The *S* Parameter on the Lattice

David Schaich (University of Colorado)

Lattice Meets Experiment: Beyond the Standard Model
Fermilab, 15 October 2011

LSD Collaboration, *Phys. Rev. Lett.* **106**:231601 (2011) [1009.5967]

Lattice Strong Dynamics Collaboration

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Yale	Thomas Appelquist, George Fleming, Meifeng Lin, Gennady Voronov

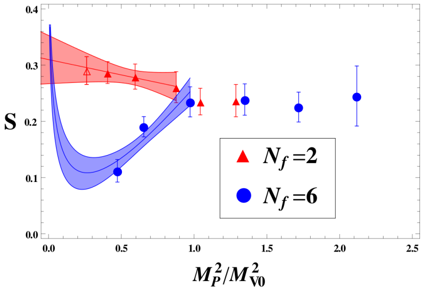
Performing non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider

Overview of this talk

- Focus on LSD Collaboration analysis in PRL **106**:231601 (2011)

Main result

Dynamical reduction in S
for 6-fermion theory
compared to scaled-up QCD



- Focus on lattice methods rather than latest results
  Similar methods used in two lattice QCD studies:
  JLQCD Collaboration, PRL **101**:242001 (2008)
  RBC-UKQCD Collaboration, PRD **81**:014504 (2010)
- First, a brief review of why S remains an important observable

The S parameter (Peskin and Takeuchi, 1991)

Constrain the physics of electroweak symmetry breaking
from its effects on vacuum polarizations $\Pi(Q)$ of EW gauge bosons

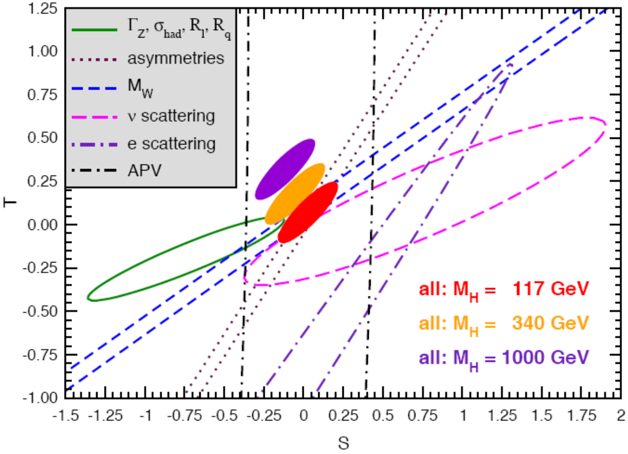
(independent of flavor physics/ETC)

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

1. $\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$
   (lattice details later)
2. $N_D \geq 1$ is the number of doublets with chiral electroweak couplings
3. ΔS_{SM} subtracted so that $S = 0$ in the standard model
   Removes three eaten modes, sets a "reference" Higgs mass scale

Experimentally, $S \lesssim 0$

For a reference Higgs mass scale of 1 TeV, $S \approx -0.15(10)$



(PDG)

- Z decay partial widths and asymmetries
- Neutrino scattering cross sections
- M_W, M_Z
- Atomic parity violation

Scaling up QCD gives $S \gtrsim 0.3$

N_f fermions in fundamental rep of SU(N_c) for $N_c \geq 3$,
all in $N_D = N_f/2$ chiral electroweak doublets

$$S \simeq 0.3 \frac{N_f}{2}\frac{N_c}{3} + \frac{1}{12\pi}\left(\frac{N_f^2}{4} - 1\right)\log\left(\frac{M_V^2}{M_P^2}\right)$$

1. Resonance contribution uses QCD phenomenology to model $R(s)$

$$4\pi \lim_{Q^2 \to 0} \frac{d}{dQ^2}\Pi_{V-A}(Q^2) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right]$$

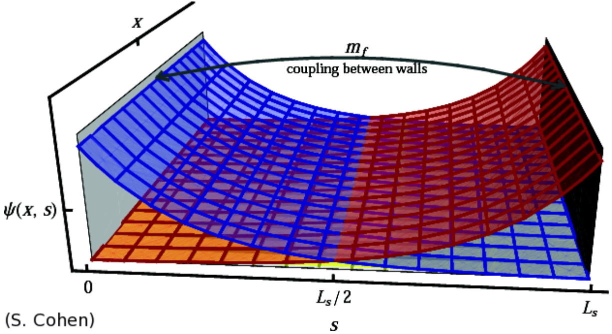
(essentially vector meson dominance with large-N_c scaling)

2. Chiral-log contribution based on leading-order chiral pert. theory

Lattice calculations needed for quantitative study of S
in theories with non-QCD dynamics (e.g., walking technicolor)

S on the lattice: general approach

- Use QCD as a baseline, explore changes as N_f increases
- Use chiral lattice fermions: significant computational expense



(S. Cohen)

- Domain wall fermions add fifth dimension of length L_s
- Exact chiral symmetry at finite lattice spacing in the limit $L_s \to \infty$
- At finite L_s, "residual mass" $m_{res} \ll m_f$; $\quad m = m_f + m_{res}$

S on the lattice: current correlators

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2} \Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

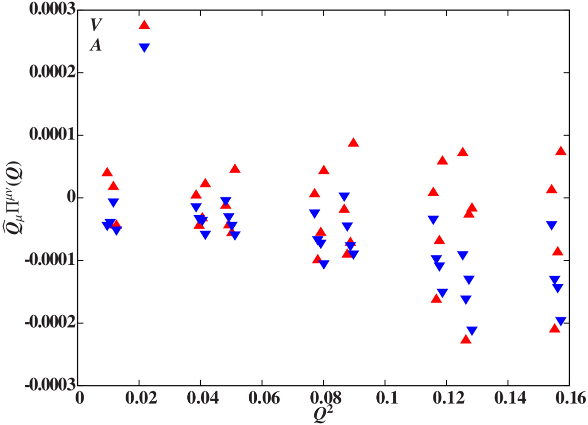
- Renormalization constant $Z = Z_A = Z_V$ for chiral fermions
  Computed non-perturbatively: $Z = 0.85$ (2f); 0.73 (6f)
- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel. . .

Lattice currents and Ward identities

$$\Pi^{\mu\nu}_{V-A}(Q) = Z\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)}\mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x)\, V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

"Local" currents are simple $\overline{q}\gamma^\mu q$, defined on domain walls

No Ward identity: $\widehat{Q}_\mu[\sum_x e^{iQ\cdot x}\left\langle V^a_\mu(x) V^a_\nu(0)\right\rangle] \neq 0$

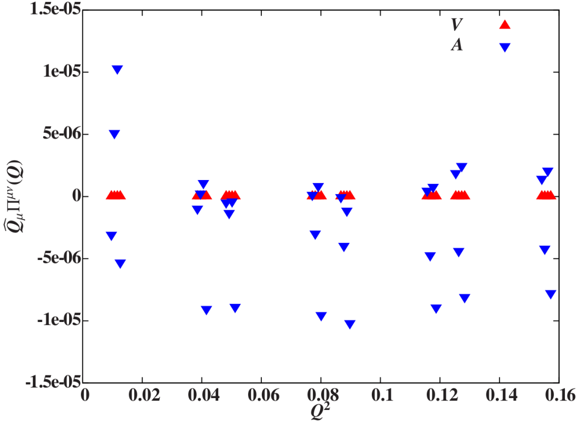


Lattice currents and Ward identities

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x)\, V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Conserved currents are point-split, summed over fifth dimension

Obey Ward identity, PCAC: $\widehat{Q}_\mu [\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x)\, V^a_\nu(0)\right\rangle] = 0$

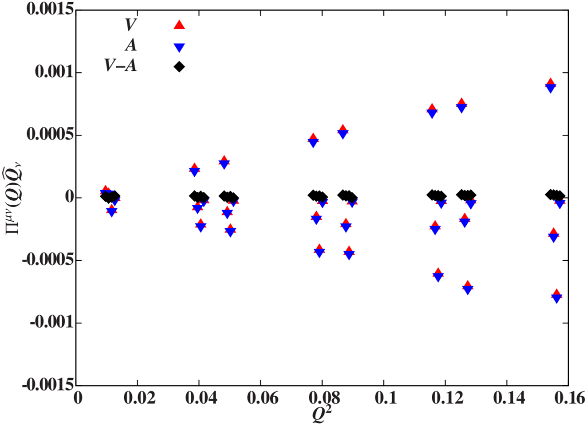


Lattice currents and Ward identities

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

Ward identity violations of mixed correlators cancel in $V\!-\!A$ difference

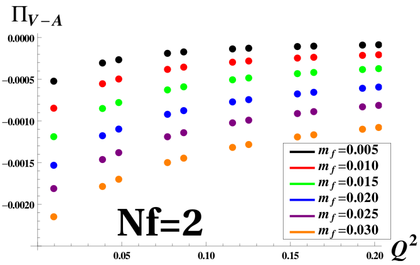
Save an order of magnitude in computing costs

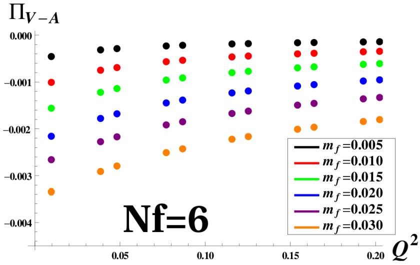


Polarization function data, $N_f = 2$ and $N_f = 6$

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right) \Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2} \Pi^L(Q^2)$$



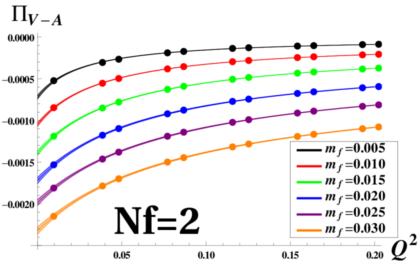


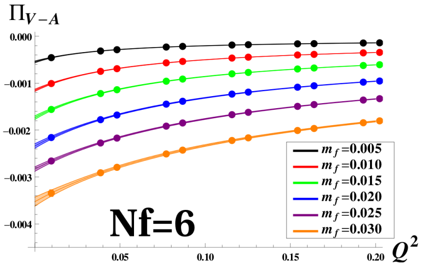
Next step: extrapolate $Q^2 \to 0$ and extract slope

$$S = 4\pi N_D \lim_{Q^2\to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

What we use: Padé rational function

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$





Very smooth data $\Rightarrow$ fit to Padé-(1, 2) functional form

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{\sum_{n=0}^{2} b_n Q^{2n}}$$

(similar to single-pole-dominance approximation)

Results stable and $\chi^2/dof \ll 1$ as Q^2 fit range varies

What others use: chiral perturbation theory (χPT)

JLQCD, RBC-UKQCD

- Effective field theory predicting dependence on M_P^2 and Q^2
- Need both M_P^2 and Q^2 smaller than ours
- $N_f > 2$ produces complications (discussed later)

For $N_f = 2$, (Gasser and Leutwyler, 1984)

$$S = \frac{1}{12\pi}\left(\bar{\ell}_5 + \log\left[\frac{M_P^2 \frac{v^2}{F_P^2}}{M_H^2}\right] - \frac{1}{6}\right)$$

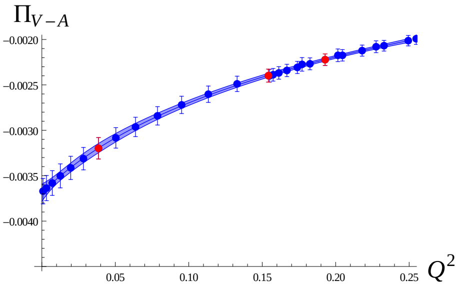
$$\Pi_{V-A}(Q^2) = -F_P^2 + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + \frac{2}{3}(1+x)\bar{J}(x)\right]$$

$$\bar{J}(x) = \frac{1}{16\pi^2}\left(\sqrt{1+x}\log\left[\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right] + 2\right), \qquad x \equiv 4M_P^2/Q^2$$

What might be used in the future

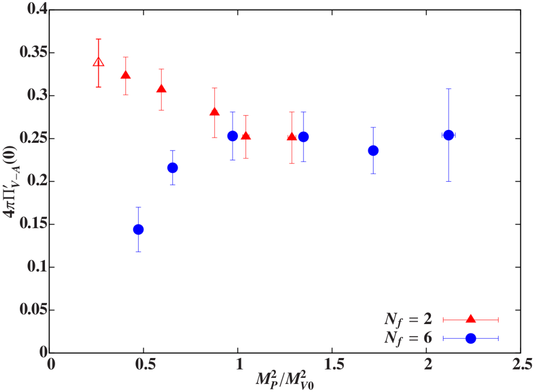
Twisted boundary conditions (TwBCs)

- Introduce external abelian field (equivalent to adding phase at lattice boundaries)
- Allows access to arbitrary Q^2, not just lattice modes $2\pi n/L$



- Correlations $\Longrightarrow$ TwBCs do not improve Padé fit results for slope
- May make it easier to apply χPT

Fit results for $\Pi'_{V-A}(0)$, $N_f = 2$ and $N_f = 6$



Vertical axis: $4\pi\Pi'_{V-A}(0)$

where

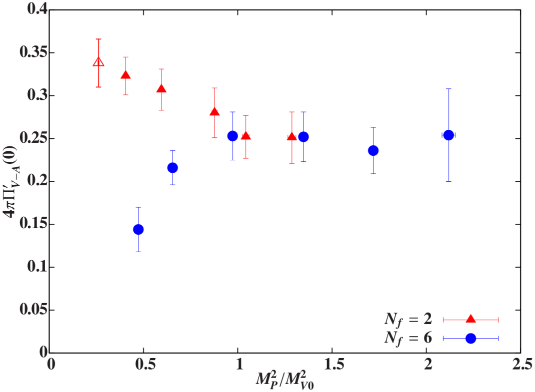
$$\Pi'(0) = \lim_{Q^2 \to 0} \frac{d}{dQ^2}\Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

Horizontal axis: M_P^2/M_{V0}^2 gives a more physical comparison than m_f
$M_{V0} \equiv \lim_{m \to 0} M_V$ is matched between $N_f = 2$ and $N_f = 6$

Expect agreement in the quenched limit $M_P^2 \to \infty$

Fit results for $\Pi'_{V-A}(0)$, $N_f = 2$ and $N_f = 6$

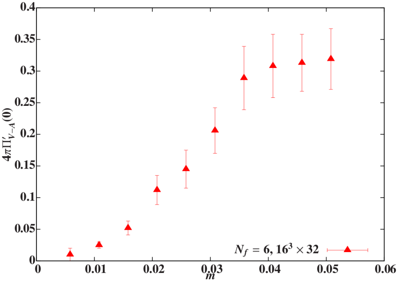


Lightest $N_f = 2$ point not included in analysis
due to potentially significant finite volume effects

Finite volume effects can produce spurious $S \longrightarrow 0$

If m too small compared to L, system deconfines
$\Longrightarrow$ chiral symmetry restored, parity doubling

$$4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right] \longrightarrow 0$$



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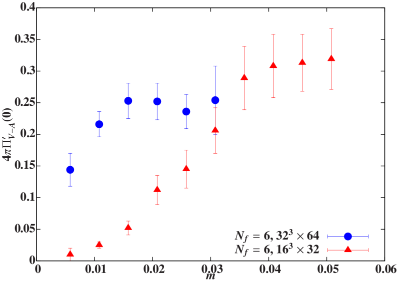

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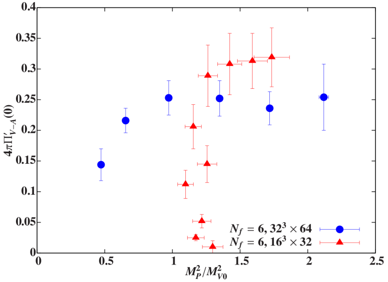


Also clearly distorts spectrum

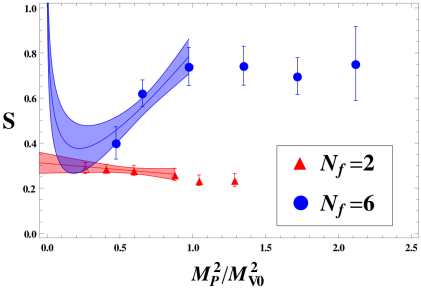
From slopes to S

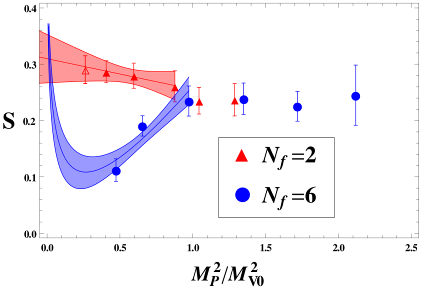
$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

1. N_D doublets with chiral electroweak couplings contribute to S
   Usually consider maximum $N_D = N_f/2$
   but only $N_D \geq 1$ is required for electroweak symmetry breaking

2. $\Delta S_{SM} = \frac{1}{4} \int_{4M_P^2}^{\infty} \frac{ds}{s} \left[1 - \left(1 - \frac{M_{V0}^2}{s} \right)^3 \Theta(s - M_{V0}^2) \right]$
   No direct dependence on N_f or N_D
   Diverges logarithmically in the limit $M_P^2 \to 0$
   to cancel contribution of three eaten modes
   Small ($\lesssim 10\%$) reduction in this work ($M_P^2 > 0$)

S parameter, $N_f = 2$ and $N_f = 6$





Maximum $N_D = N_f/2$
$C = N_f^2 - 1 - 3$

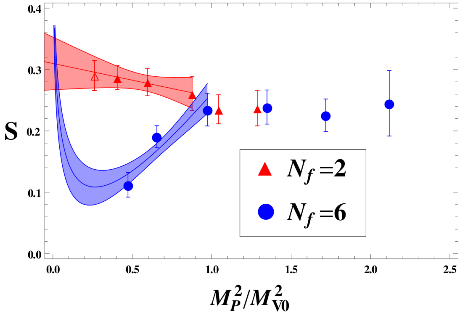
Minimum $N_D = 1$
$C = 4N_f - 5 - 3$

Linear fit to light points guides the eye,
accounts for known $M_P^2 \to 0$ log divergence remaining after ΔS_{SM}

$$S = A + B\frac{M_P^2}{M_{V0}^2} + \frac{C}{48\pi}\log\left(\frac{M_{V0}^2}{M_P^2}\right) \longrightarrow \lim_{M_P^2 \to 0} S = 0.31(4) \text{ for } N_f = 2$$

Connecting lattice results to phenomenology

- Lattice calculation involves $N_f^2 - 1$ degenerate pseudoscalars
- Only three massless NGBs eaten in Higgs mechanism, $N_f^2 - 4$ must be massive PNGBs (complicating χPT)



Imagine freezing $N_f^2 - 4$ PNGB masses at the blue curve's minimum, and taking only three to zero mass. . .

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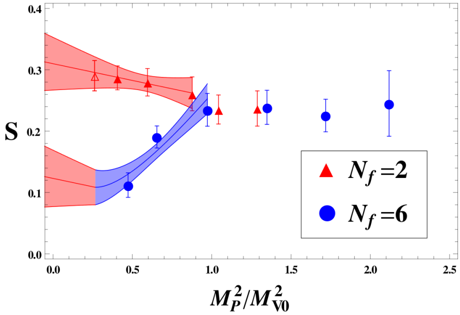

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Thank you!

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Collaborators

Tom Appelquist, Ron Babich, Rich Brower, Mike Buchoff, Michael Cheng, Mike Clark, Saul Cohen, George Fleming, Fu-Jiun Jiang, Joe Kiskis, Meifeng Lin, Heechang Na, Ethan Neil, James Osborn, Claudio Rebbi, Sergey Syritsyn, Pavlos Vranas, Gennady Voronov, Joe Wasem, Oliver Witzel

Funding and computing resources

Backup: Polarization functions for the S parameter

$$= i g_2 g_1 \cos\theta_w \sin\theta_w \Pi_{ee} \delta^{\mu\nu} + \ldots$$

$$= i g_2 g_1 \left(\Pi_{3e} - \sin^2\theta_w \Pi_{ee}\right) \delta^{\mu\nu} + \ldots$$

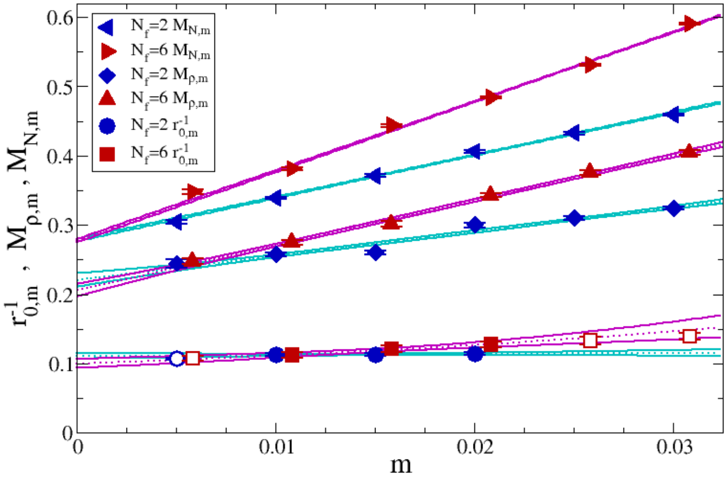
$$= \frac{i g_2 g_1}{\cos\theta_w \sin\theta_w} \left(\Pi_{33} - 2\sin^2\theta_w \Pi_{3e} + \sin^4\theta_w \Pi_{ee}\right) \delta^{\mu\nu} + \ldots$$

$$= i g_2^2 \Pi_{11} \delta^{\mu\nu} + \ldots$$

$$\Pi_{VV} = 2\Pi_{3e} \qquad\qquad \Pi_{AA} = 4\Pi_{33} - 2\Pi_{3e}$$

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \left[\Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)\right] - \Delta S_{SM}$$

Backup: matching IR scales in the chiral limit



Vector mass, nucleon mass, and inverse Sommer scale

all match at 10% level between $N_f = 2$ and $N_f = 6$

$M_{V0} = 0.215(3)$ [2f]; 0.209(3) [6f]

Backup: Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^{\mu a}(x) = \sum_{s=0}^{L_s-1} j^{\mu a}(x,s) \qquad \mathcal{A}^{\mu a}(x) = \sum_{s=0}^{L_s-1} \text{sign}\left(s - \frac{L_s-1}{2}\right) j^{\mu a}(x,s)$$

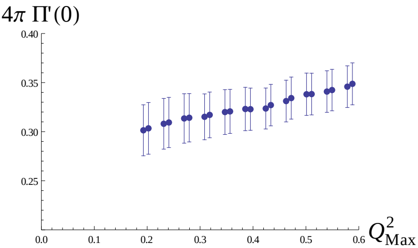
$$\begin{aligned} j^{\mu a}(x,s) = {} & \overline{\Psi}(x+\widehat{\mu},s)\frac{1+\gamma^\mu}{2}\tau^a U^\dagger_{x,\mu}\Psi(x,s) \\ & -\overline{\Psi}(x,s)\frac{1-\gamma^\mu}{2}\tau^a U_{x,\mu}\Psi(x+\widehat{\mu},s) \end{aligned}$$

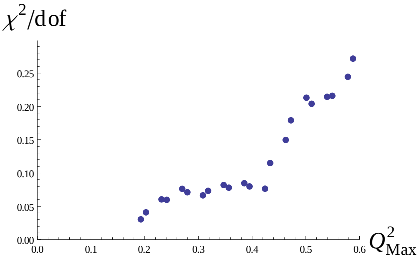
Local currents:

$$V^\mu(x) = \overline{q}(x)\gamma^\mu\tau^a q(x) \qquad A^\mu(x) = \overline{q}(x)\gamma^\mu\gamma^5\tau^a q(x)$$

$$q(x) = \frac{1-\gamma^5}{2}\Psi(x,0) + \frac{1+\gamma^5}{2}\Psi(x,L_s-1)$$

Backup: Padé fit Q^2-range dependence



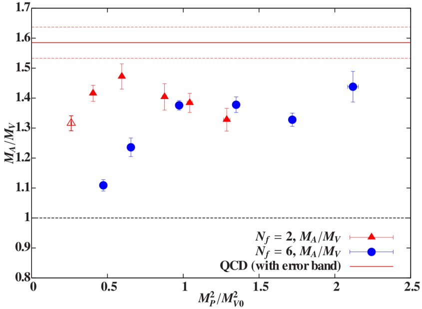


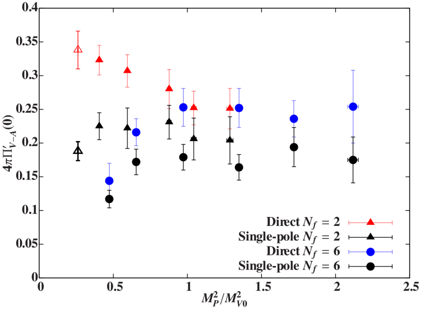
Results reported here used $Q_{Max} = 0.4$

Backup: Connection to parity-doubling

$$4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}[R_V(s) - R_A(s)] \longrightarrow 4\pi\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right]$$

$$\text{for } R(s) \longrightarrow 12\pi F^2\delta(s - M^2)$$





Signs of $N_f = 6$ parity-doubling consistent with reduced S
Direct checks of finite-volume effects underway

Backup: More parity-doubling plots