Lattice study of a technicolor dark matter candidate

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in collaboration with

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Quantum numbers of the techniquarks

In the SU(2) technicolor model,

- U is a technicolor doublet (fundamental representation).
- D is a technicolor doublet (fundamental representation).

- U and D do not carry QCD color.

- U_L and D_L form a weak doublet.
- U_R and D_R are weak singlets.

- U has electric charge $+1/2$.
- D has electric charge $-1/2$.

This model has an extra Goldstone boson,
which is a natural candidate for light asymmetric dark matter.

Extensions of this simple model would be straightforward to implement.

Chiral symmetry breaking in QCD

$\mathrm{SU(2)_L \times SU(2)_R} \quad \rightarrow \quad \mathrm{SU(2)_V}$

- Neglecting u and d quark masses, the Lagrangian has global $\mathrm{SU(2)_L \times SU(2)_R}$:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

- With degenerate u and d quark masses, the Lagrangian has global $\mathrm{SU(2)_V}$:

$$\delta\mathcal{L} = -m\bar{q}_R q_L + \mathrm{h.c.}$$

DYNAMICAL SYMMETRY BREAKING:

- Even without u and d quark masses, the hadron spectrum is $\mathrm{SU(2)_V}$ multiplets, not $\mathrm{SU(2)_L \times SU(2)_R}$ multiplets.

For example, $\quad m_{\rho^\pm} \approx m_{\rho^0} \quad \neq \quad m_{a_1^\pm} \approx m_{a_1^0}$.

Chiral symmetry breaking in SU(2) technicolor

SU(4) $\rightarrow$ Sp(4)

Appelquist, Rodriques da Silva & Sannino, PRD60, 116007 (1999)
Ryttov & Sannino, PRD78, 115010 (2008)

- Neglecting U and D quark masses, the Lagrangian has global SU(4):

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{Q}\gamma^\mu D_\mu Q$$

$$Q = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 C\overline{U}_R^T \\ -i\sigma^2 C\overline{D}_R^T \end{pmatrix}$$

- With degenerate U and D quark masses, the Lagrangian has global Sp(4):

$$\delta\mathcal{L} = \frac{m}{2}Q^T(-i\sigma^2 C)EQ + \mathrm{h.c.}, \qquad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

HYPOTHESIS (dynamical symmetry breaking):

- Even without U and D quark masses, the hadron spectrum is Sp(4) multiplets,
not SU(4) multiplets.

Seeing the SU(4) and Sp(4) symmetries

Act on the Lagrangian with an infinitesimal SU(4) transformation defined by

$$Q \to \left(1 + i \sum_{n=1}^{15} \alpha^n T^n\right) Q$$

- The kinetic terms are invariant because the fundamental representation is real. This is true for $N_{\mathrm{TC}} = 2$ but not for $N_{\mathrm{TC}} > 2$.

- The mass terms are not invariant under SU(4):

$$\mathcal{L} \to \mathcal{L} + \frac{im}{2} \sum_{n=1}^{15} \alpha^n Q^T(-i\sigma^2 C)\left(ET^n + T^{nT}E\right)Q + \mathrm{h.c.}$$

Only 10 of the 15 generators leave $\mathcal{L}$ invariant: those that obey $ET^n + T^{nT}E = 0$. These 10 generators define an Sp(4) Lie algebra.

Recall: $E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$.

Anticipating dynamical symmetry breaking

In QCD, the nonzero vacuum expectation value is $\langle \overline{u}u \rangle = \langle \overline{d}d \rangle \neq 0.$
This has the same form as the (degenerate) explicit mass terms.
Therefore dynamical breaking has the same structure as explicit breaking:

$$\mathrm{SU(2)_L \times SU(2)_R} \quad \rightarrow \quad \mathrm{SU(2)_V}$$

The 3 broken generators require 3 Goldstone bosons: π^+, π^-, π^0.

In the SU(2) technicolor model, we expect $\langle \overline{U}U \rangle = \langle \overline{D}D \rangle \neq 0.$
This has the same form as the (degenerate) explicit mass terms.
Therefore dynamical breaking would have the same structure as explicit breaking:

$$\mathrm{SU(4)} \quad \rightarrow \quad \mathrm{Sp(4)}$$

The 5 broken generators would require 5 Goldstone bosons.
We see these 5 Goldstone bosons in our lattice simulations.

Technihadron operators

local operators for technimesons:

$$
\begin{aligned}
\mathcal{O}^{(\Gamma)}_{\overline{U}D}(x) &= \overline{U}(x)\Gamma D(x) \\
\mathcal{O}^{(\Gamma)}_{\overline{D}U}(x) &= \overline{D}(x)\Gamma U(x) \\
\mathcal{O}^{(\Gamma)}_{\overline{U}U\pm\overline{D}D}(x) &= \frac{1}{\sqrt{2}}\Big(\overline{U}(x)\Gamma U(x) \pm \overline{D}(x)\Gamma D(x)\Big)
\end{aligned}
$$

local operators for technibaryons (techni-diquarks):

$$
\begin{aligned}
\mathcal{O}^{(\Gamma)}_{UD}(x) &= U^T(x)(-i\sigma^2 C)\Gamma D(x) \\
\mathcal{O}^{(\Gamma)}_{DU}(x) &= D^T(x)(-i\sigma^2 C)\Gamma U(x) \\
\mathcal{O}^{(\Gamma)}_{UU}(x) &= U^T(x)(-i\sigma^2 C)\Gamma U(x) \\
\mathcal{O}^{(\Gamma)}_{DD}(x) &= D^T(x)(-i\sigma^2 C)\Gamma D(x)
\end{aligned}
$$

Note: $\Gamma = 1$ or γ^5 or γ^μ or $\ldots$ is any Dirac structure.

Technihadron correlation functions

Put a creation operator at time t_x and an annihilation operator at time t_y.

Technimeson example:

$$\begin{aligned} C^{(\Gamma)}_{\overline{U}D}(t_x-t_y) &= \sum_{\vec{x}}\sum_{\vec{y}} \mathcal{O}^{(\Gamma)}_{\overline{U}D}(y)\left(\mathcal{O}^{(\Gamma)}_{\overline{U}D}(x)\right)^\dagger \\ &= \sum_{\vec{x}}\sum_{\vec{y}} \mathrm{Tr}\left[\Gamma D(y)\overline{D}(x)\gamma^0\Gamma^\dagger\gamma^0 U(x)\overline{U}(y)\right] \end{aligned}$$

Technibaryon example:

$$\begin{aligned} C^{(\Gamma)}_{UD}(t_x-t_y) &= \sum_{\vec{x}}\sum_{\vec{y}} \mathcal{O}^{(\Gamma)}_{UD}(y)\left(\mathcal{O}^{(\Gamma)}_{UD}(x)\right)^\dagger \\ &= \sum_{\vec{x}}\sum_{\vec{y}} \mathrm{Tr}\left[\Gamma D(y)\overline{D}(x)\gamma^0\Gamma^\dagger(-i\sigma^2C)^\dagger\gamma^{0T}\overline{U}^T(x)U^T(y)(-i\sigma^2C)\right] \\ &= \sum_{\vec{x}}\sum_{\vec{y}} \mathrm{Tr}\left[\Gamma D(y)\overline{D}(x)\gamma^0\Gamma^\dagger\gamma^0 U(x)\overline{U}(y)\right] \\ &= C^{(\Gamma)}_{\overline{U}D}(t_x-t_y) \end{aligned}$$

In this way, each technibaryon is mass-degenerate with a technimeson.

Parity partners and Goldstone bosons

The degenerate pairs have equal angular momentum but opposite parities:

$$J\left(\mathcal{O}_{UD}^{(\Gamma)}\right) = J\left(\mathcal{O}_{\overline{U}D}^{(\Gamma)}\right)$$
$$P\left(\mathcal{O}_{UD}^{(\Gamma)}\right) = -P\left(\mathcal{O}_{\overline{U}D}^{(\Gamma)}\right)$$

Because of this, we expect the 5 Goldstone bosons to be

- the pseudoscalars Π^+ and Π^-
- their degenerate scalars Π_{UD} and $\Pi_{\overline{UD}}$
- the neutral pseudoscalar Π^0 (which has no technibaryon partner)

The 3 pseudoscalars will be eaten by W and Z.

The 2 scalars are our dark matter candidate and its antiparticle.

Choices for our lattice explorations

The Wilson action is used for technicolor:

$$S_W = \frac{\beta}{2}\sum_{x,\mu,\nu}\left(1-\frac{1}{2}\mathrm{ReTr}U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)\right)+(4+m_0)\sum_x\bar{\psi}(x)\psi(x)$$
$$-\frac{1}{2}\sum_{x,\mu}\left(\bar{\psi}(x)(1-\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu})+\bar{\psi}(x+\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\psi(x)\right)$$

Two choices for β gives two lattice spacings.

Six choices of m_0 give six techniquark masses per β.

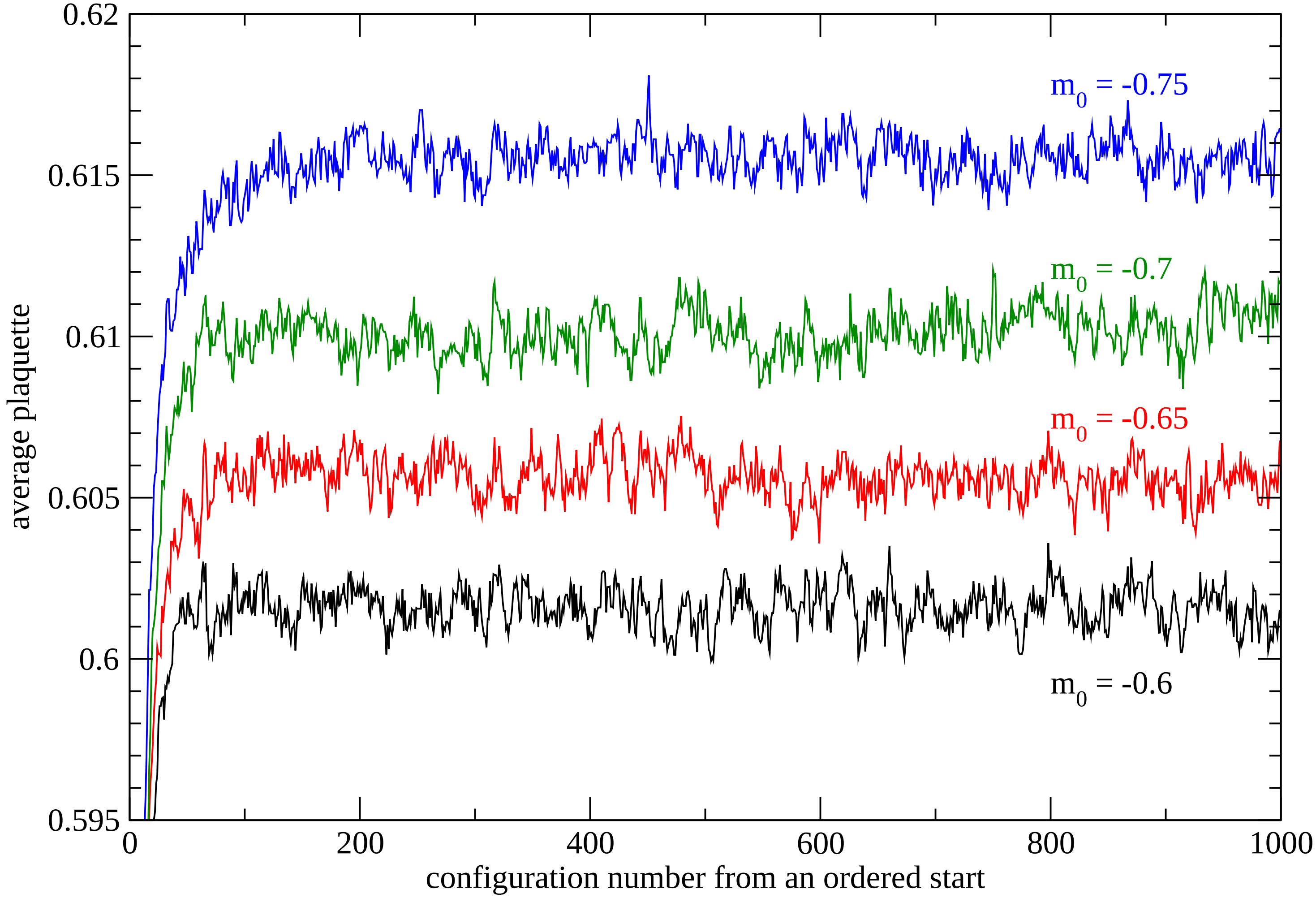
β	m_0
2.0	-0.85, -0.90, -0.94, -0.945, -0.947, -0.949
2.2	-0.60, -0.65, -0.68, -0.70, -0.72, -0.75

All lattices are $16^3 \times 32$.

Electroweak interactions are omitted from the simulations.

Creating ensembles of gauge fields

Raw data for $\beta = 2.2$.



We define each ensemble = {cfg320, cfg340, cfg360, . . . cfg1000}.

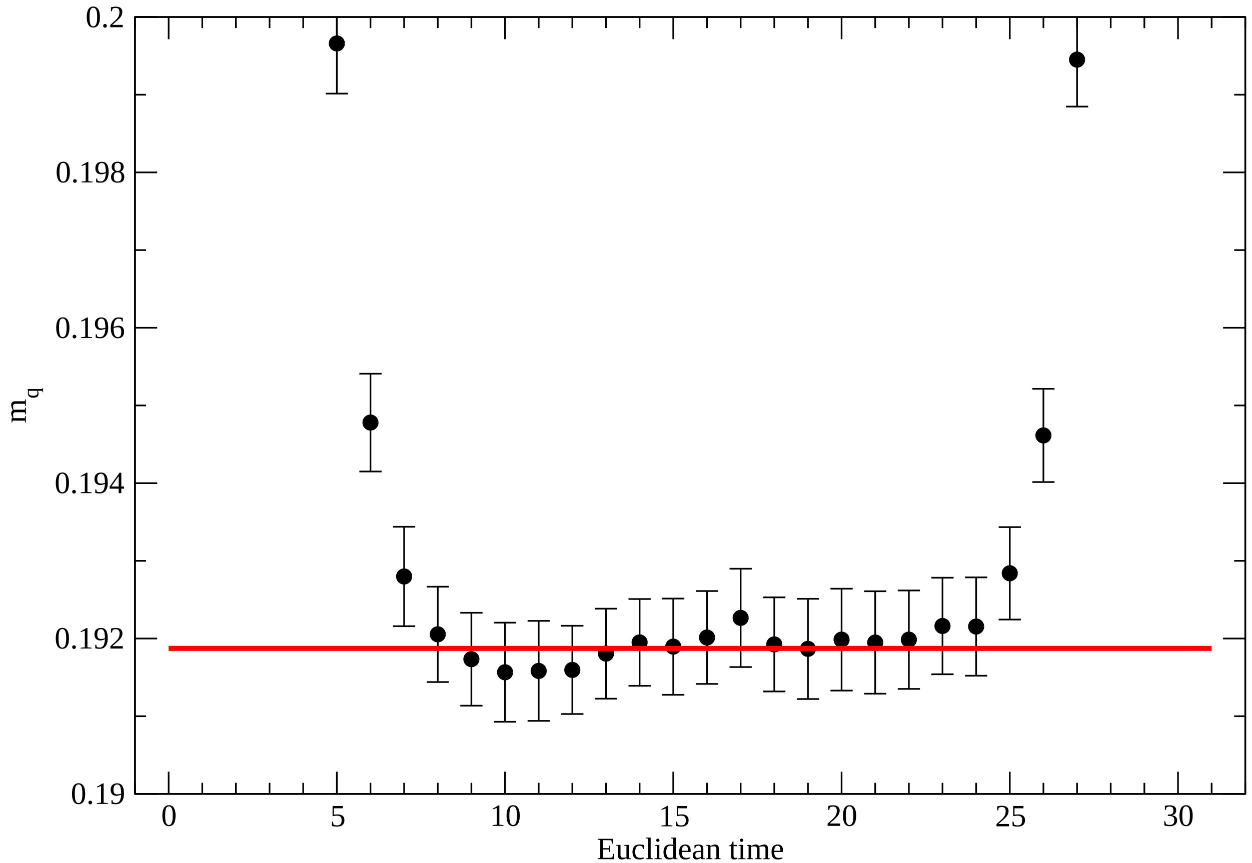
Techniquark propagators are random U(1) wall sources, averaged over each time step.

Determining a techniquark mass

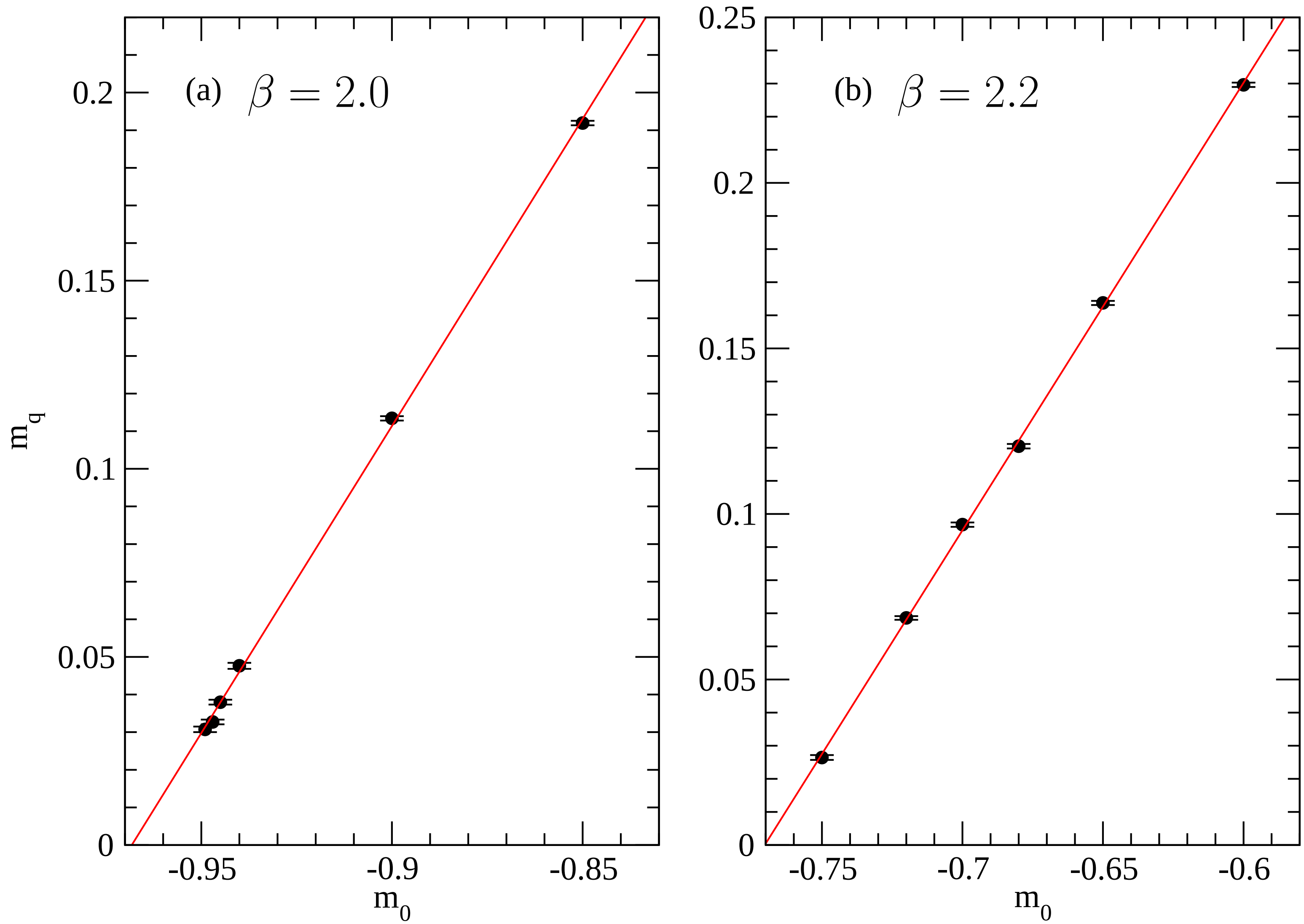
For lattice, a convenient definition of quark mass comes from PCAC:

$$m_q = \lim_{t\to\infty}\left(\frac{\langle A_4(t+1)P(0)\rangle - \langle A_4(t-1)P(0)\rangle}{4\langle P(t)P(0)\rangle}\right),$$

Here is the example of $\beta = 2.2$ and $m_0 = -0.75$:

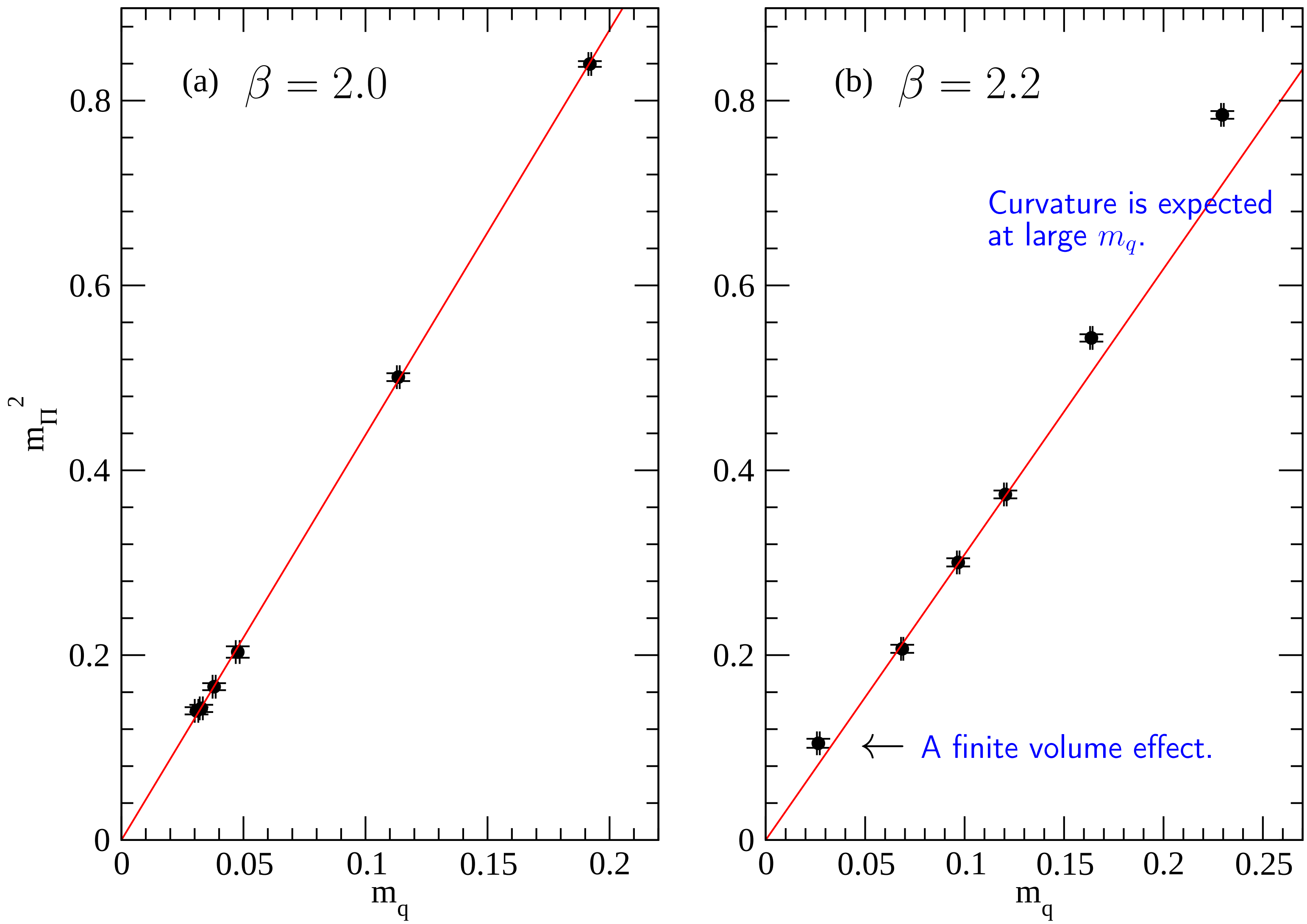


The PCAC mass is linear in the bare mass



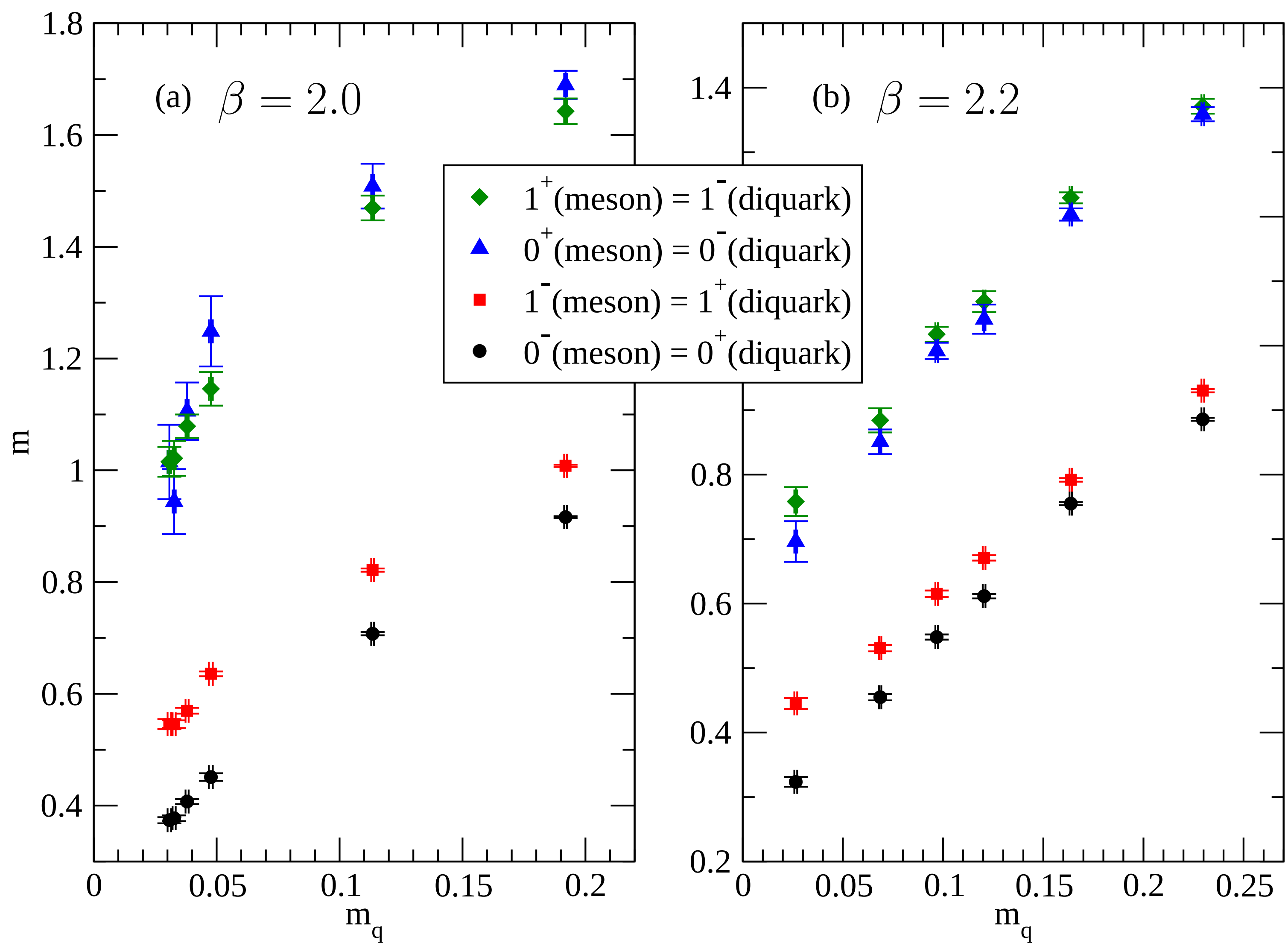
Observing the Goldstone bosons

The expected behavior, $m_\Pi^2 \propto m_q$ for small m_q, is observed.
These plots apply to all five Goldstone bosons.



Note: These are 3-state fits to all time steps (except the source).

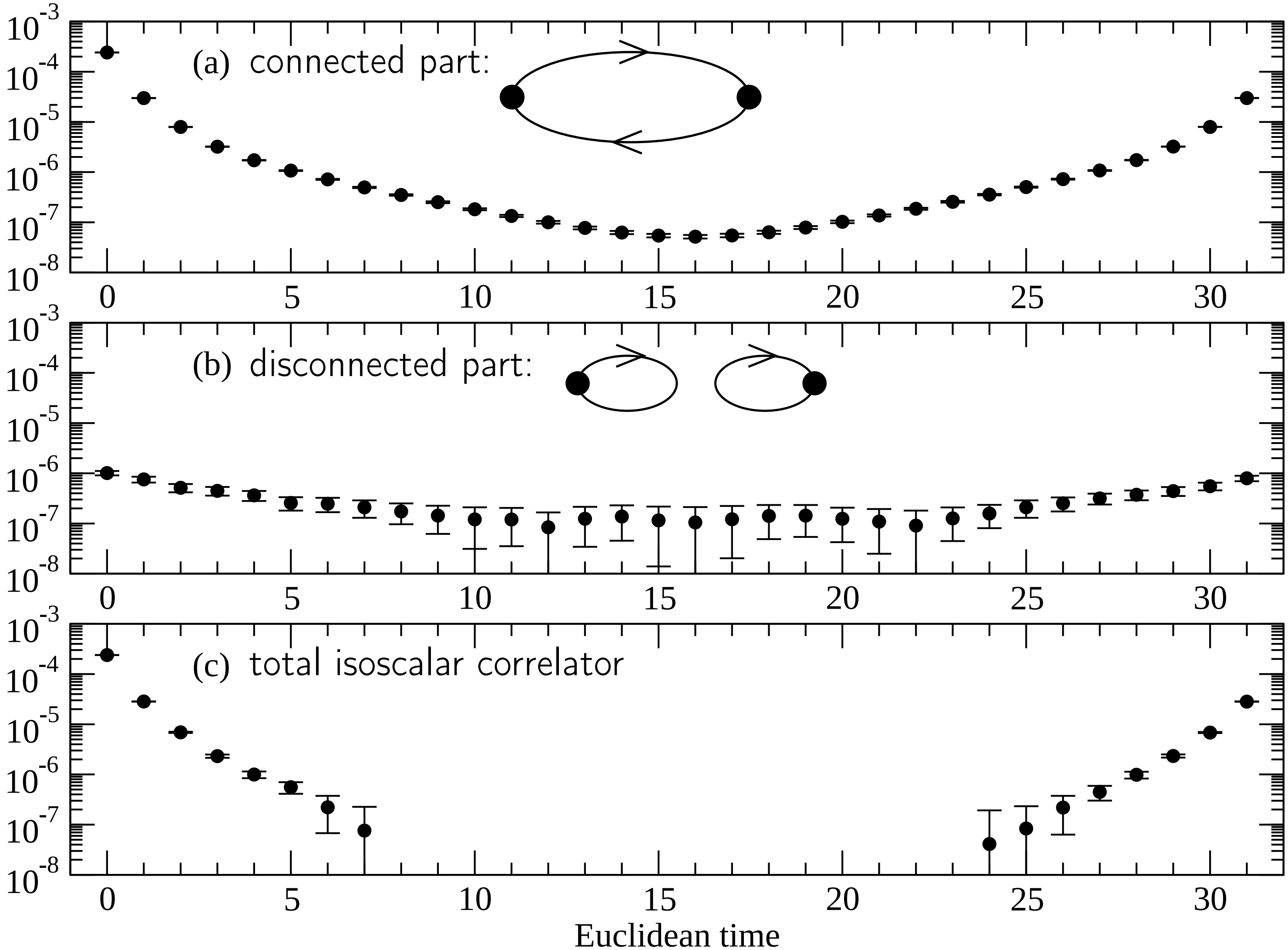
Exploring the spectrum of technihadron masses



Extrapolation to $m_q = 0$ gives nonzero masses to non-Goldstones.

For $\beta = 2.2$, all masses extrapolate to below the lattice cutoff, $m \sim 1$.

Isoscalar pseudoscalar meson

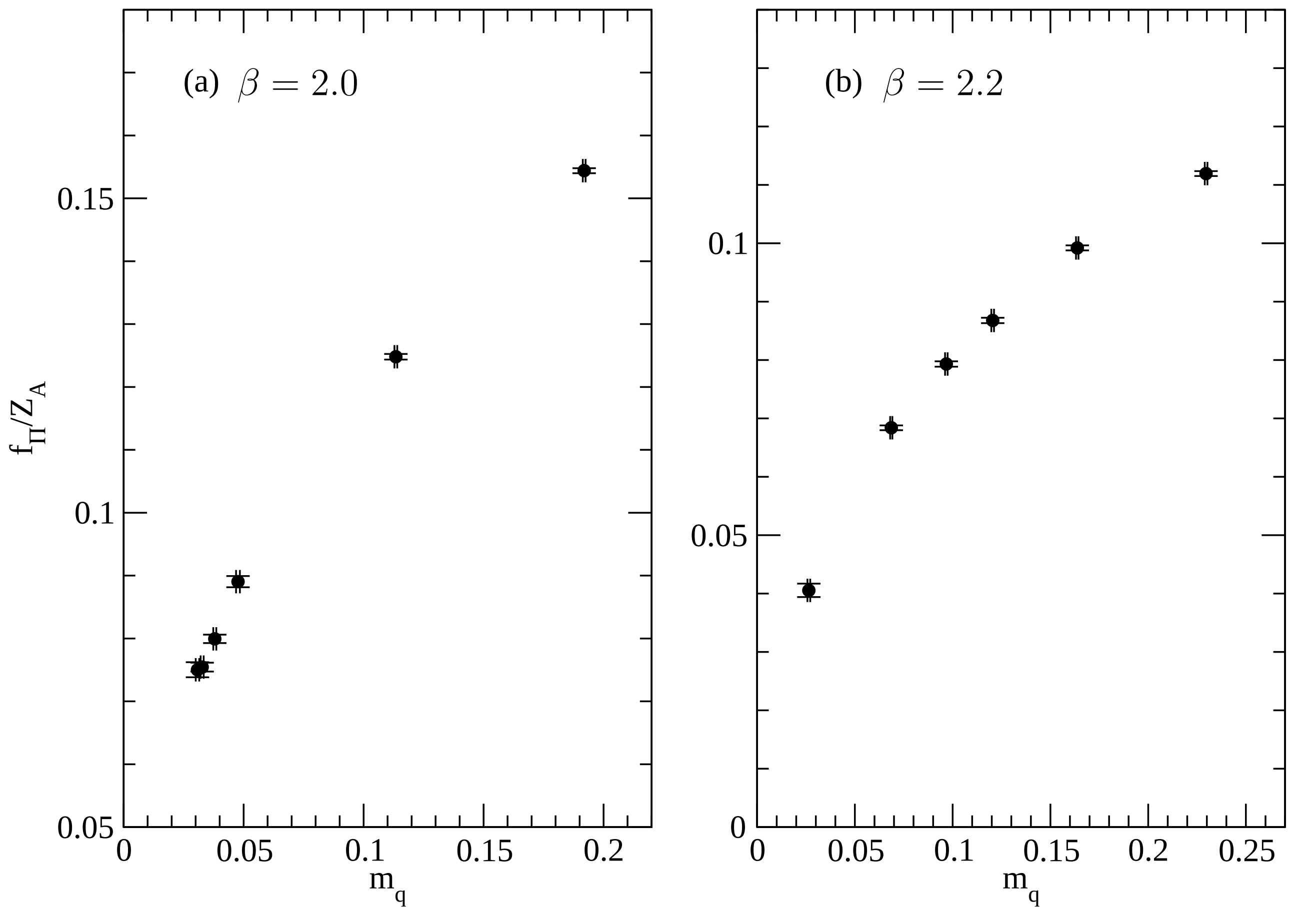


(a) shows our signal for the isovector Goldstone boson.

(c) shows no signal for any isoscalar Goldstone boson.

Goldstone decay constant (up to renormalization)

$$\begin{aligned}
\langle A_4(t)A_4(0)\rangle &= \sum_{j=1}^{n} \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right) \\
\langle A_4(t)P(0)\rangle &= \sum_{j=1}^{n} \left(\frac{m_j}{2m_q Z_P}\right) \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right) \\
\langle P(t)P(0)\rangle &= \sum_{j=1}^{n} \left(\frac{m_j}{2m_q Z_P}\right)^2 \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right)
\end{aligned}$$



An effective theory for the five Goldstones

Since $\frac{m_\rho}{(f_\Pi/Z_A)} = O(10)$, we can integrate out all but the five Goldstones.

The effective Lagrangian couple according to

$$
\begin{aligned}
\delta\mathcal{L}_{\mathcal{G}} &= \sum_{n=1}^{5}\left(Q^T(-i\sigma^2 C)\gamma^5 T^n Q\right)\Pi^n \\
&= Q^T(-i\sigma^2 C)\gamma^5 \mathcal{G} Q \quad \text{with} \quad \mathcal{G} = \frac{i}{2}\begin{pmatrix} 0 & \sqrt{2}\Pi_{UD} & \Pi^0 & \sqrt{2}\Pi^+ \\ -\sqrt{2}\Pi_{UD} & 0 & \sqrt{2}\Pi^- & -\Pi^0 \\ -\Pi^0 & -\sqrt{2}\Pi^- & 0 & -\sqrt{2}\Pi_{\overline{UD}} \\ -\sqrt{2}\Pi^+ & \Pi^0 & \sqrt{2}\Pi_{\overline{UD}} & 0 \end{pmatrix}
\end{aligned}
$$

Similar to Ryttov&Sannino,PRD78(2008)115010, the resulting effective Lagrangian is

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\mathrm{Tr}\left[D_\mu \mathcal{G} D^\mu \mathcal{G}^\dagger\right] + \cdots \quad \text{(linear form)} \\
\mathcal{L} &= f_\Pi^2 \mathrm{Tr}\left[\omega_\mu^\perp \omega^{\perp\mu}\right] + \cdots \quad \text{(nonlinear form)}
\end{aligned}
$$

where $\omega^\perp$ contains an exponential of the Goldstone fields as well as a covariant derivative.

Summary

SU(2) technicolor with two techniquarks contains an extra Goldstone boson, which is a natural candidate for light asymmetric dark matter.

Our lattice exploration

- confirmed the symmetry-breaking pattern: SU(4)$\rightarrow$Sp(4).
- explored the mass spectrum of the lightest technihadrons.
- established an effective field theory.

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