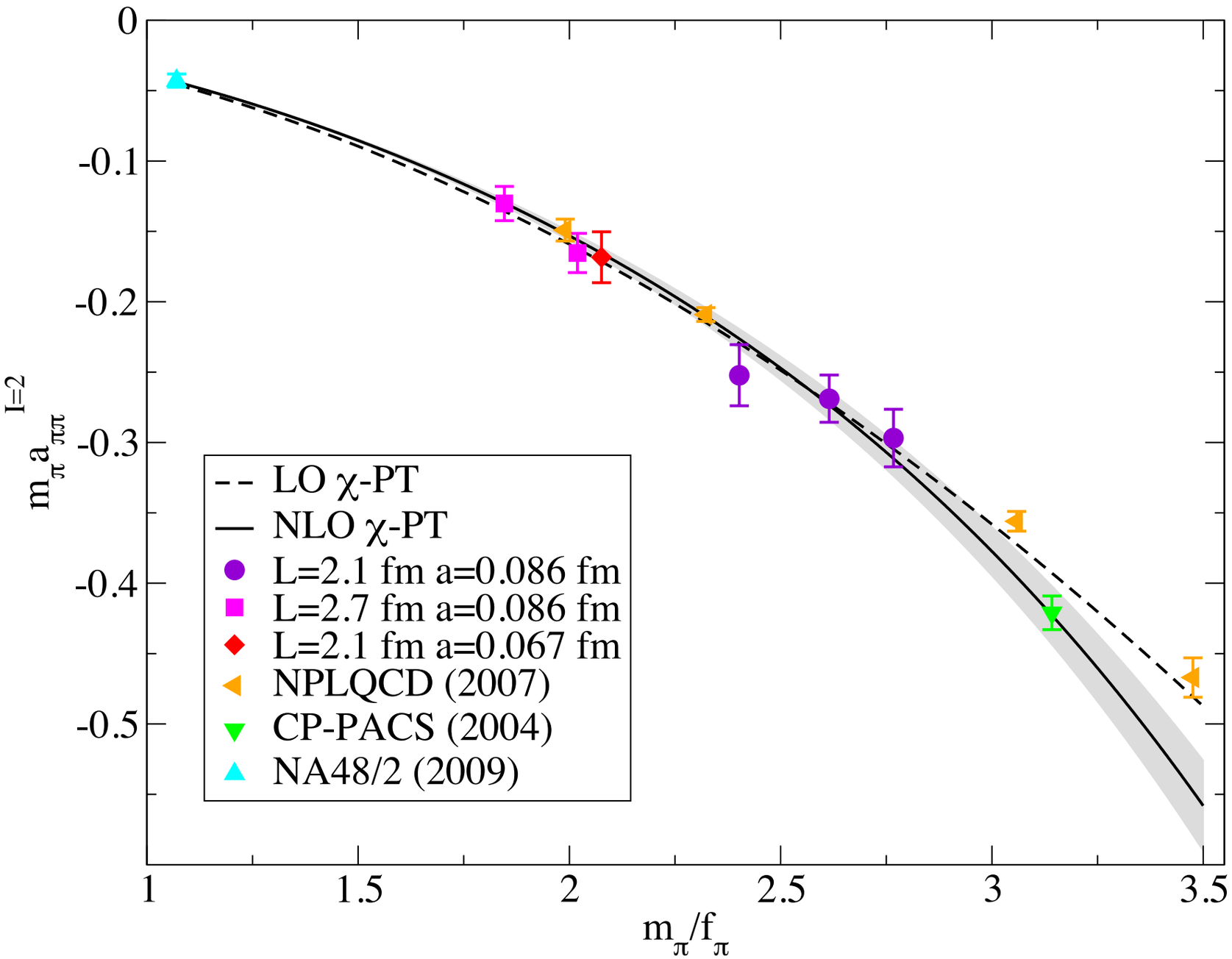
Lattice QCD Results



arXiv:0909.3255

LO Weinberg works well!

Lattice QCD Results



arXiv:0909.3255

LO Weinberg works well!

...TOO WELL!

Lattice Details

10 DWF Ensembles:

- $32^3 \times 64 \times 16$ lattices

$$am_\rho \sim \frac{1}{5}$$

2 flavor: m_q = 0.010 - 0.030

6 flavor: m_q = 0.010 - 0.030

Table 1: 2 Flavor

m_q	# Configs	# Meas
0.010	564	564
0.015	148	444
0.020	131	131
0.025	67	268
0.030	39	154

Table 1: 6 Flavor

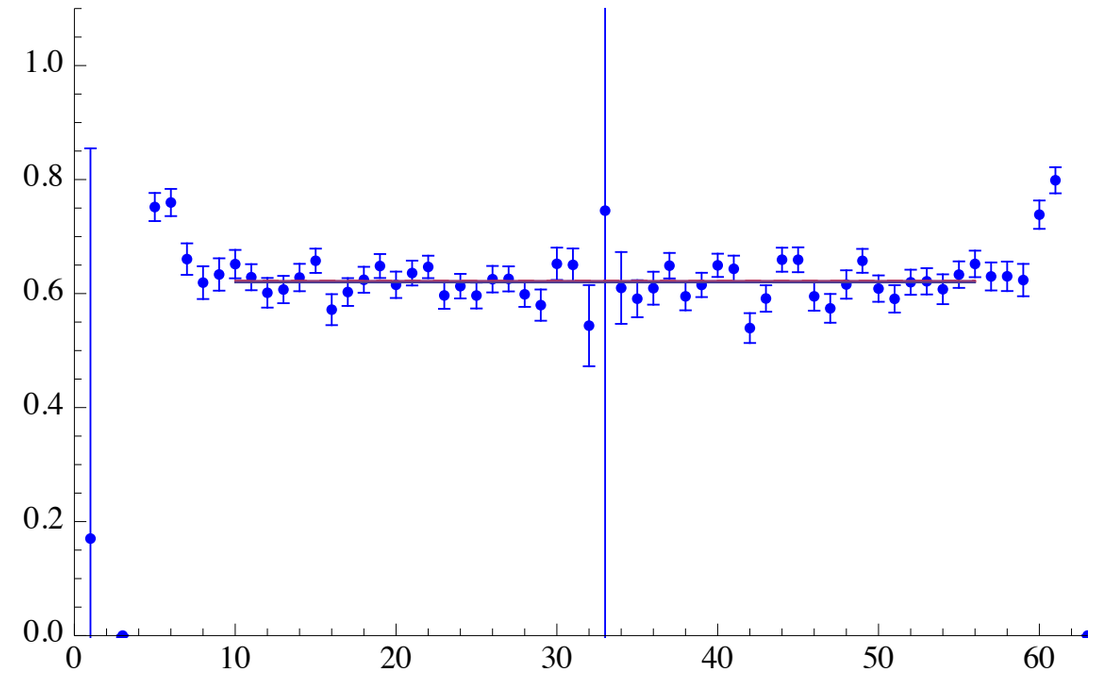
m_q	# Configs	# Meas
0.010	221	882
0.015	112	414
0.020	81	324
0.025	89	267
0.030	72	259

Analysis Snippet

$$C_{\pi\pi}(t) = A + B\cosh(E_{\pi\pi}t)$$

$$E_{\pi\pi} = \frac{C(t+2) - C(t-2)}{2(C(t+1) - C(t-1))}$$

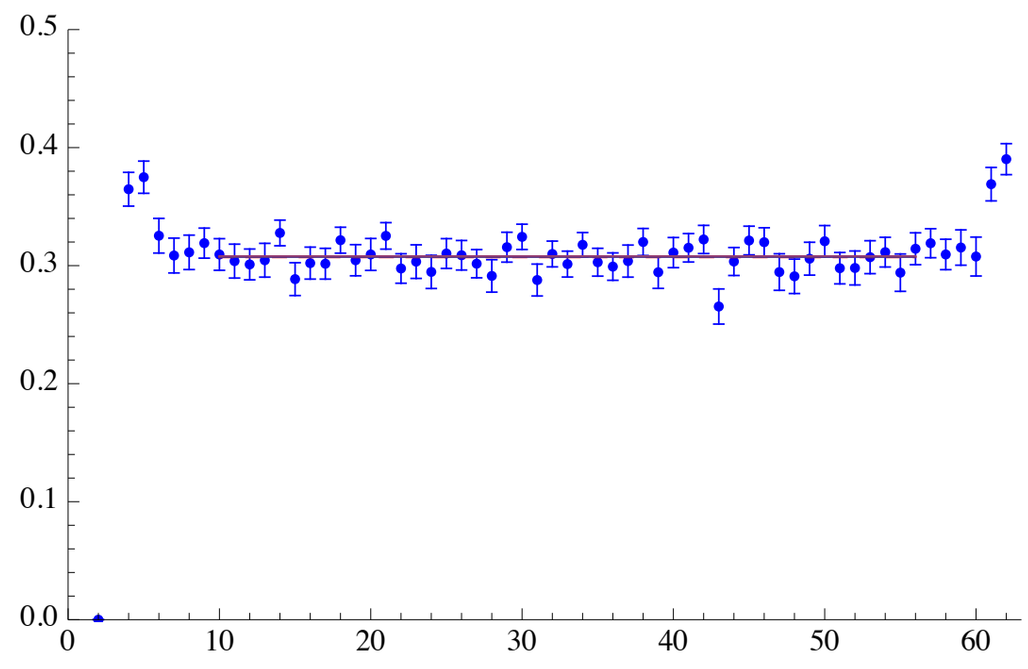
$$E_{\pi\pi} = 0.621 \pm 0.001$$



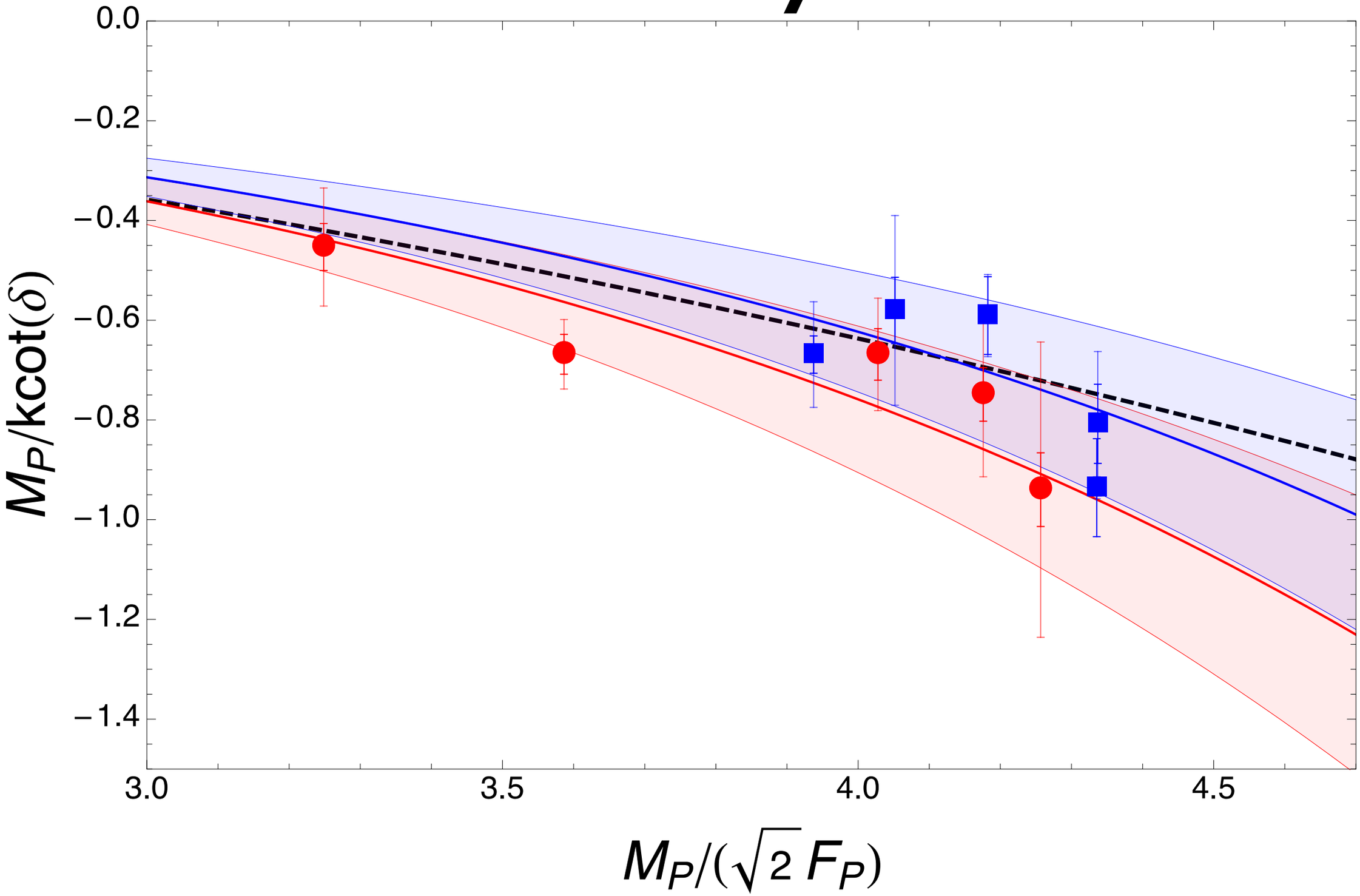
$$C_{\pi}(t) = B\cosh(m_{\pi}t)$$

$$m_{\pi} = \frac{C(t+1) + C(t-1)}{2C(t)}$$

$$m_{\pi} = 0.3075 \pm 0.0005$$

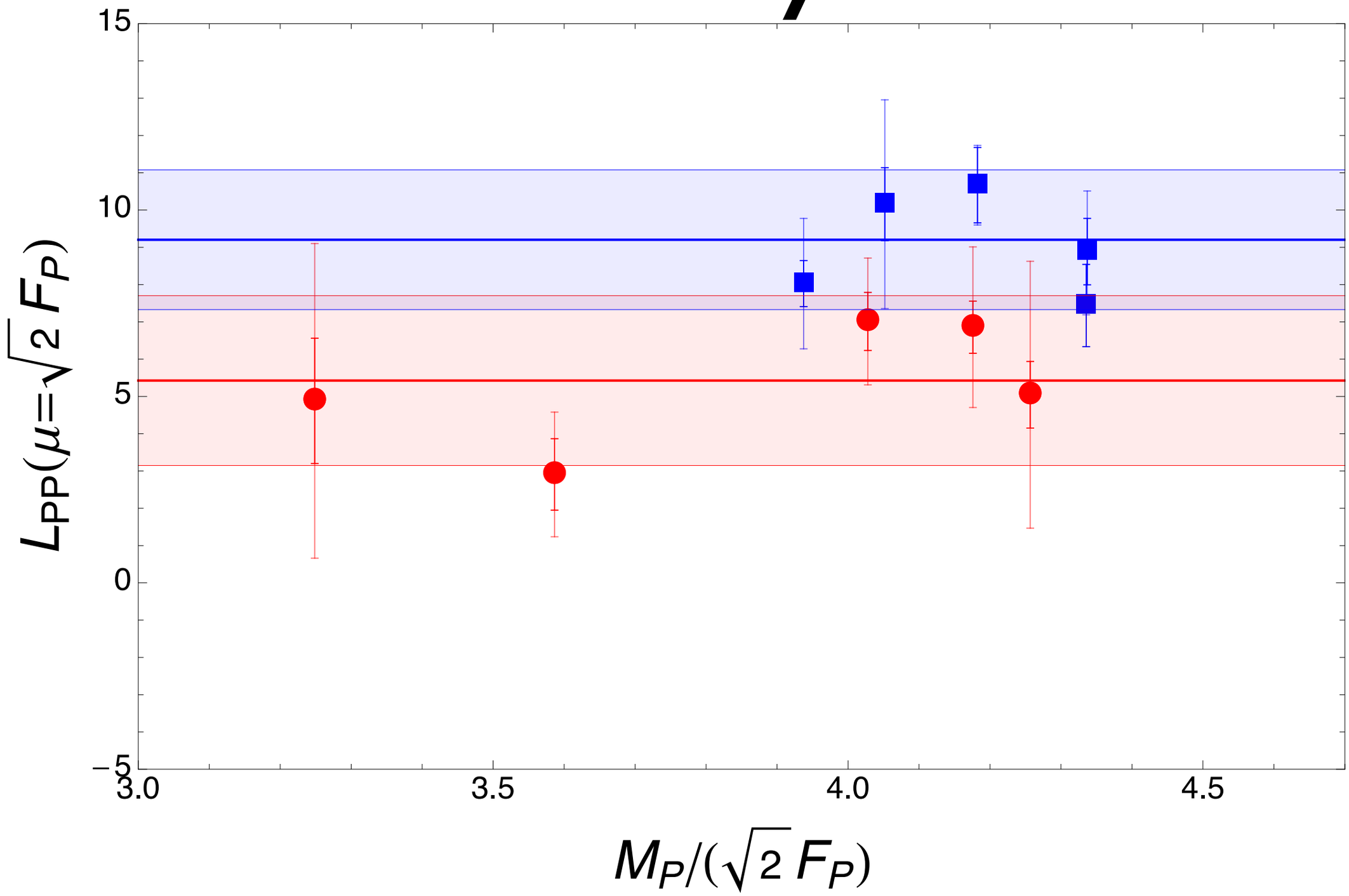


Preliminary Results



LO Weinberg dominance persists!!!!

Preliminary Results



$$L^{2F}_{\pi\pi}(\mu=\sqrt{2}F_P) = 5.42 \pm 2.28 \quad \chi^2/\mathrm{d.o.f} = 0.71$$

$$L^{6F}_{\pi\pi}(\mu=\sqrt{2}f_P) = 9.20 \pm 1.87 \quad \chi^2/\mathrm{d.o.f} = 0.54$$

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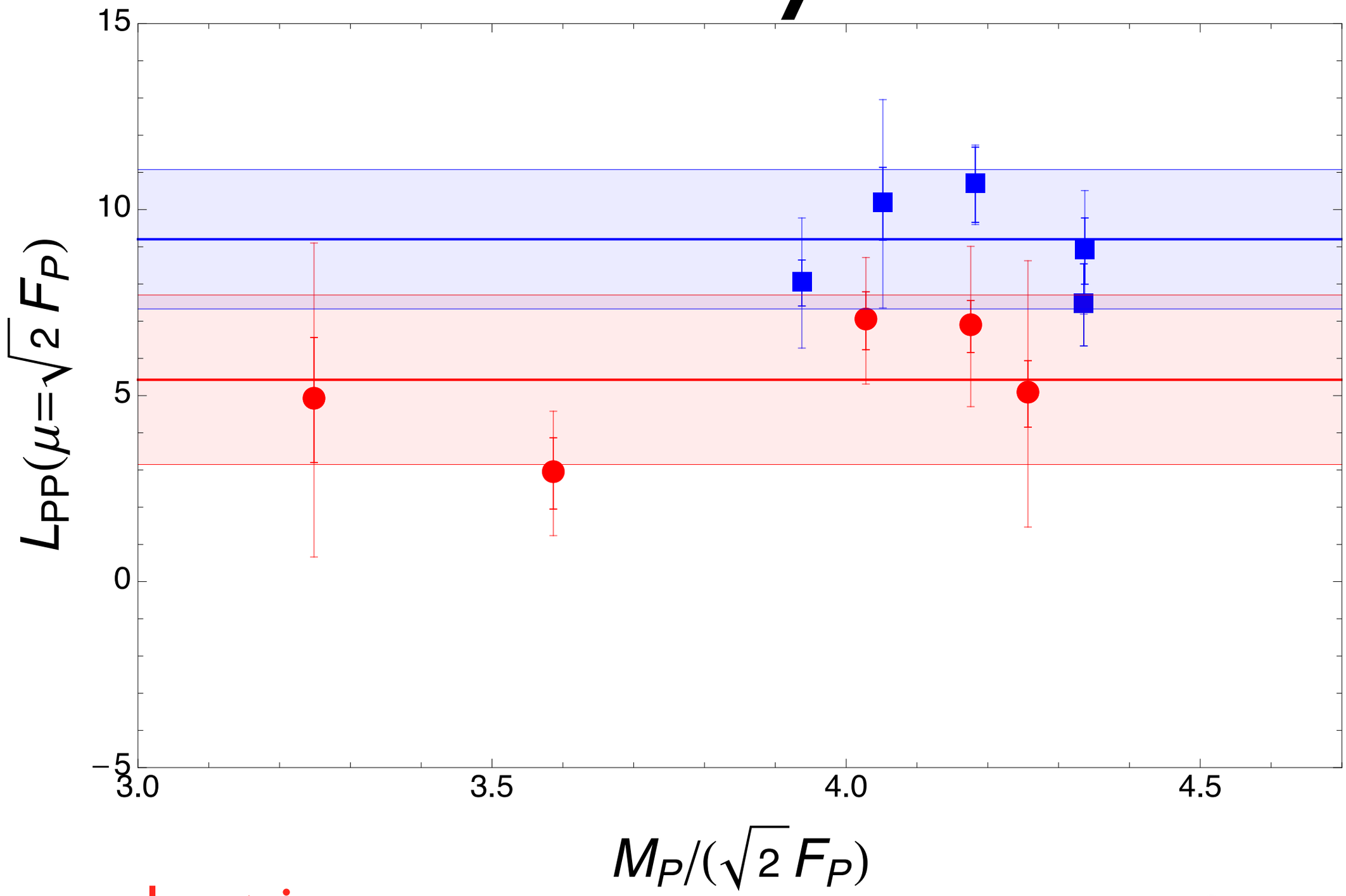

How robust is result?

$$L^{2F}_{\pi\pi}(\mu=\sqrt{2}F_P) = 5.42 \pm 2.28 \quad \chi^2/\mathrm{d.o.f} = 0.71$$

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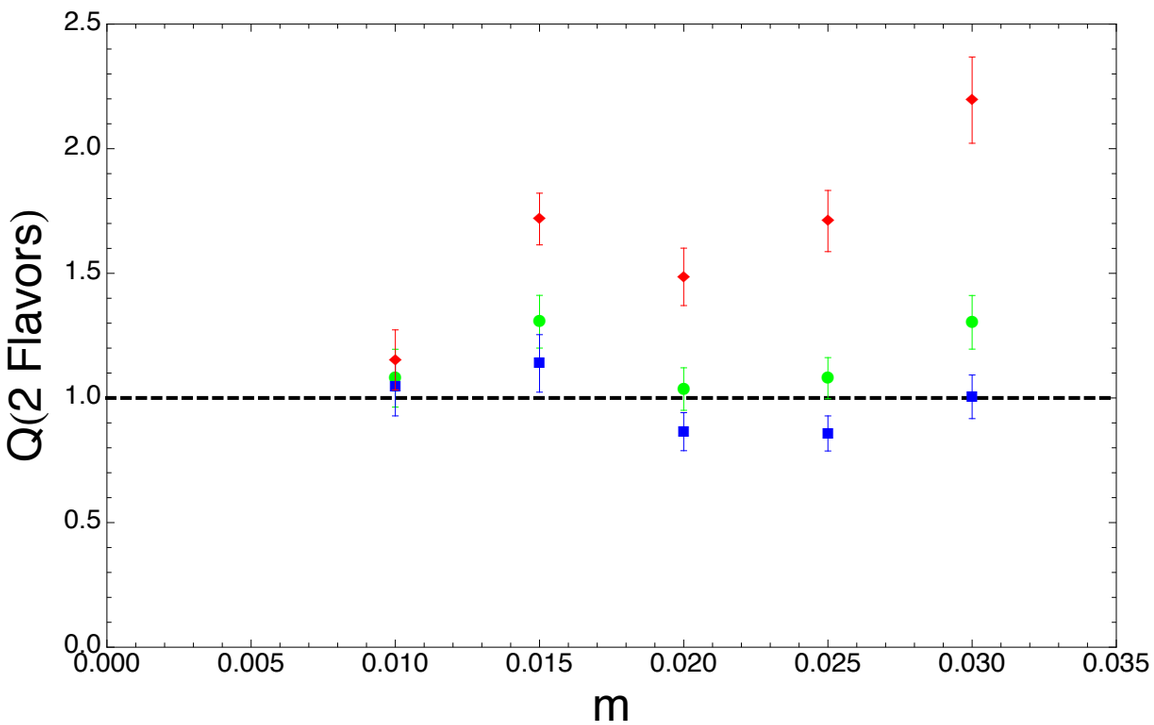
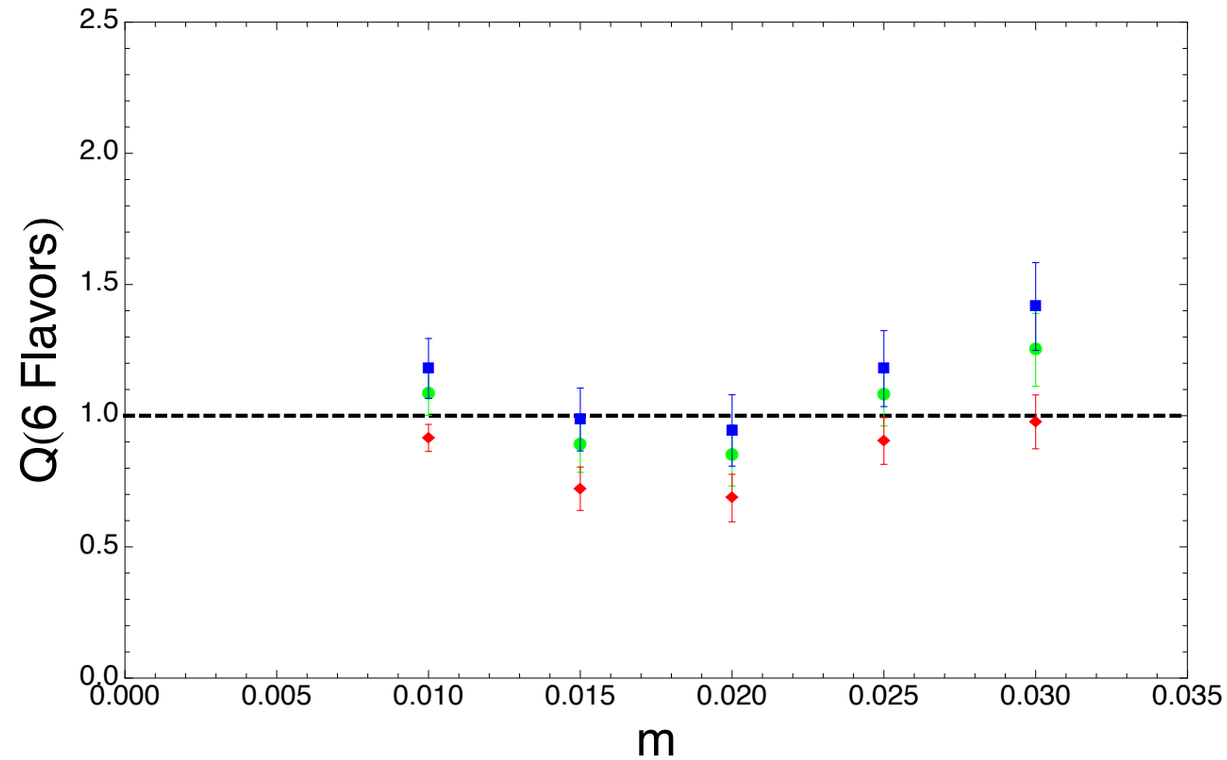
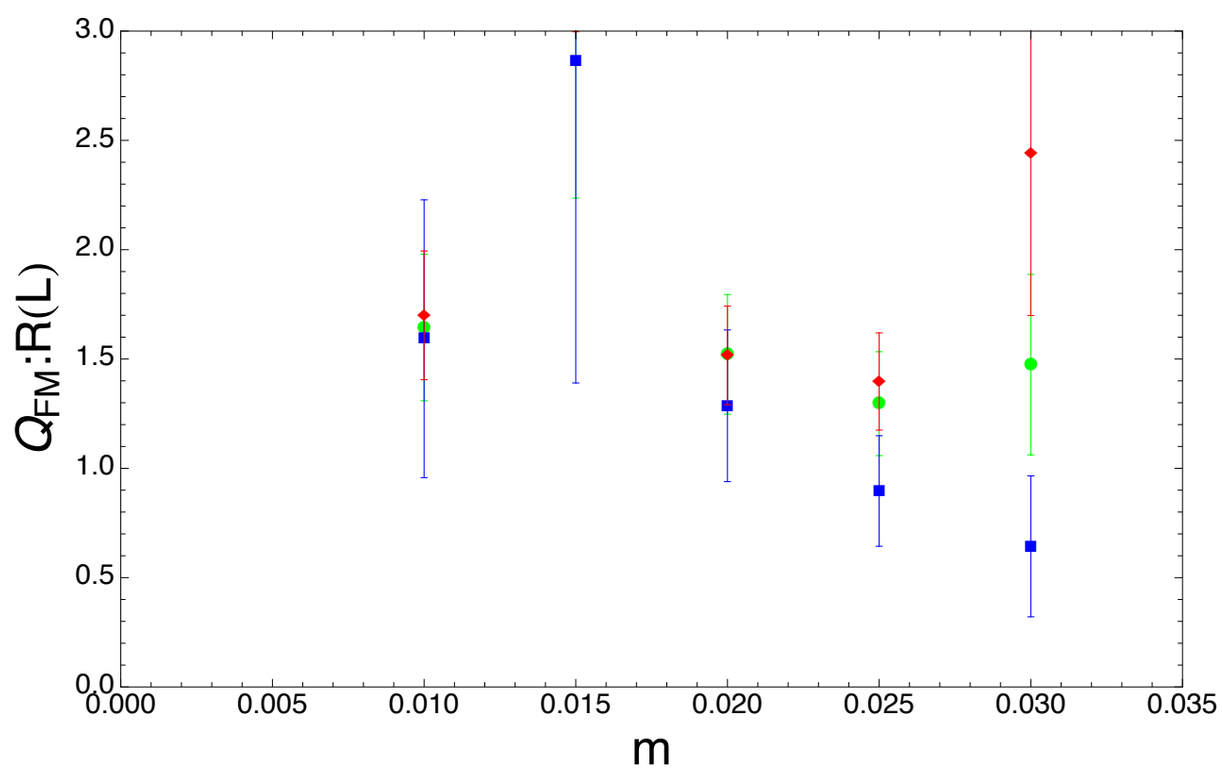
Early Six Flavors Test





$$Q = -\frac{8\pi F_P^2}{M_P} a_{PP}$$

$$\to 1$$



$$\frac{L_{PP}^{6F}}{L_{PP}^{2F}} \gtrsim 1.6$$

Two Flavor WW LECs

NLO calculations of M_P F_P $M_P a_{PP}$

Three EQs, four unknowns

$$\alpha_4 + \alpha_5 = \begin{cases} (3.43 \pm 0.31) \times 10^{-3} & \mu \sim 246 \text{ GeV} \\ (0.15 \pm 0.31) \times 10^{-3} & \mu \sim 2 \text{ TeV} \end{cases}$$

(Preliminary)

Complications for more flavors

- Results directly useful for techni-pion scattering...
  ...but not for W-W
  - Cannot disentangle L_0-L_4 from L_5-L_8

- Use LECs from pion mass, pion decay const., chiral cond.
  - Works for two flavor!
  ...not so for general flavor...

$$b_M = 8N_F(2L_6 - L_4) + 8(2L_8 - L_5)$$

$$b_F = 4N_F L_4 + 4L_5$$

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Where we Stand

Estimates for 99% CL bounds for 100 inverse fb: $\mu \sim 2\ \mathrm{TeV}$

$$-7.7 \times 10^{-3} < \alpha_4 < 15 \times 10^{-3}$$
$$-12 \times 10^{-3} < \alpha_5 < 10 \times 10^{-3}$$

Eboli et. al. 2006

Two Flavor results: (Preliminary)

$$\alpha_4 + \alpha_5 = \begin{cases} (3.43 \pm 0.31) \times 10^{-3} & \mu \sim 246\ \mathrm{GeV} \\ (0.15 \pm 0.31) \times 10^{-3} & \mu \sim 2\ \mathrm{TeV} \end{cases}$$

Six flavor shows early signs for enhanced values, but is currently inconclusive

Future Directions

1) Ultimately need:

- Different volume(s)
- More statistics & 0.0075 mass point

2) Get W-W parameters in other ways!

- I=2 pi-pi D-wave scattering (more stats, operators)
- Pion form factors (more stats, mass points)
- Eff. Range & Shape Param. (more stats, volumes)
- NNLO analysis (more stats, mass points)
- PQ analysis (more inversions, volumes) ← DWF GPU Inverter

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$$\mathcal{L}_{tree} = \mathcal{L}_{tree}^{W^2} + \mathcal{L}_{tree}^{W^4} + \cdots$$

$$\mathcal{L}_{tree}^{W^2} = (\partial_\mu W_\mu - \partial_\nu W_\mu)^2$$

$$\begin{aligned}\mathcal{L}_{tree}^{W^4} = &- g^2[W_\mu, W_\nu]^2 + L_0 \mathrm{tr}(u^\mu u^\nu u_\mu u_\nu) + L_1 \mathrm{tr}(u^\mu u_\mu)\mathrm{tr}(u^\nu u_\nu) \\ &+ L_2 \mathrm{tr}(u^\mu u^\nu)\mathrm{tr}(u_\mu u_\nu) + L_3 \mathrm{tr}(u^\mu u_\mu u^\nu u_\nu)\end{aligned}$$

$$u_\mu = -(u^\dagger \vec{G} \cdot \vec{W}_\mu u - u \vec{G} \cdot \vec{W}_\mu u^\dagger)$$