Four fermion interactions on the lattice

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Motivations

- Technicolor models need to be embedded in ETC type extensions to allow for masses of SM fermions and additional Goldstones ...
- These ETC interactions generate 4 fermion ops at low energy.
- Hence important for making detailed predictions on TC/ETC models ...
- Additionally - if start in the conformal window serve to break conformal invariance. Four fermi coupling tunable parameter to control walking ?
- Required by other composite Higgs scenarios e.g.
    - top quark condensate models ...(Bardeen, Hill, Lindner)
    - (Gauged) NJL model ...

Two exploratory (lattice) studies ...

- Phase structure of gauged NJL model using *reduced* staggered fermions (with R. Galvez, A. Veernala, D. Mehta and J. Hubisz).
- Toy "top quark condensate" model using Wilson fermions (with J. Hubisz)

Aim is exploratory. Lattices modest (just CPU/GPU). Expand range of models lattice can handle
Preliminary results ...

Gauged NJL model

Continuum: consider doublet of Dirac fermions in fundamental rep of $SU(2)$.
Chirally invariant action:

$$S = \int d^4x\, \bar{\psi}(i\partial\!\!\!/ - A\!\!\!/)\psi + \frac{G^2}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2] - \frac{1}{2g^2}Tr[F_{\mu\nu}F^{\mu\nu}]$$

$\gamma_5\tau^a, a = 1\ldots3$ generators of $SU(2)_L \times SU(2)_R$ flavor.
Implement four fermion terms via Yukawas

$$S = \int d^4x\, G\left(\bar{\psi}\psi\phi_4 + \bar{\psi}i\gamma_5\tau^a\psi\phi_a\right) + \frac{1}{2}\left(\phi_4^2 + \phi_a^2\right)$$

Twisting ...

First rewrite in twisted variables (better discretization) Global symmetries: $SO_{\rm Lorentz}(4) \times SO_{\rm flavor}(4)$

$$\psi_{i\alpha} = R_{ij}\ \psi_{j\beta}\ F^T_{\beta\alpha}$$

Under diagonal subgroup $R = F$ fermions $\rightarrow$ matrices Ψ

$$S_{\rm kin} = \int {\rm Tr}\left(\overline{\Psi}\gamma\,.D\Psi\right)$$

2 Dirac fermions by projection:

$$\Psi \rightarrow \frac{1}{2}\left(\Psi - \gamma_5\Psi\gamma_5\right), \qquad \overline{\Psi} \rightarrow \left(\overline{\Psi} + \gamma_5\Psi\gamma_5\right)$$

Explicitly (chiral basis):

$$\Psi = \left(\begin{array}{cc} 0 & \psi_R \\ \psi_L & 0 \end{array}\right), \qquad \overline{\Psi} = \left(\begin{array}{cc} \overline{\psi}_L & 0 \\ 0 & \overline{\psi}_R \end{array}\right).$$

Twisted action

$$\int \mathrm{Tr}(\overline{\Psi}\gamma_\mu\partial_\mu\Psi) = \overline{\psi}_L\sigma_\mu\partial_\mu\psi_L + \overline{\psi}_R\sigma_\mu\partial_\mu\psi_R$$

Note: ψ_L, ψ_R are 2×2 matrices – 2 flavors of Weyl spinor
Rewrite Yukawas in matrix form:

$$\int \mathrm{Tr}\left(\overline{\Psi}\Psi\Phi\right)$$

where

$$\Phi = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix} = \phi_\mu\gamma_\mu,$$

with the 2×2 matrix $\phi = \phi_4 I + i\phi_i\tau_i$.

Discretization

Map matrices to staggered lattice fields

$$\Psi(x) = \frac{1}{8}\sum_b \gamma^{x+b}\chi(x+b),$$

$$\overline{\Psi}(x) = \frac{1}{8}\sum_b (\gamma^{x+b})^\dagger \overline{\chi}(x+b) \ \text{with} \ \gamma^{x+b} = \prod_{i=1}^{4} \gamma_i^{x_i+b_i}$$

Projection just restricts $\overline{\chi}$ and χ to even and odd parity sites.

$$S_L = \sum_{x,\mu} \chi^T(x)\,\mathcal{U}_\mu(x)\,\chi(x+a_\mu)\,[\eta_\mu(x) + G\,\overline{\phi}_\mu(x)\,\epsilon(x)\,\xi_\mu(x)]$$

$$\mathcal{U}_\mu(x) = \frac{1}{2}[1+\epsilon(x)]\,U_\mu(x) + \frac{1}{2}[1-\epsilon(x)]\,U^*_\mu(x).$$

$$\overline{\phi}_\mu(x) = \frac{1}{16}\sum_b \phi_\mu(x-b)$$

Lattice symmetries

- $\chi \to \epsilon(x)\chi(x)$ U(1) fermion number.
- Discrete subset of continuum chiral-flavor.

$$\Psi \to \Psi\gamma_\rho$$

$$\begin{aligned} \chi(x) &\to \xi_\rho(x)\chi(x+\rho), \\ U_\mu(x) &\to U^*_\mu(x+\rho), \\ \phi_\mu(x) &\to (-1)^{\delta_{\mu\rho}}\phi_\mu(x+\rho). \end{aligned}$$

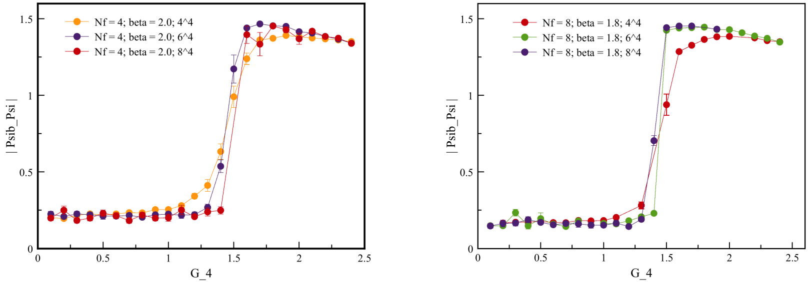
$\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu-1} x_i}$, $\xi_\mu(x) = (-1)^{\sum_{i=\mu+1}^{4} x_i}$

- Massless theory: spontaneous breaking generates vev for ϕ

Simulations

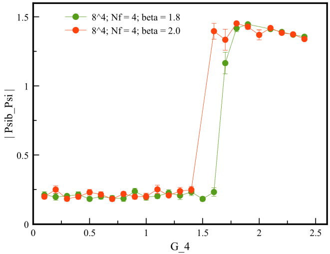
- $SU(2)$ pseudoreal – Pf real. For N_f flavors use $N_f/2$ doublets. No sign problem for $N_f = 4$ and $N_f = 8$.
- Wilson gauge action. RHMC alg (GPU acceleration solver). Lattices: 6^4, 8^4 and $8^3 \times 16$.
- Range of $\beta = 1.8 - 2.1$ $G = 0.1 - 2.5$. Range of β determined by finite volume phase transition on small lattices.
- 4 fermion term allows running for $m = 0$!
- Condensate $< \epsilon(x)\chi(x)\chi(x + e_4) >$. Average abs value on each config. Behavior as $V \to \infty$ determines phase ...
- Smallest eigenvalue $\lambda \sim 1/V$. Large condition number. Small dt needed to handle large force in leapfrog.

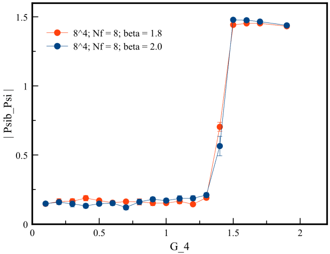
Condensate - finite volume effects



Transition sharpens with inc V. 1st order ? Note: see only vev for ϕ_4 - Dirac mass term. Expected χ-symmetry breaking pattern.

Condensate - versus β

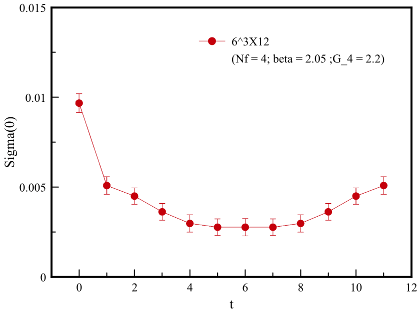




Phase transition - mass generation – throughout window in β.
Spontaneous χ-symmetry breaking even as $G \to 0$ for both N_f.
Similar to pure NJL model.

Goldstones

$< \phi_1(0)\phi_1(t) >$ $6^3 \times 12$ lattice $G = 2.2$ $N_f = 4$ $\beta = 1.8$



ϕ_1 becomes dynamical in broken phase - would be pion

Observations

- Reduced staggered fermions allow for 2 flavor simulations at $m = 0$ for $SU(2)$
- NJL type Yukawa interactions can be added in minimal way while preserving exact discrete chiral symmetries
- As expected see condensation at strong four fermi coupling - like pure NJL
- See expected pattern χ-symmetry breaking - Dirac mass term and scalars propagate (pions) in broken phase
- Transition appears first order - no new UV fixed point. Change this by expanding terms in bare action ?

Gauging the scalar auxiliaries ...

Continuum:

- Flavor doublet of Dirac fields: ψ^i , χ^i, $i = 1, 2$
- ψ in fundamental rep of $SU(2)$, χ gauge singlet.
- Scalar ϕ in fundamental $SU(2)$.

$$S = S_1 + S_2$$

where

$$S_1 = \int \overline{\psi}^i \gamma.D\psi^i + \overline{\chi}^i \gamma.\partial\chi^i + S_{\rm YM}$$

and

$$S_2 = G\overline{\psi}^i (i\tau_3)ij\chi^j\phi + \overline{\chi}^i (i\tau_3)^{ij}\psi^j\phi^\dagger + \phi^\dagger\phi$$

Analogy to top quark condensate model

- Replace $\chi \to t_R$. $\psi^1, \psi^2 \to (t_L, b_L)$
- $SU(2) \to SU(2) \times U(1)$
- Two $\to$ one flavor. $G \to iG$.

Lattice

- Wilson fermions $D \rightarrow D_W$. OK since focus on spontaneous breaking of gauge invariance not chiral invariance ....
- Fermion determinant real positive definite since

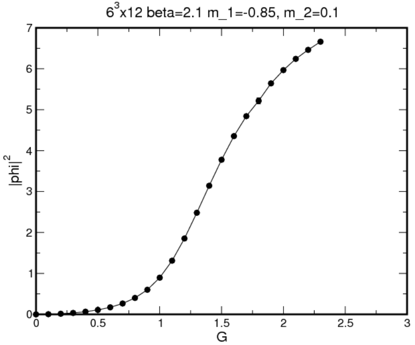
$$\tau_1 \gamma_5 M^\dagger \gamma_5 \tau_1 = M$$

  (Yukawas have twisted mass structure)
- Action gauge invariant. Yet can arrange for fermion loops to generate negative contribution to $\phi^\dagger \phi$ - drive ϕ away from origin. Dynamical mass generation ...
- Flavor symmetry explicitly broken .. Vafa-Witten *does not* apply ...

Simulations

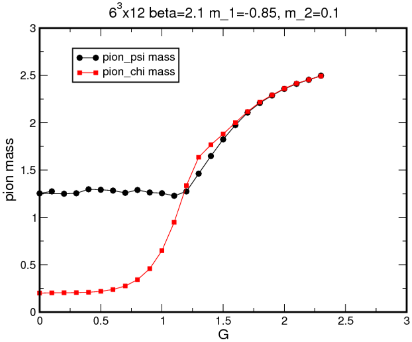
- $6^3 \times 12$ at $\beta = 2.1$.
- Tune fermions to be light at $G = 0$: $m^0_\psi \sim m^{\rm crit}_\psi = -0.85$, $m^0_\chi = 0.1$
- $G = 0.1 - 2.5$. GPU code.
- Measure:
  - Gauge action, condensate $\phi^\dagger \phi$
  - Flavor non singlet mesons in both gauged and ungauged sectors.
- See $\boxed{\text{strong}}$ evidence for symmetry breaking for $G > 1$.

Condensate



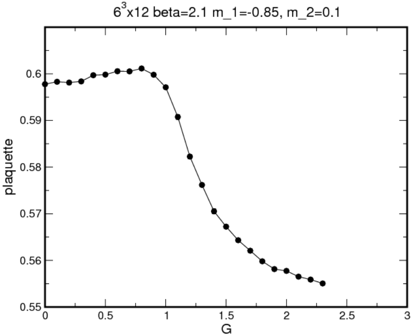
Smooth dependence on G. Phase transition ?

Pions



Gauge singlet and doublet become heavy (and degenerate) for $G > 1.2$

Gauge action



Fermions decouple at large G due to large dynamical mass; gauge coupling increases and P falls

What is happening ...

- In top quark condensate model formation of condensate implies spontaneous breaking of gauge symmetry ...
- Is this happening here ?.. certainly have dynamical mass generation and $V_{\rm min}(\phi^\dagger\phi)$ away from origin.
- Implies $<\overline{\psi}\tau\chi\overline{\chi}\tau\psi>\neq 0$. Only non-zero Wick contractions (Elitzur) $<\overline{\psi}\tau\psi>\neq 0$ and $<\overline{\chi}\tau\chi>\neq 0$ - gauge singlets
- To see definite sign for breaking of gauge symmetry requires e.g. observation of perimeter law in wilson loop .... still to be done

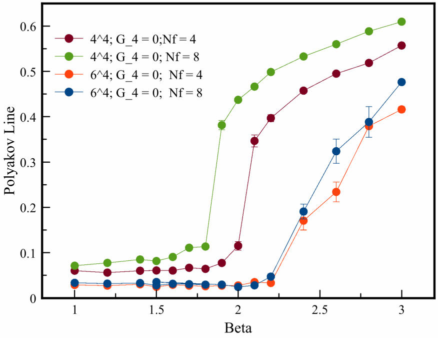
Conclusions

- Investigated two (lattice) models incorporating 4 fermion ops;
- Gauged NJL:
  - Use reduced staggered fermions - multiples of 2 flavors - no sign problem plus exact lattice chiral symmetries
  - See phase transition for strong 4 fermi coupling G with dynamical mass generation and spontaneous breaking of (exact) lattice chiral symmetry
  - BUT transition looks 1st order - like pure NJL - no new UV fixed pt at which to define renormalizable theory – need to keep finite cut-off
- Toy model for top quark condensation:
  - Use doublet of Wilson Dirac fields. No sign problem.
  - See dynamical mass generation for strong G (meson spectra).

Fin

Lattice can be used to study other models of composite Higgs!

Finite volume transition - NJL model



Polyakov line