What did we learn about SU(2) adj?

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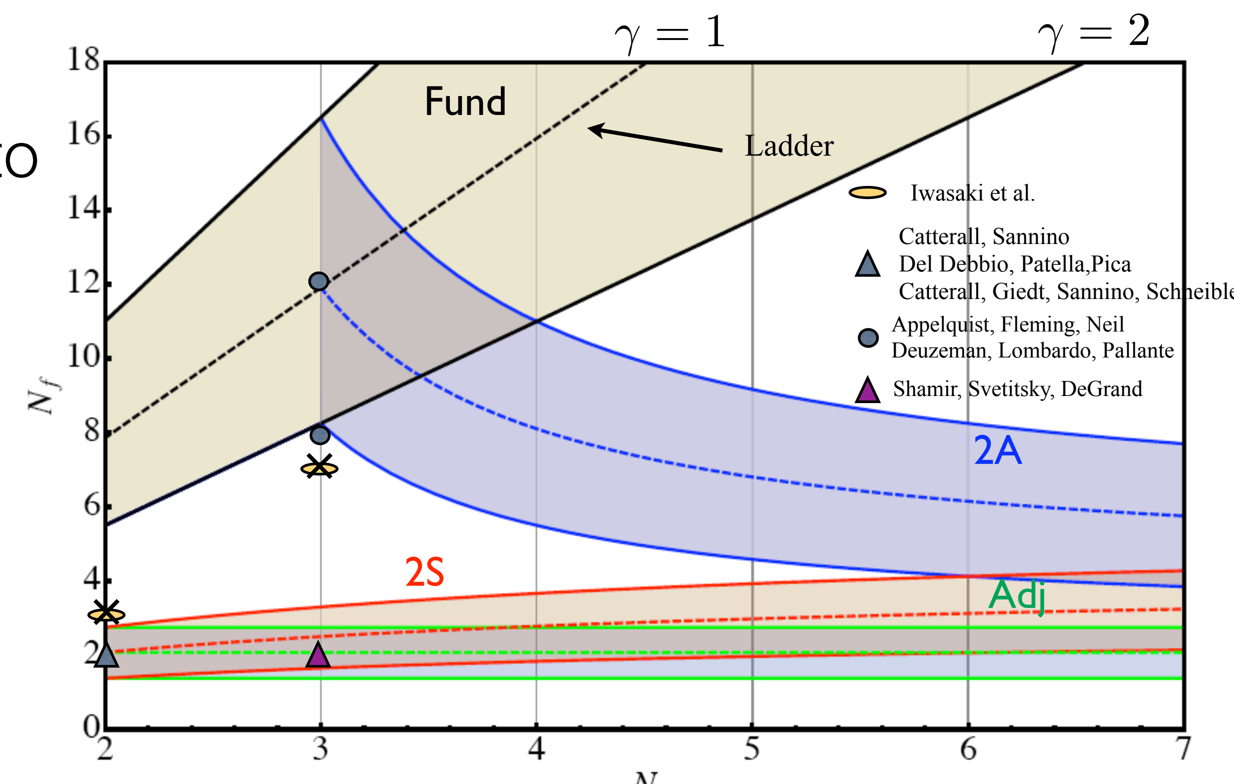
IRFP for BSM phenomenology

- IR fixed point: scale invariance at large distances
- identify the boundaries of the conformal window
- theories inside the conformal window to build models of conformal technicolor
- theories "just outside" the conformal window to build "walking TC"
- scheme/scale dependence
- lattice artefacts

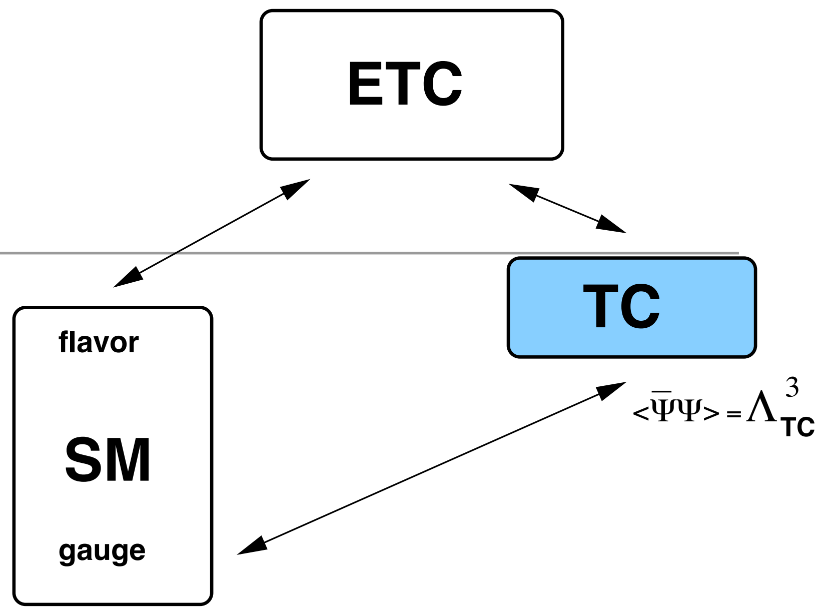
Non-SUSY Phase Diagram Bound



All Orders Beta Function

Ryttov and F.S. 07

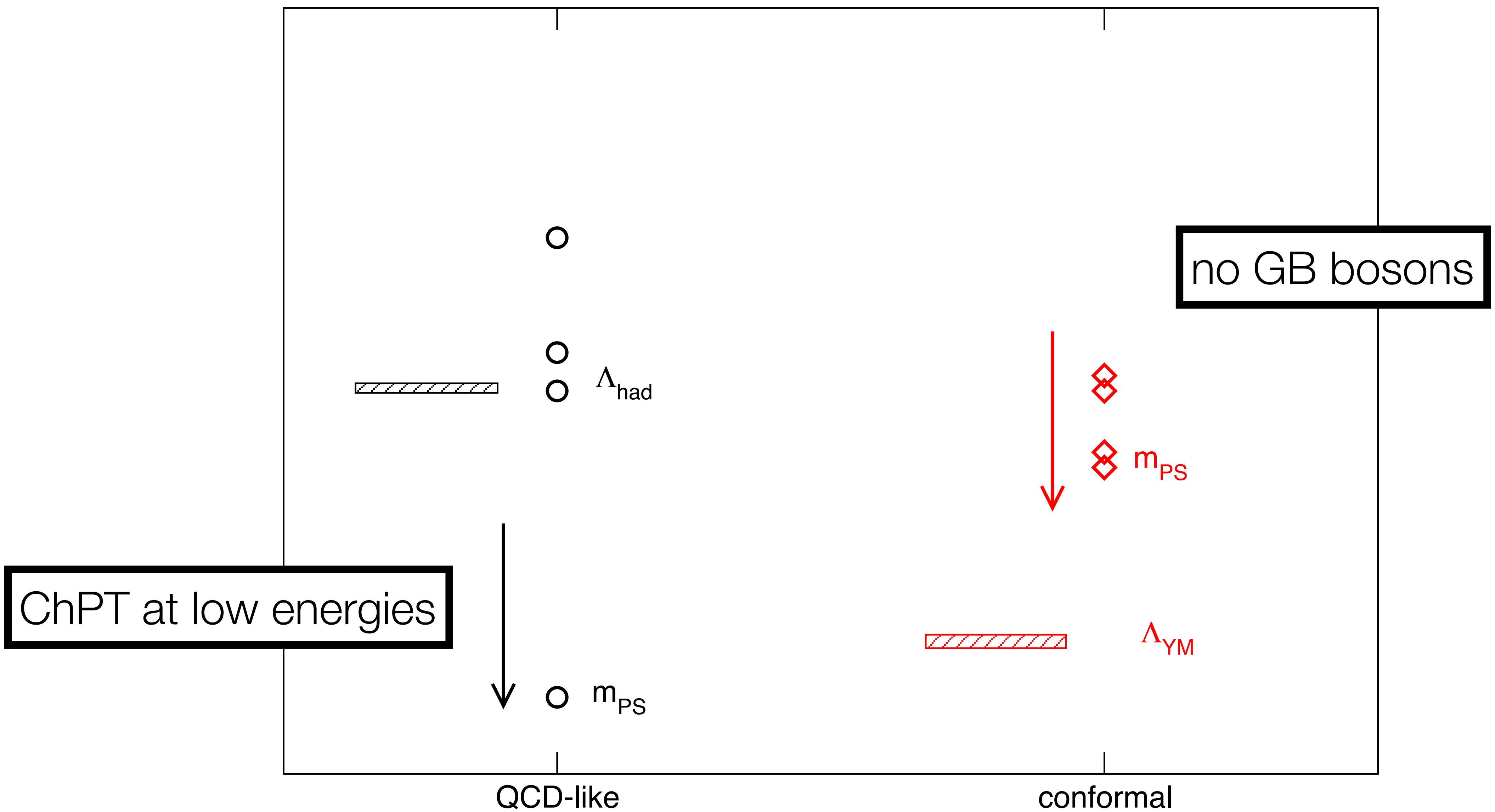
Lattice tools



- Scaling of the spectrum in a mass deformed CGT
- Define NP schemes to study the running of the couplings
  - Schrodinger functional
  - MCRG
  - Potential schemes
- Identify the fixed points, compute the anomalous dimensions
- Control the systematic errors!

Conformal spectrum

- Different qualitative behaviours in the chiral limit



Conformal scaling

- Existence of an IRFP dictates the scaling of physical observables

$$\boxed{\begin{aligned} g \to 0: \quad & \beta(g) \simeq -\beta_0 g^3 \\ & \gamma(g) \simeq \gamma_0 g^2 \\ \\ g \to g^*: \quad & \beta(g) \simeq \beta_*(g - g^*) \\ & \gamma(g) \simeq \gamma_* \end{aligned}}$$

$$Z(\mu, \mu_0) = \frac{\tilde{Z}(\mu/\Lambda)}{\tilde{Z}(\mu_0/\Lambda)}$$

- Running mass: $m(\mu) = Z(\mu, \mu_0) m(\mu_0)$

$$\tilde{Z}_m(\mu/\Lambda) = [g_* - g(\mu)]^{-\frac{\gamma_*}{\beta_*}} g(\mu)^{\frac{\gamma_0}{\beta_0}} \exp\left\{ \int_{g(\mu)}^{g_*} \left(\frac{\gamma(z)}{\beta(z)} - \frac{\gamma_*}{\beta_*(z - g_*)} + \frac{\gamma_0}{\beta_0 z} \right) dz \right\}$$

- RGI mass:

$$\boxed{m(M) = M}$$

Large mass limit

- theory is defined by the values of M & Λ

for $M \gg \Lambda$

$$m(\mu) = A_\infty \tilde{Z}(\mu/\Lambda)\ M \left(\log \frac{M}{\Lambda}\right)^{\gamma_0/\beta_0}$$

- fermions decouple - pure gauge theory at low energies

$$\bar{m} = M + \sum_{n=1}^{\infty} B_n \left(\frac{\bar{m}^2}{M^2}, 1\right) g^{2n}(M)\ .$$

$$\boxed{\begin{array}{c} M_{\mathrm{mes}} = 2M\ ; \\ M_{\mathrm{glue}} = B_{\mathrm{glue}}\Lambda\ . \end{array}}$$

Chiral limit

- conformal symmetry: no spontaneous symmetry breaking of chiral symmetry

for $M \ll \Lambda$

$$m(\mu) = A_0 \tilde{Z}(\mu/\Lambda)\ \Lambda^{-\gamma_*}\ M^{1+\gamma_*}$$

- for a physical quantity: $M_X = \mathcal{F}\left(\mu, g(\mu), m(\mu)\right)$

for $\mu = M \ll \Lambda$

$$\begin{aligned} M_X &= \mathcal{F}\left(M, g_* - A_g (M/\Lambda)^{\beta_*}, M\right) \\ &= M \tilde{\mathcal{F}}\left(1, g_* - A_g (M/\Lambda)^{\beta_*}, 1\right) \\ &\simeq M \tilde{\mathcal{F}}\left(1, g_*, 1\right) \end{aligned}$$

Hyperscaling hypothesis

- Scaling with the quark mass: $M_X \sim m^{1/(1+\gamma_*)}$

Low-energy effective theories

- early scaling:

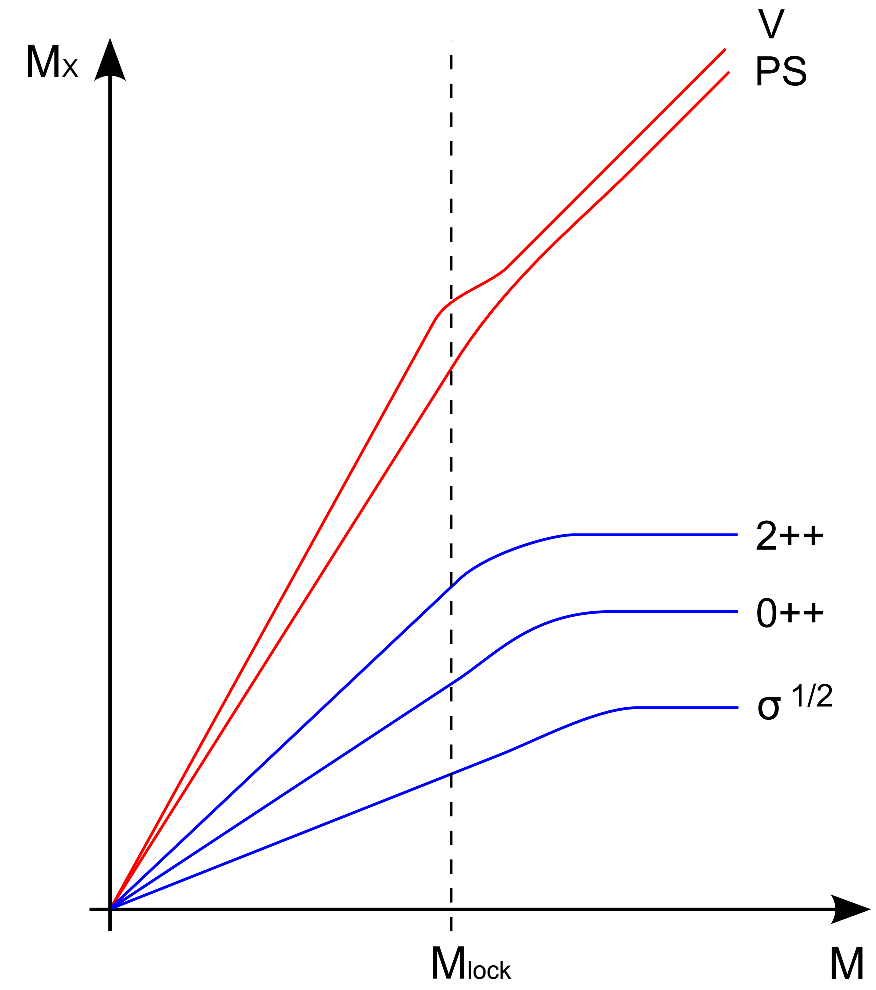
$$\mathcal{L}_{\text{eff}} = -\frac{1}{2g^2}\text{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \sum_{i=1,2,3} \frac{a_i}{M^2} S_i + O(M^{-4}) \ .$$

$$S_1 = \sum_{\mu,\nu,\rho} \text{tr}\left(J_{\mu\nu\rho}J^{\mu\nu\rho}\right) \ ,$$

$$S_2 = \sum_{\mu,\nu,\rho} \text{tr}\left(J^{\mu}{}_{\mu\rho}J_{\nu}{}^{\nu\rho}\right) \ ,$$

$$S_3 = \sum_{\mu,\nu,\rho} \text{tr}\left(J_{\mu\nu\rho}J^{\nu\mu\rho}\right) \ ,$$

$$J_{\mu\nu\rho} = \partial_\mu F_{\nu\rho} - i[A_\mu, F_{\nu\rho}]$$



$$M_{\text{PS}} \gg M_\sigma$$

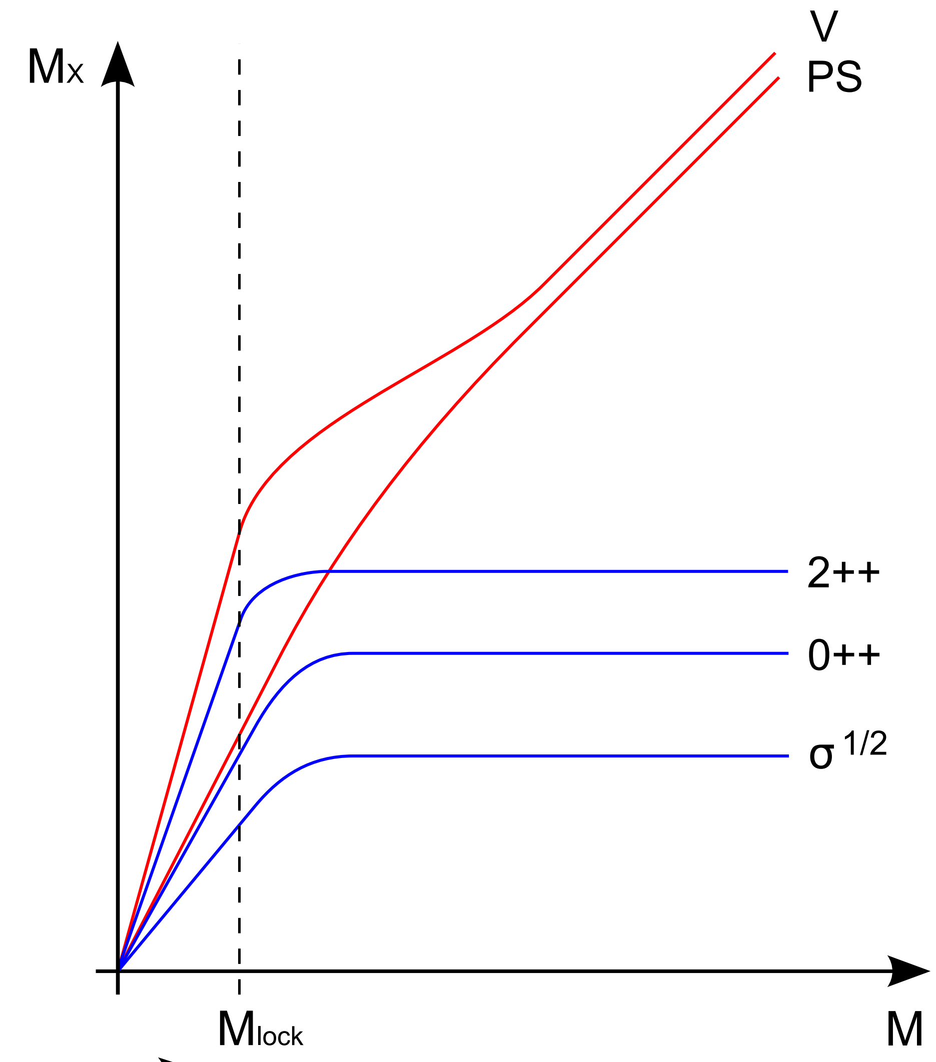
Low-energy effective theories

- late scaling:

  complicated lagrangian combining mesonic and gluonic degrees of freedom

  degeneracy of the glueball and meson spectra in the chiral limit

  scaling with the fermion mass as discussed above



$$M_{\mathrm{PS}} \simeq M_\sigma$$

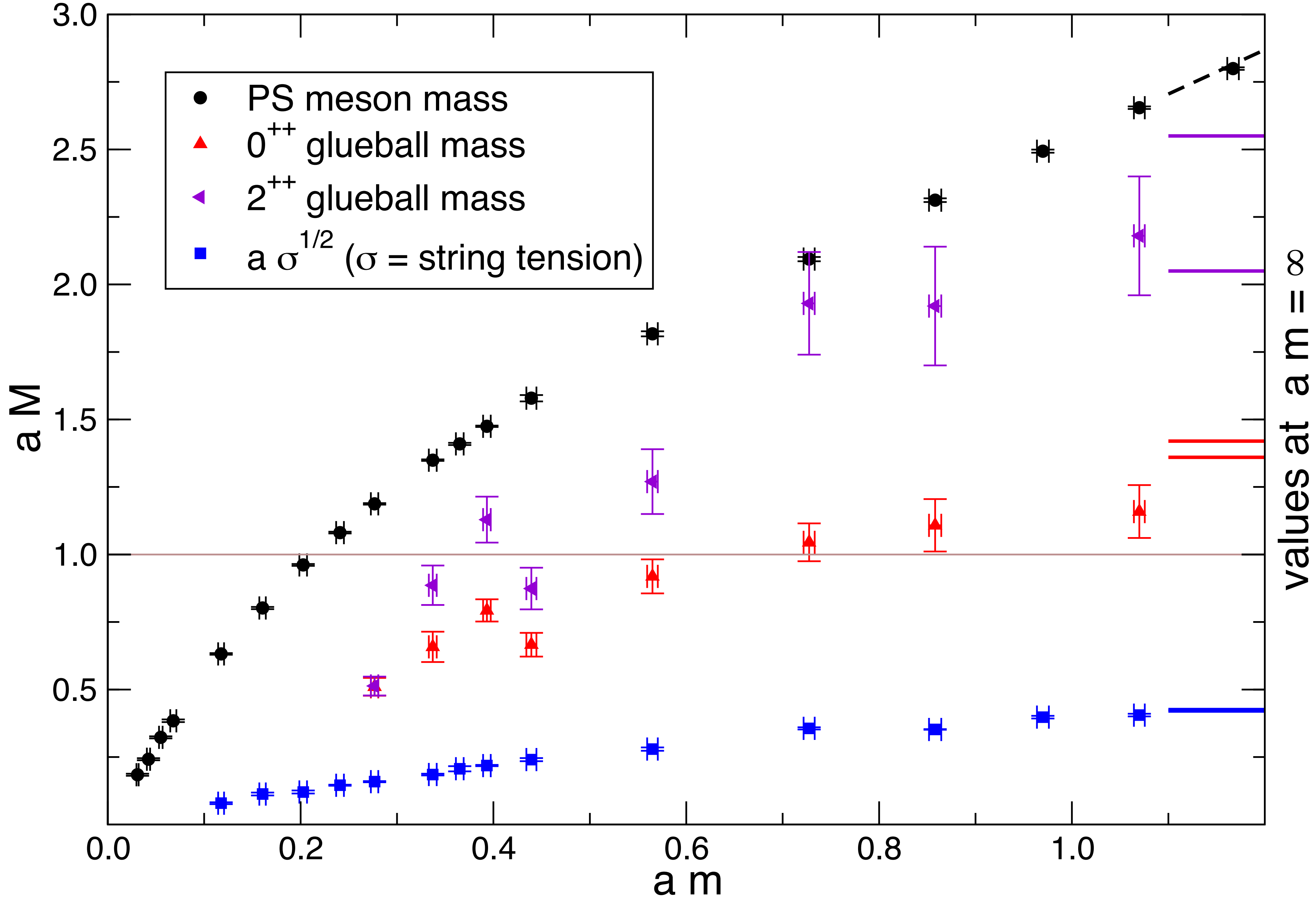
Finite-volume effects

- Finite size of the system breaks IR scale invariance
- If there exists a fixed point, these effects can be taken into account via FSS relations
- Two caveats:
  - FSS works if $1/L \ll \Lambda$
  - finite-volume effects are not described by ChPT!!
- Careful studies on large lattices are necessary

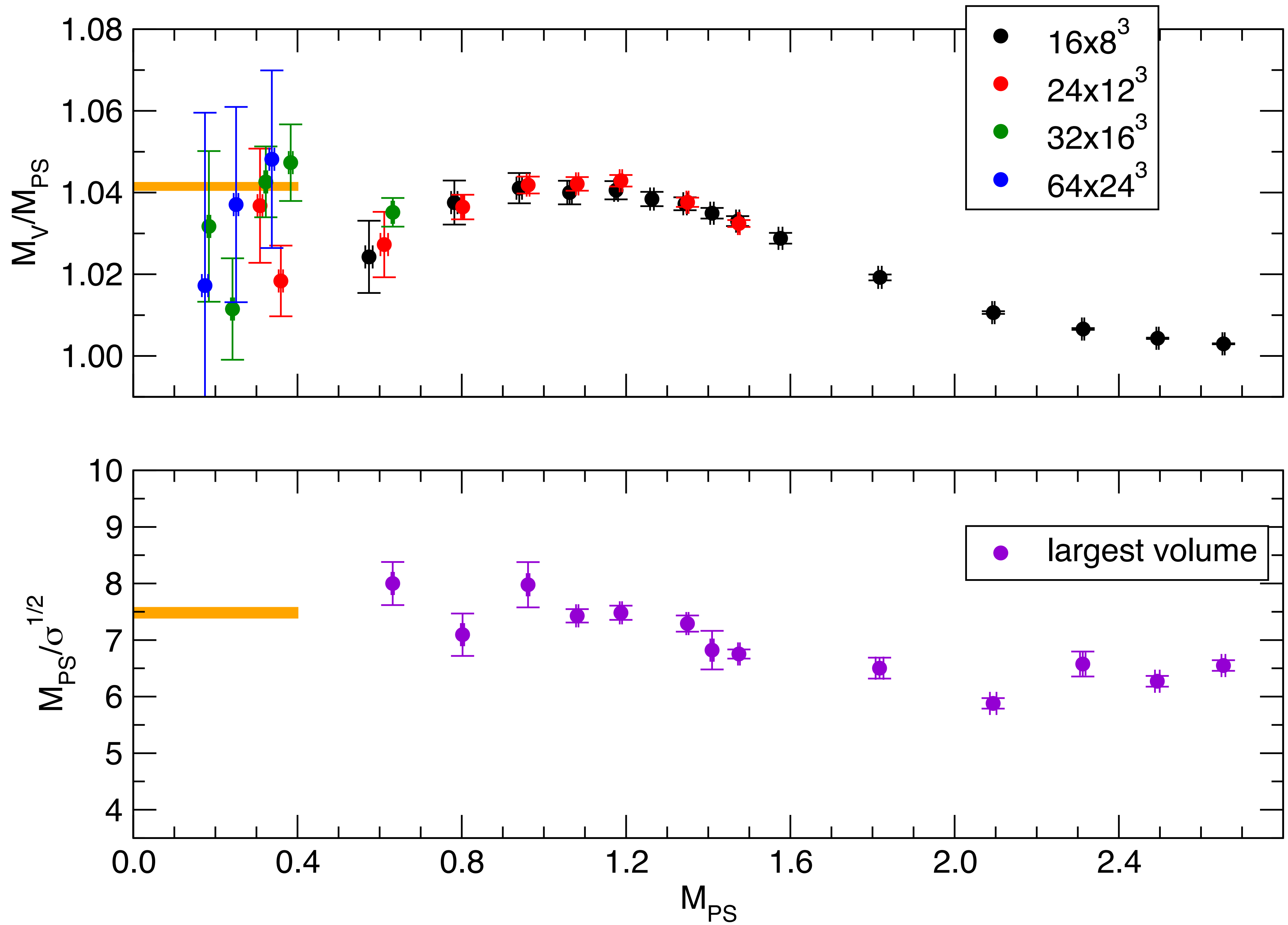
what is large?

Spectrum

- Overall picture

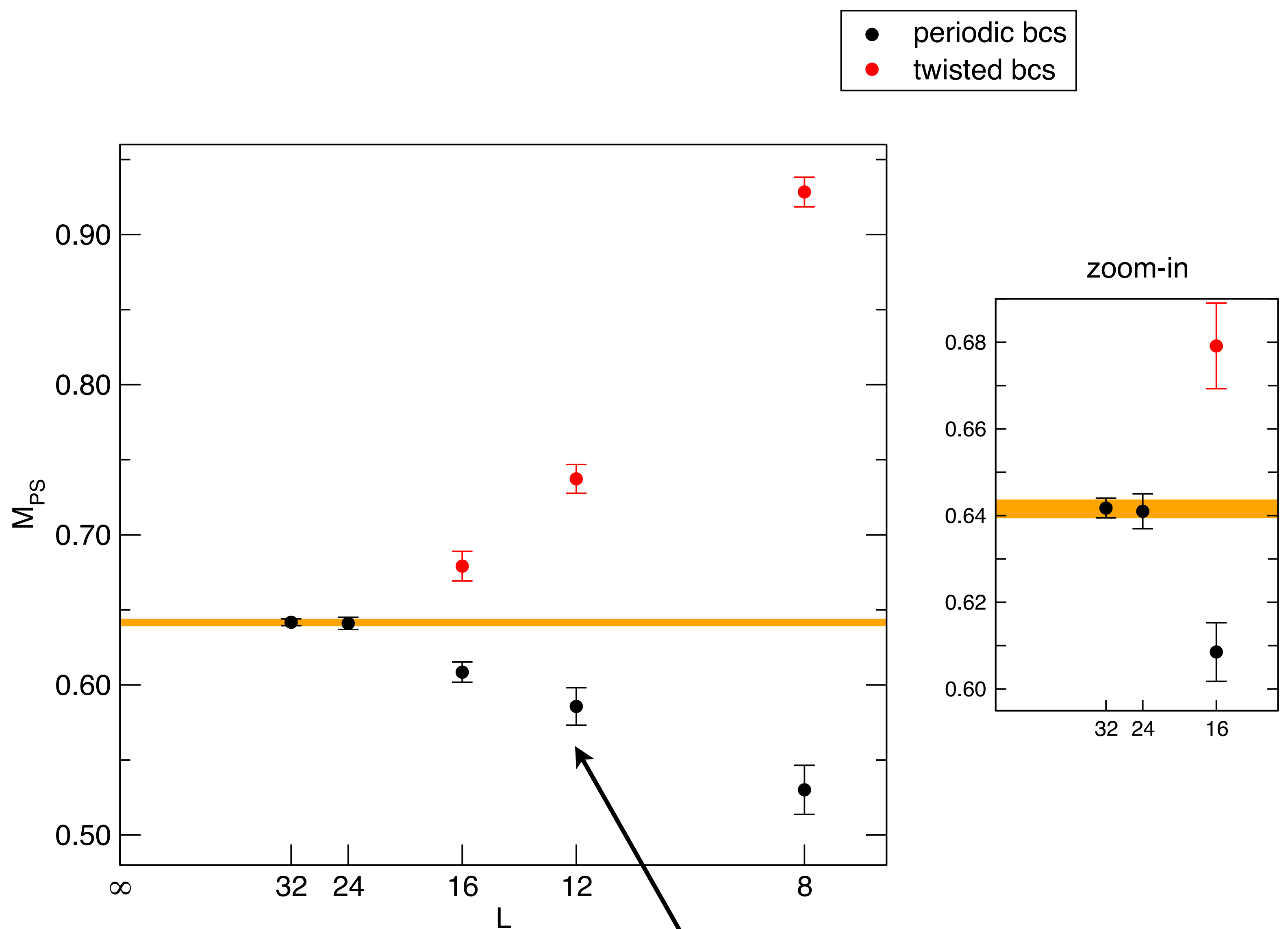


Spectrum



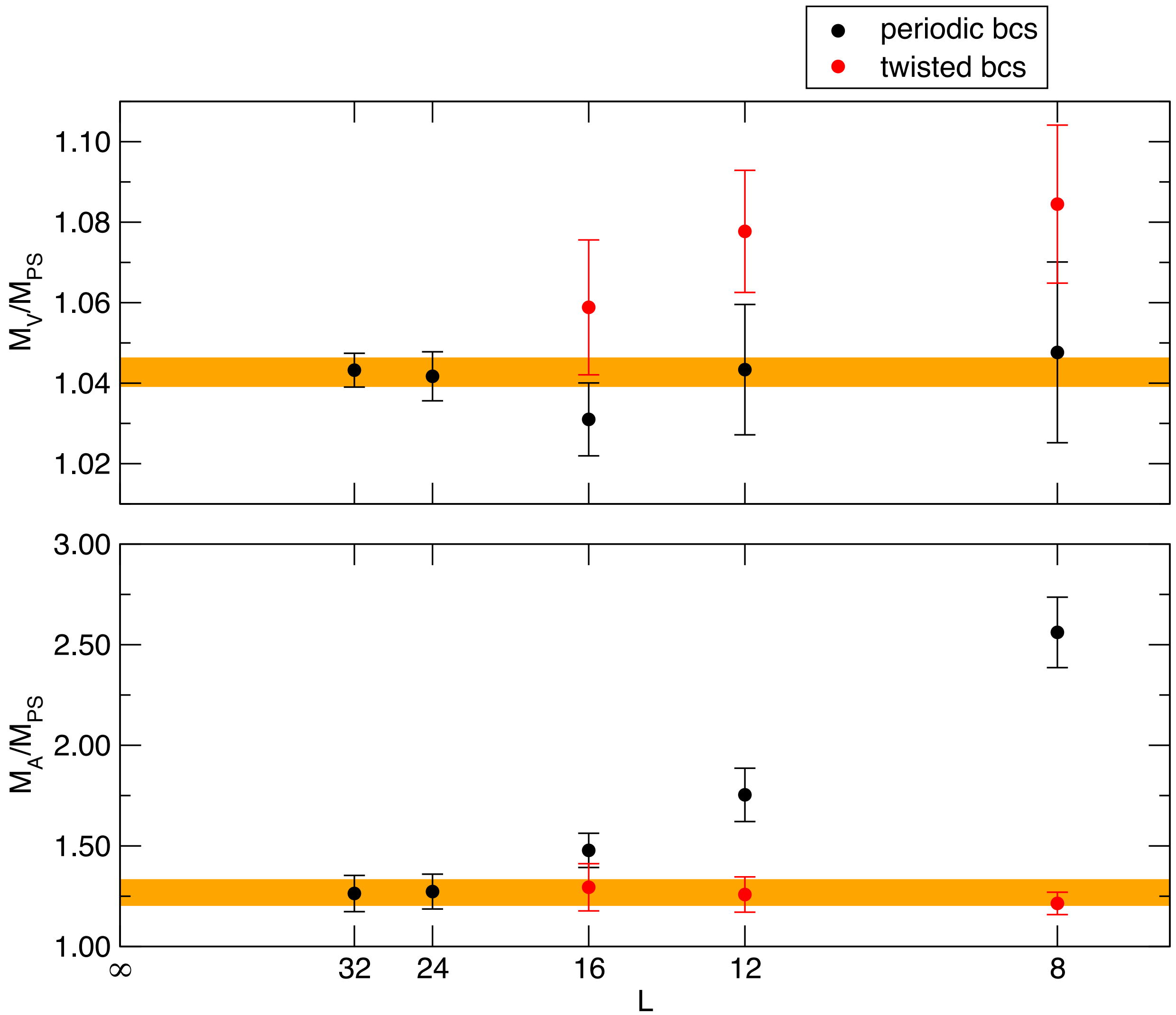
1.08
1.06
1.04
1.02
1.00
M_V/M_{PS}
$16x8^3$
$24x12^3$
$32x16^3$
$64x24^3$
10
9
8
7
6
5
4
$M_{PS}/\sigma^{1/2}$
largest volume
0.0
0.4
0.8
1.2
1.6
2.0
2.4
M_{PS}

Spectrum

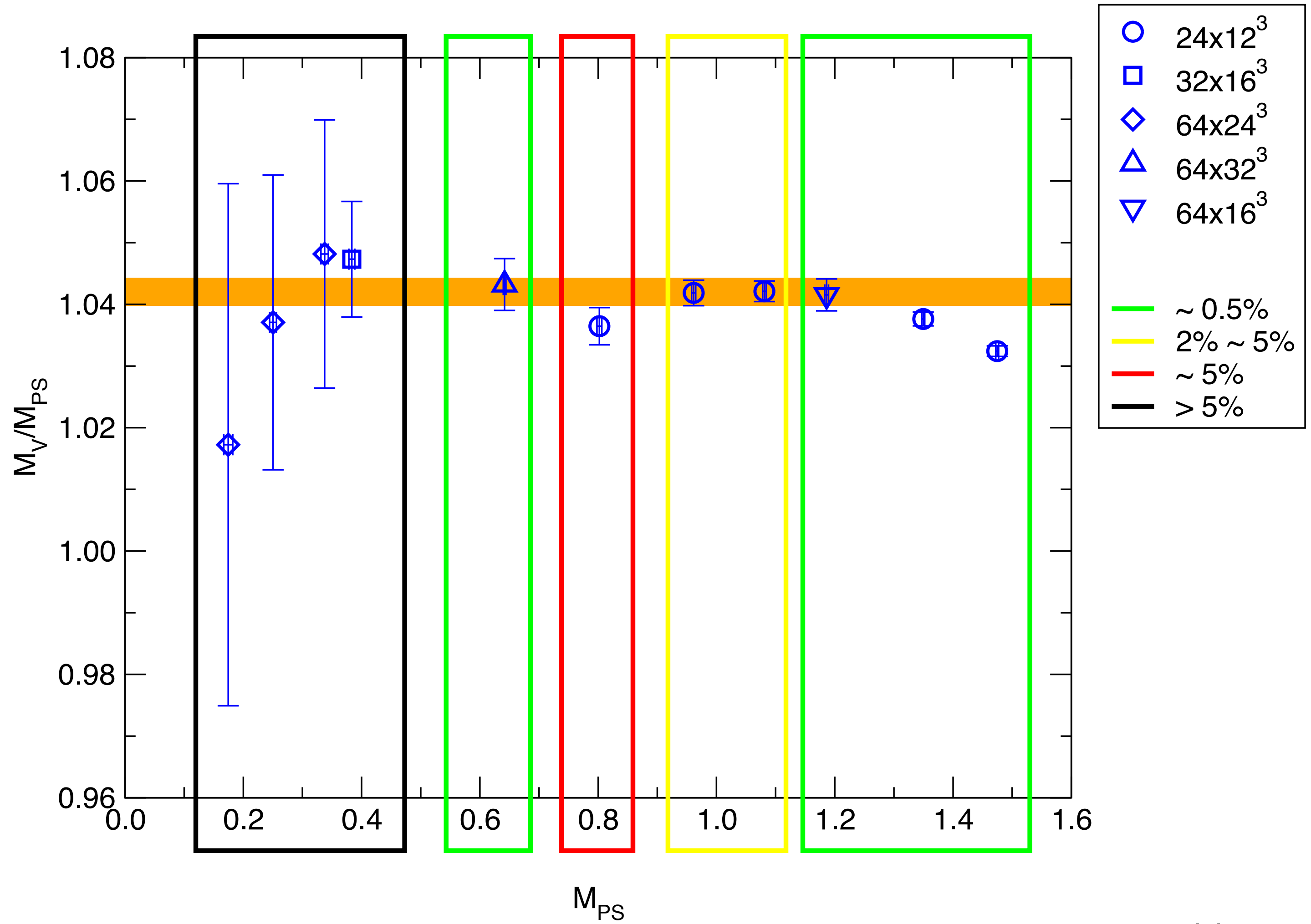


finite volume effects

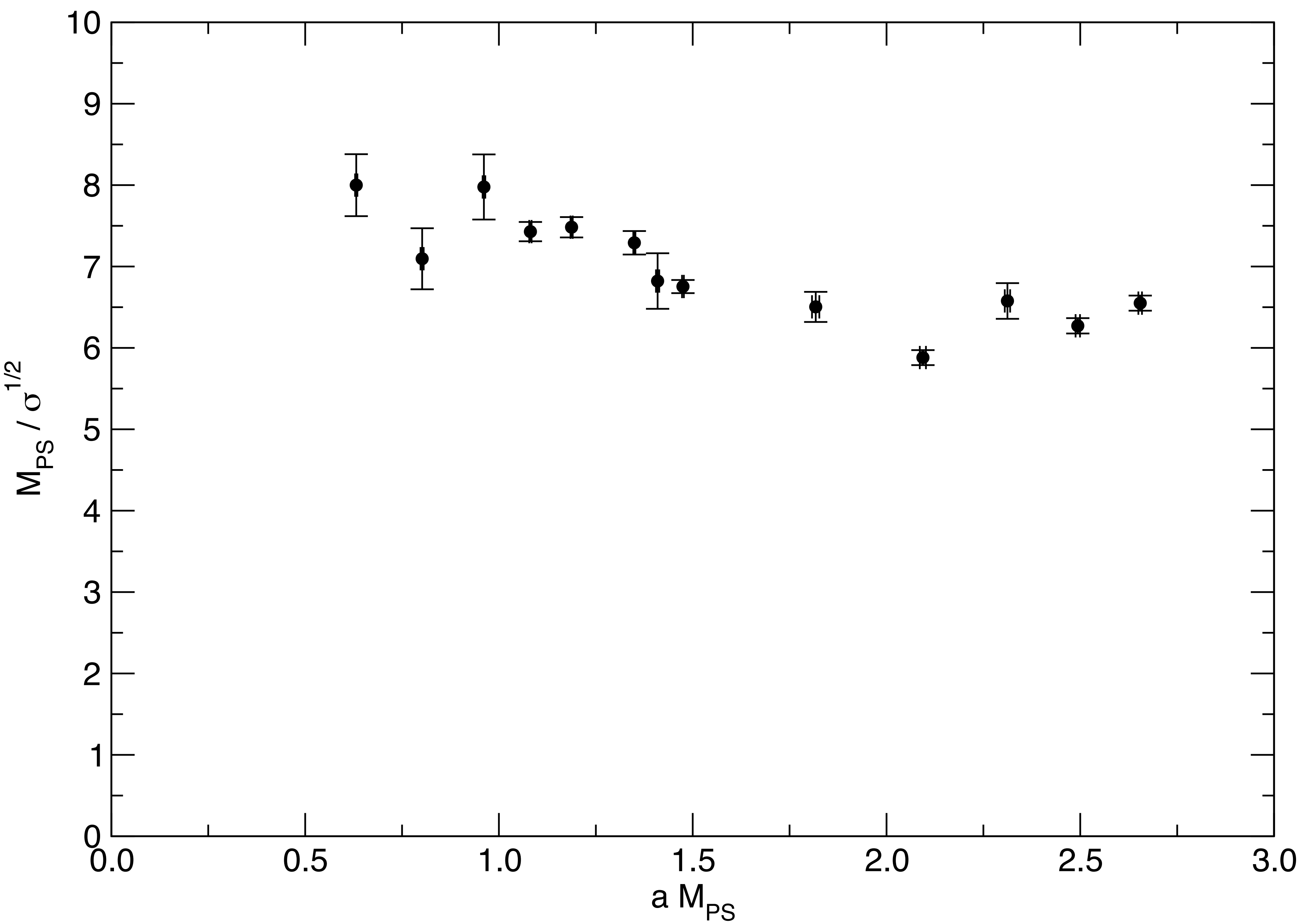
Spectrum



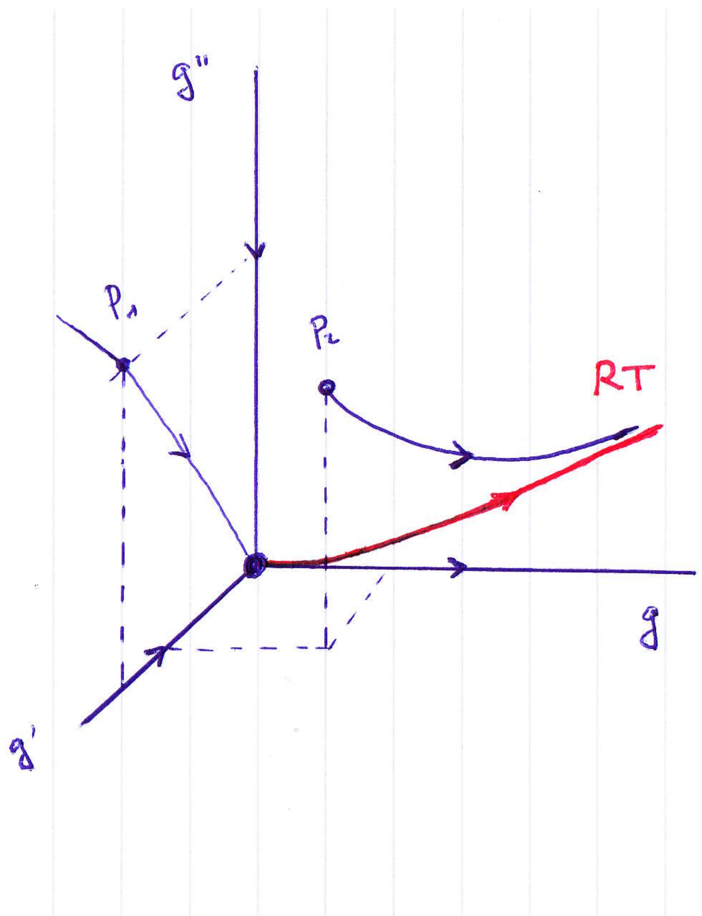
1% accuracy requires $M_{PS}L \simeq 15$



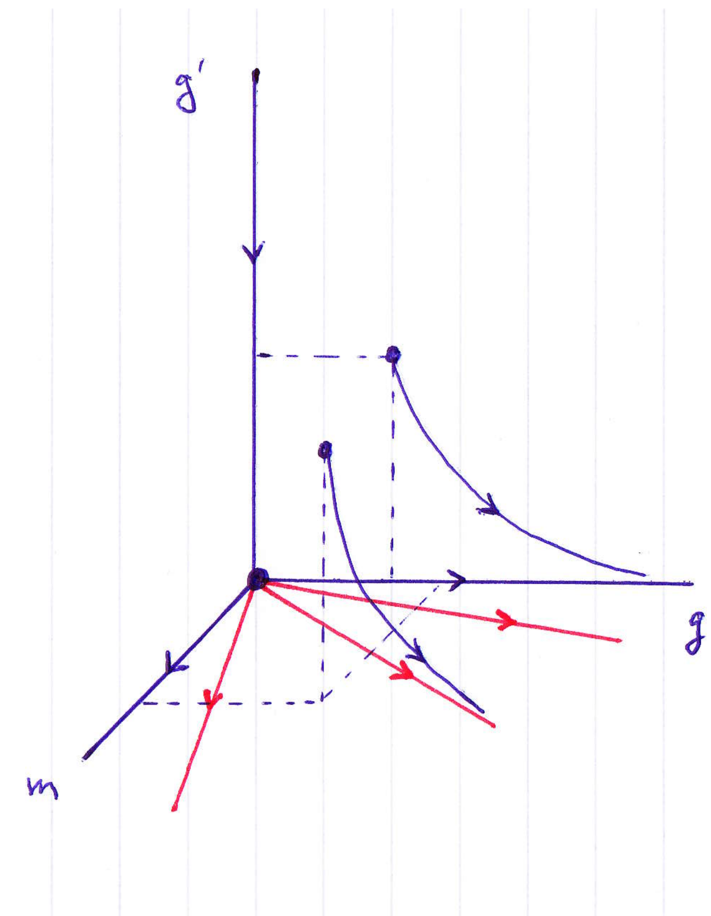
Spectrum



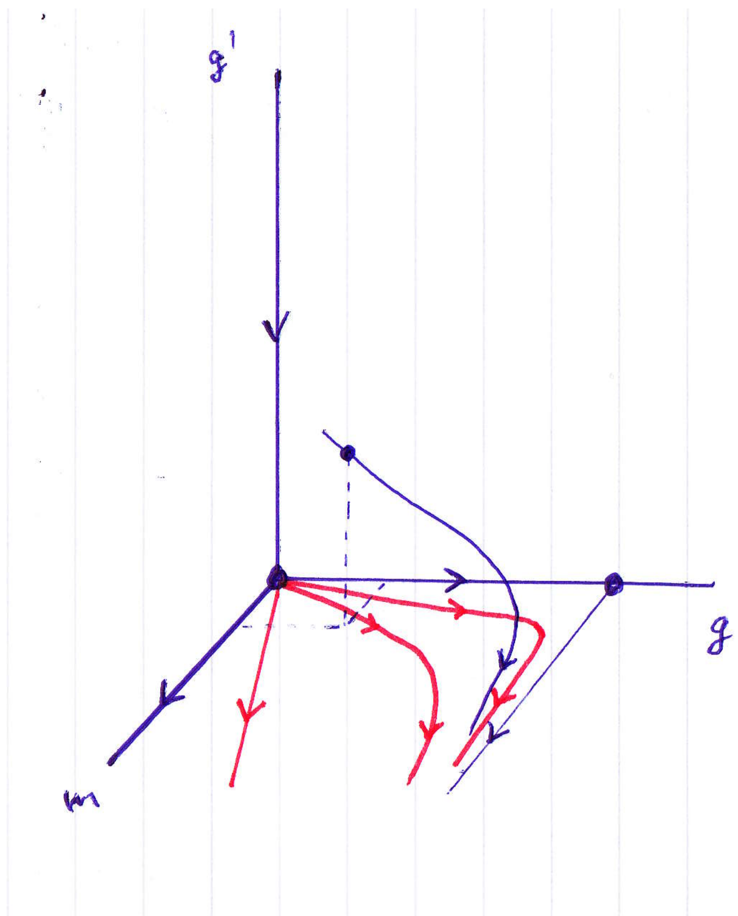
RG flows



pure gauge



QCD-like



IR conformal

Running in the SF

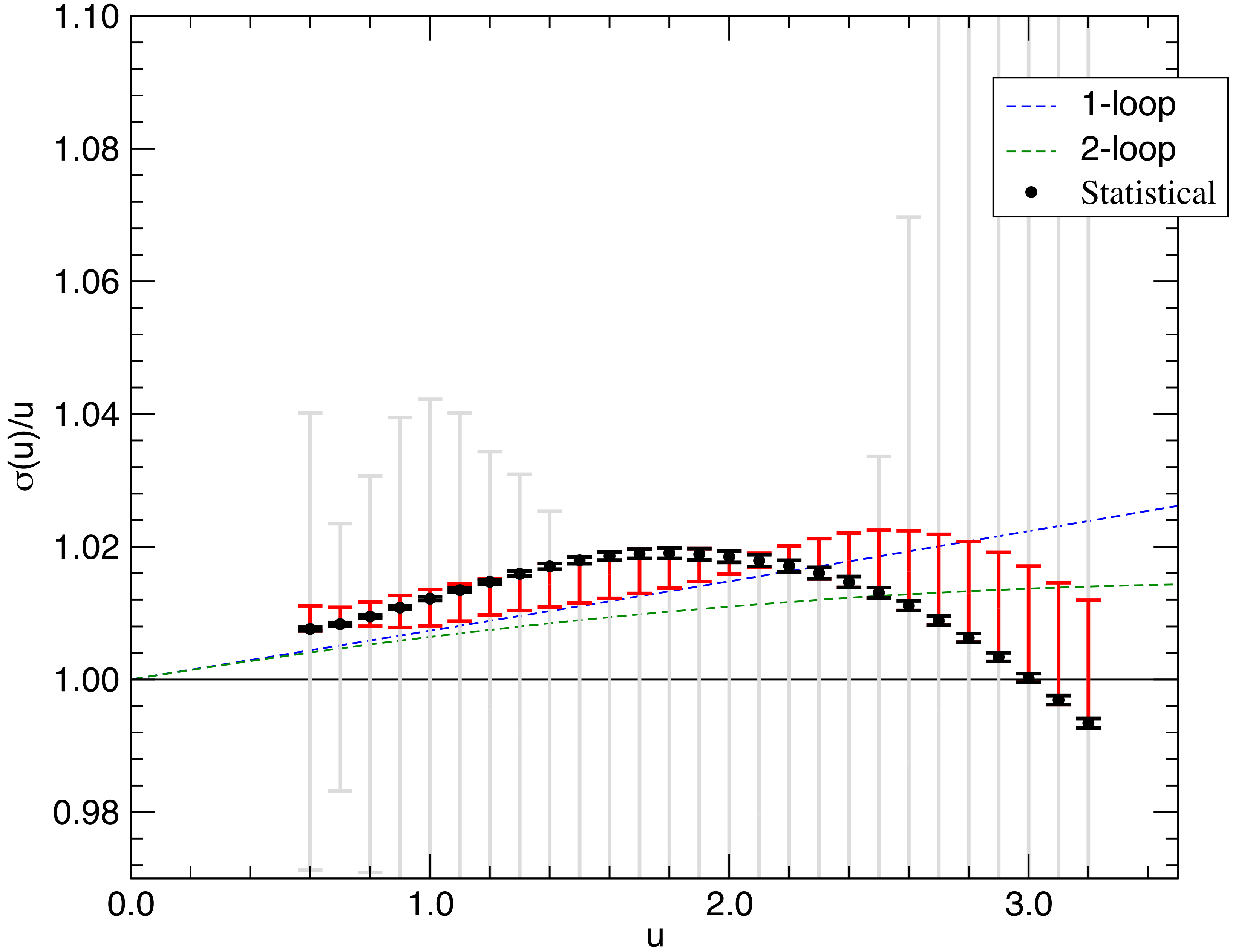
- Schrodinger functional is a NP renormalization scheme
- Renormalization scale is set by the volume of the box L
- Continuum limit is well defined: disentangle lattice artefacts from running
- Running of the coupling and the mass are encoded in the step-scaling functions:

$$\Sigma(u, s, a/L) = \bar{g}^2(g_0, sL/a)\Big|_{\bar{g}^2(g_0, L/a)=u}$$

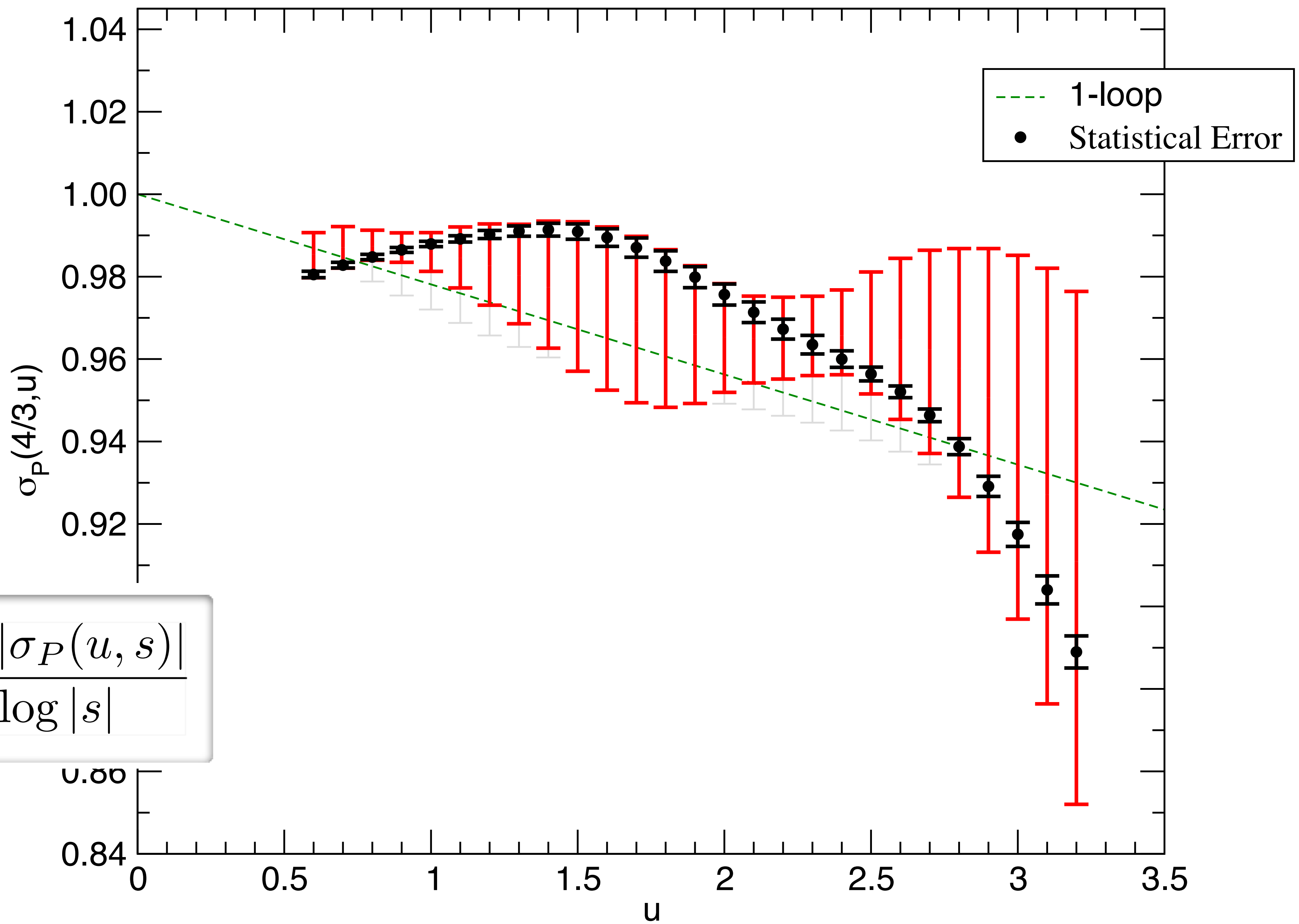
$$\Sigma_P(u, s, a/L) = \left.\frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)}\right|_{\bar{g}^2(L)=u}$$

"resolution"

Running of the coupling



Running of the mass

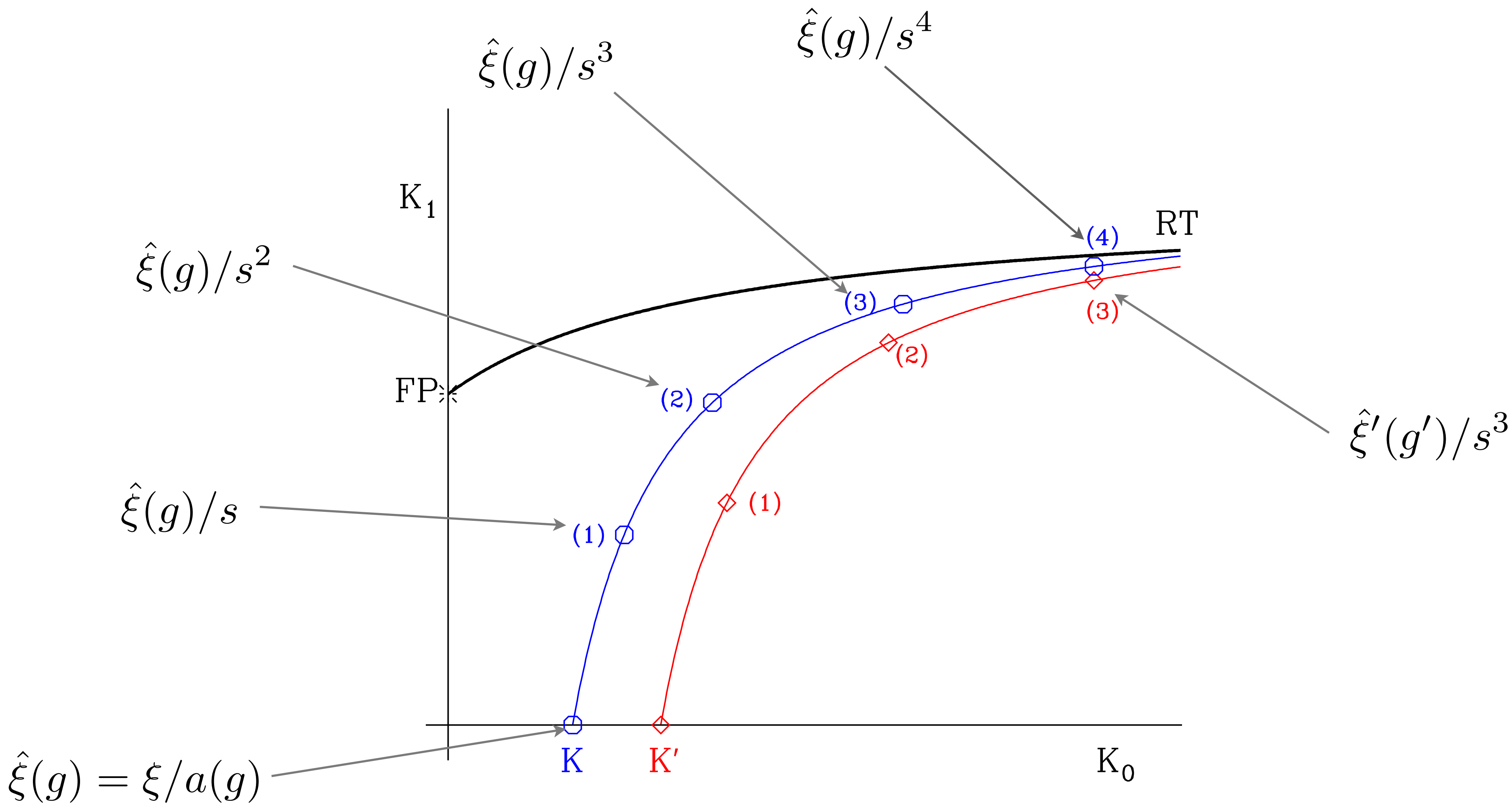


$$\hat{\gamma}(u) = -\frac{\log|\sigma_P(u,s)|}{\log|s|}$$

$$\gamma < 0.6$$

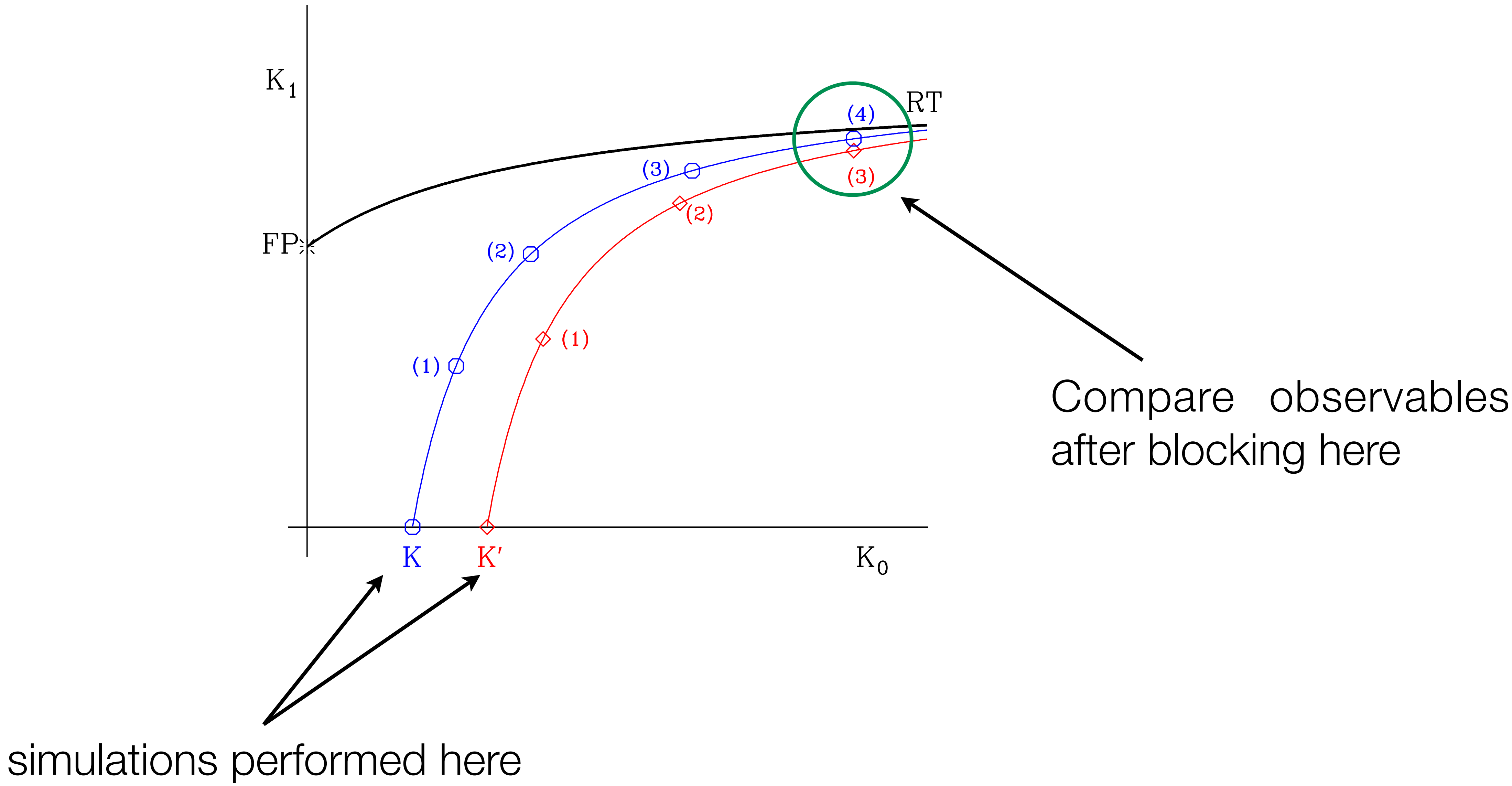
Two-lattice matching

Let us consider **first** for simplicity a theory that has only one relevant parameter flowing out from an UV fixed point; this is the common situation in **pure gauge theories**. In this case the RG trajectories converge towards a one-dimensional renormalized trajectory (RT).



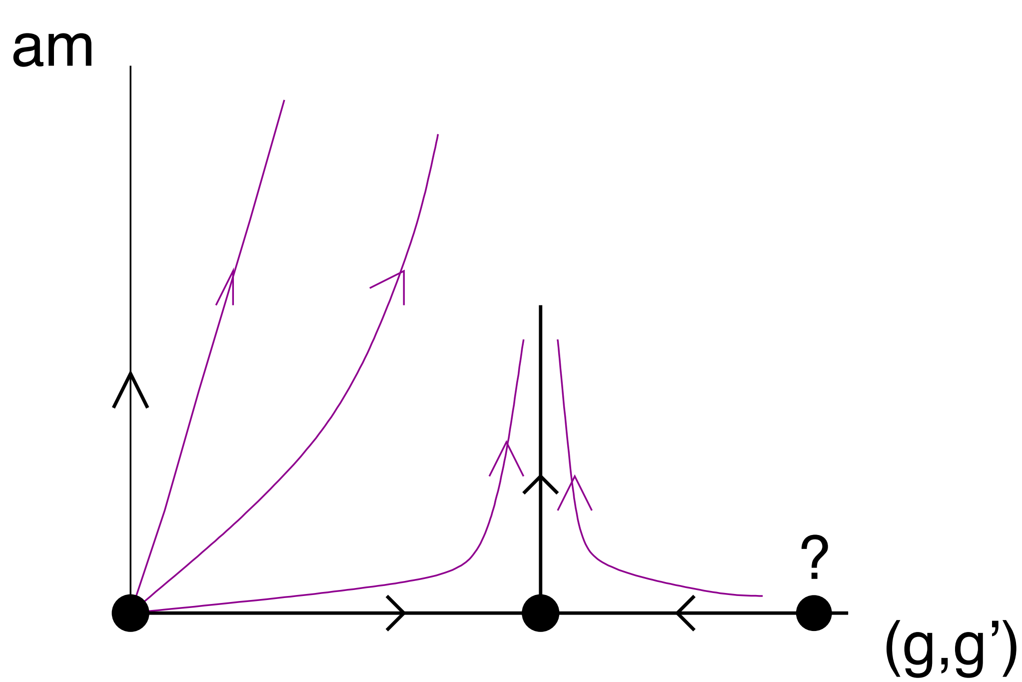
At each blocking step, the **lattice** correlation length is divided by s.

Two-lattice matching



IRFP using 2lat matching

Assume there is only one relevant direction: mass m



Neglect the running of the gauge coupling

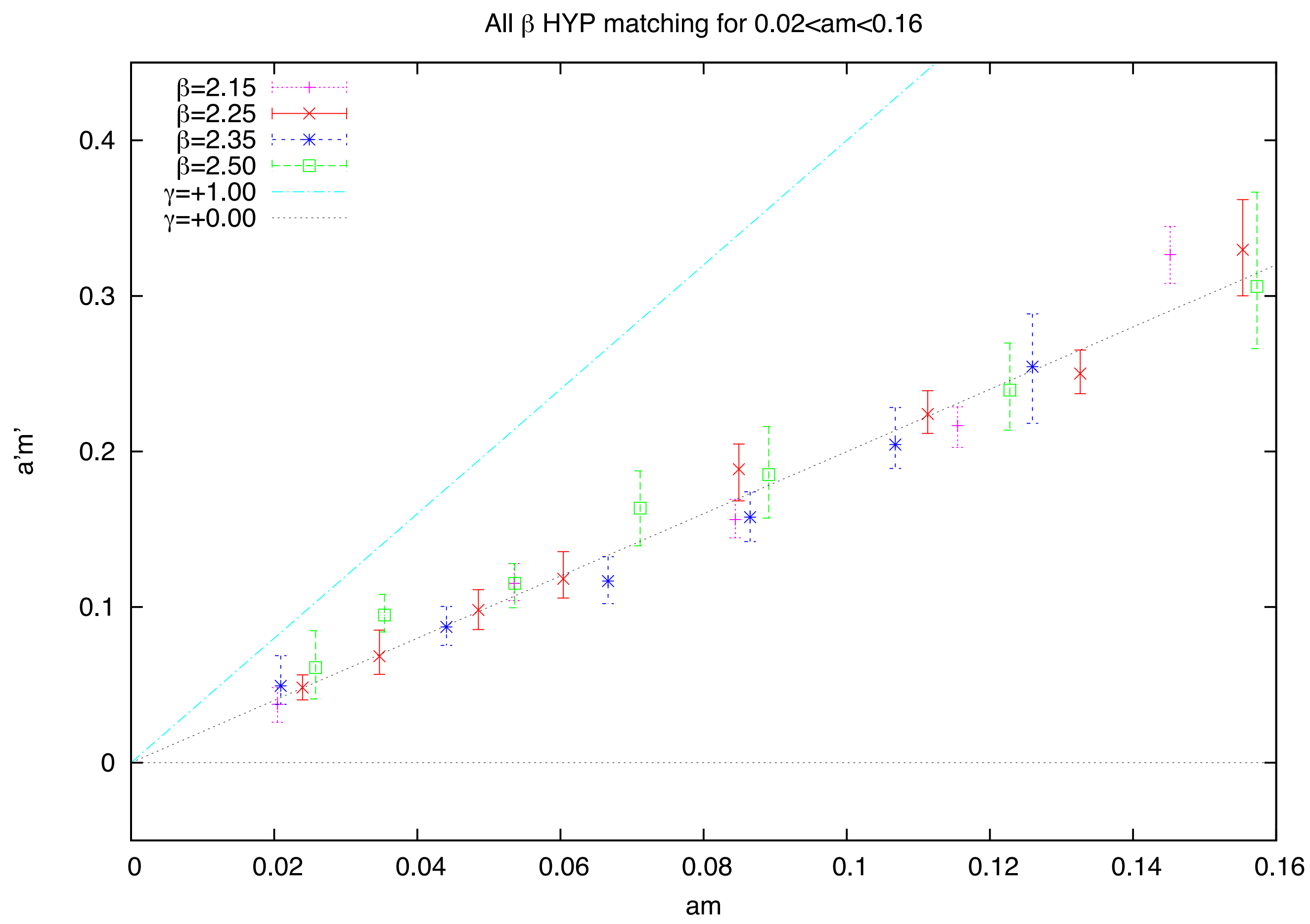
Two-lattice matching to compute the running of the mass near the fixed point.

$$m, m' \text{ such that } a(m') = sa(m)$$

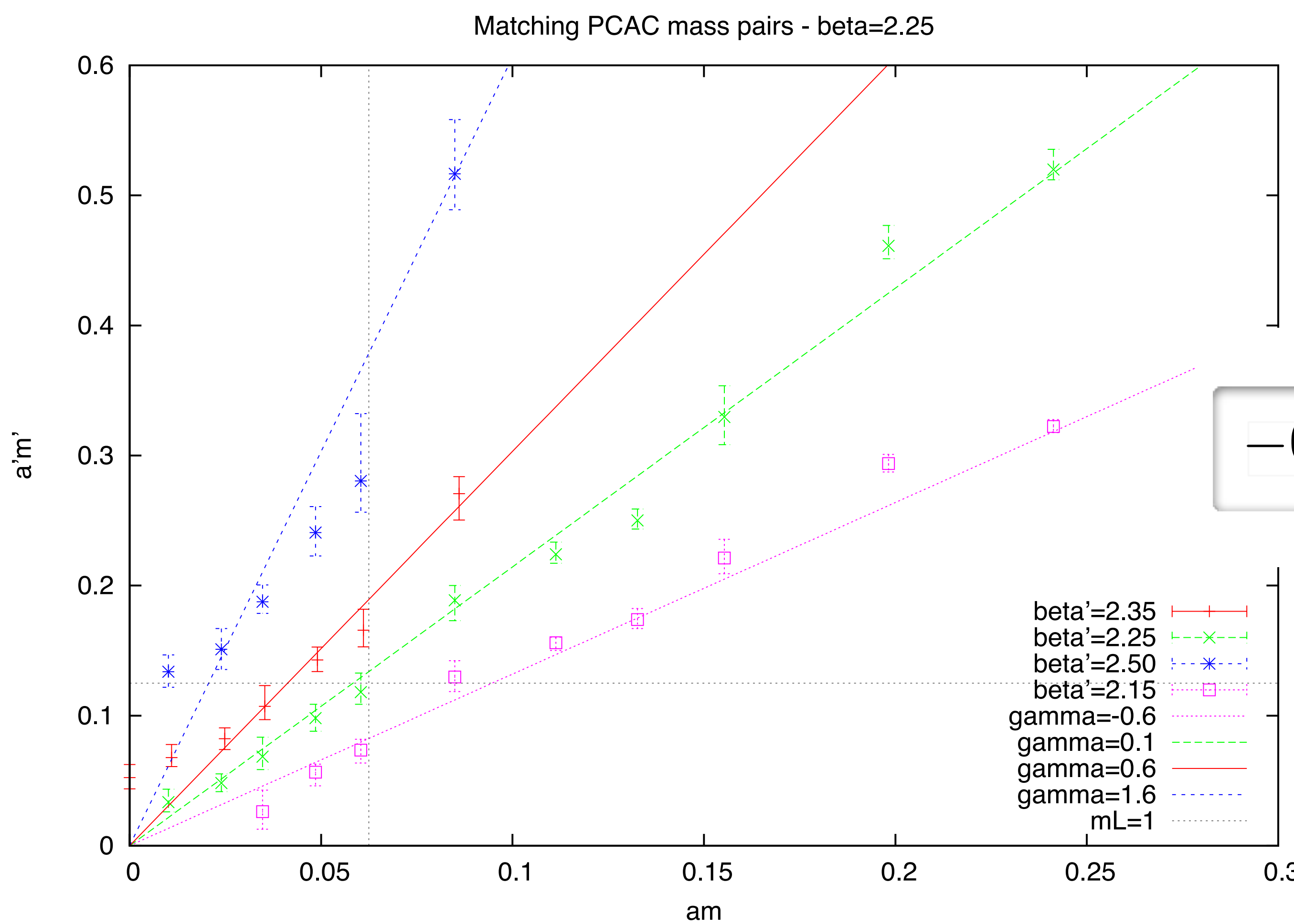
$$\frac{am'}{am} = 2^{1+\gamma_*}$$

Flow of the bare mass

Matching fermion masses at constant coupling:



Running of the coupling?



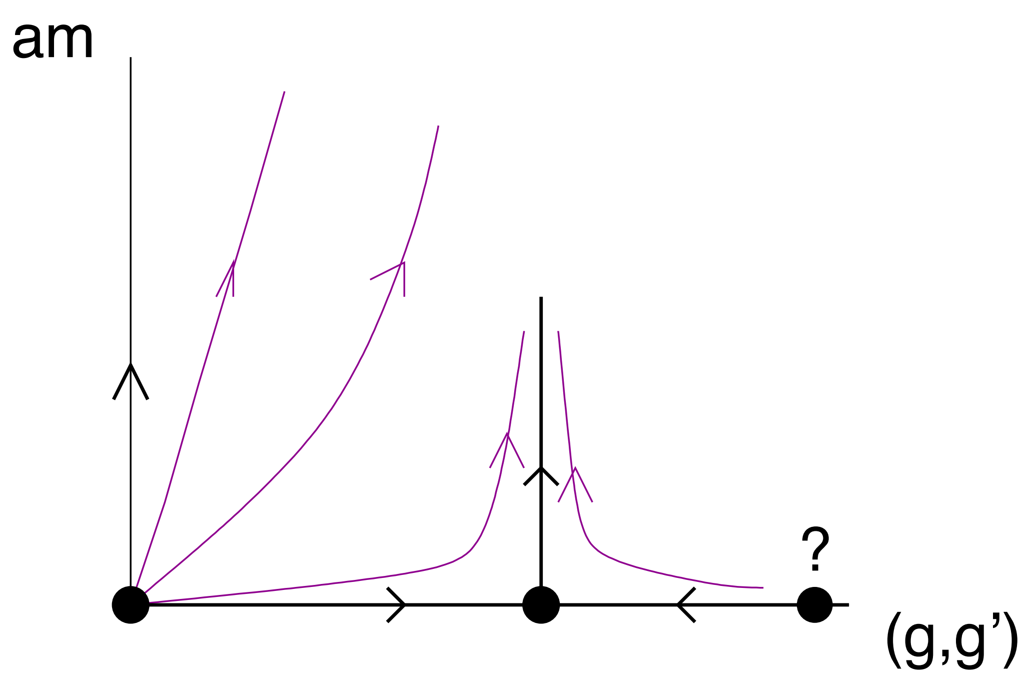
$$-0.6 < \gamma < 0.6$$

More observables to get a more constrained matching

Larger lattices

In order to really probe the conformal dynamics $m \gg 1/L$

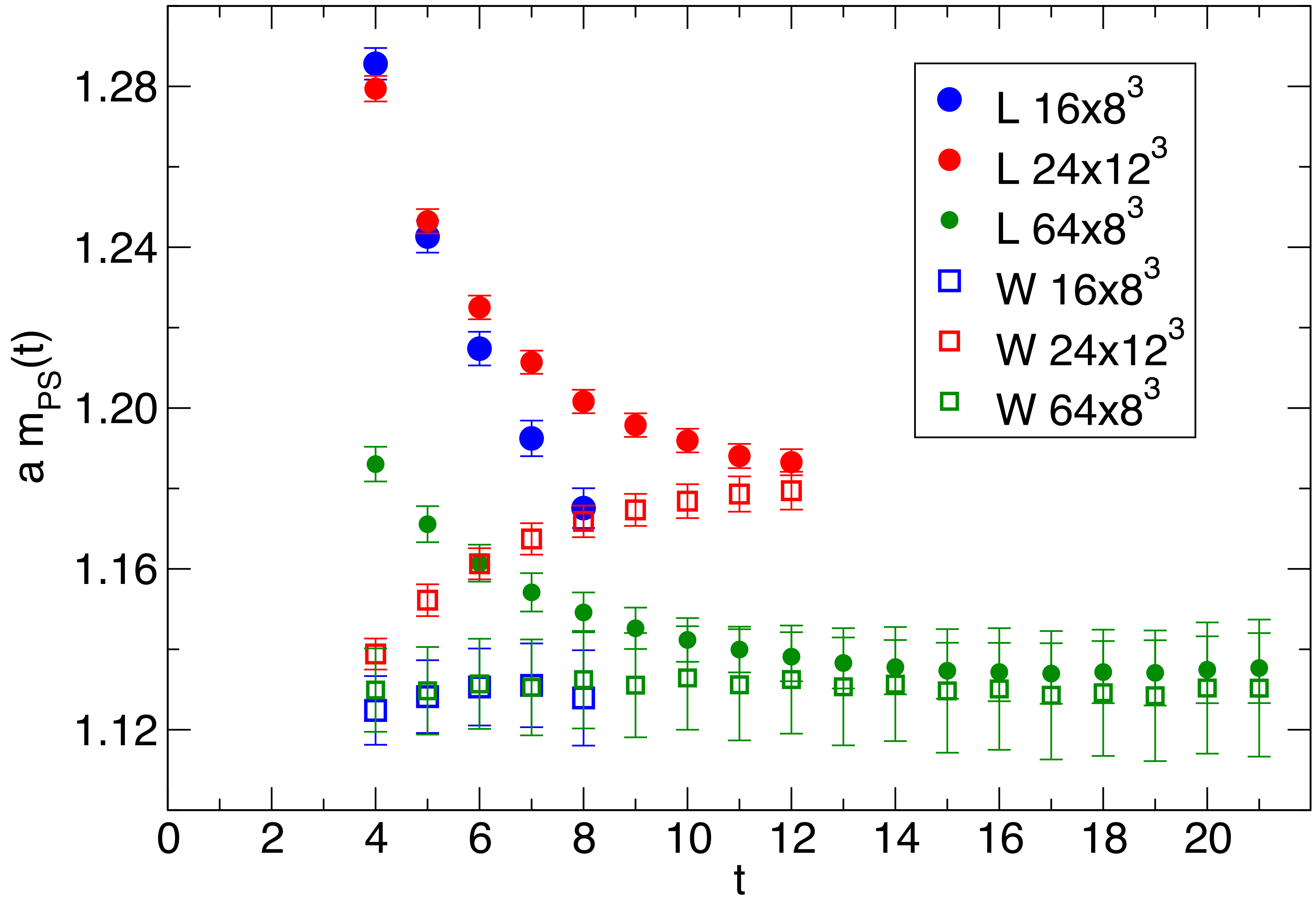
Large volumes needed to reach the small mass regime



Conclusions

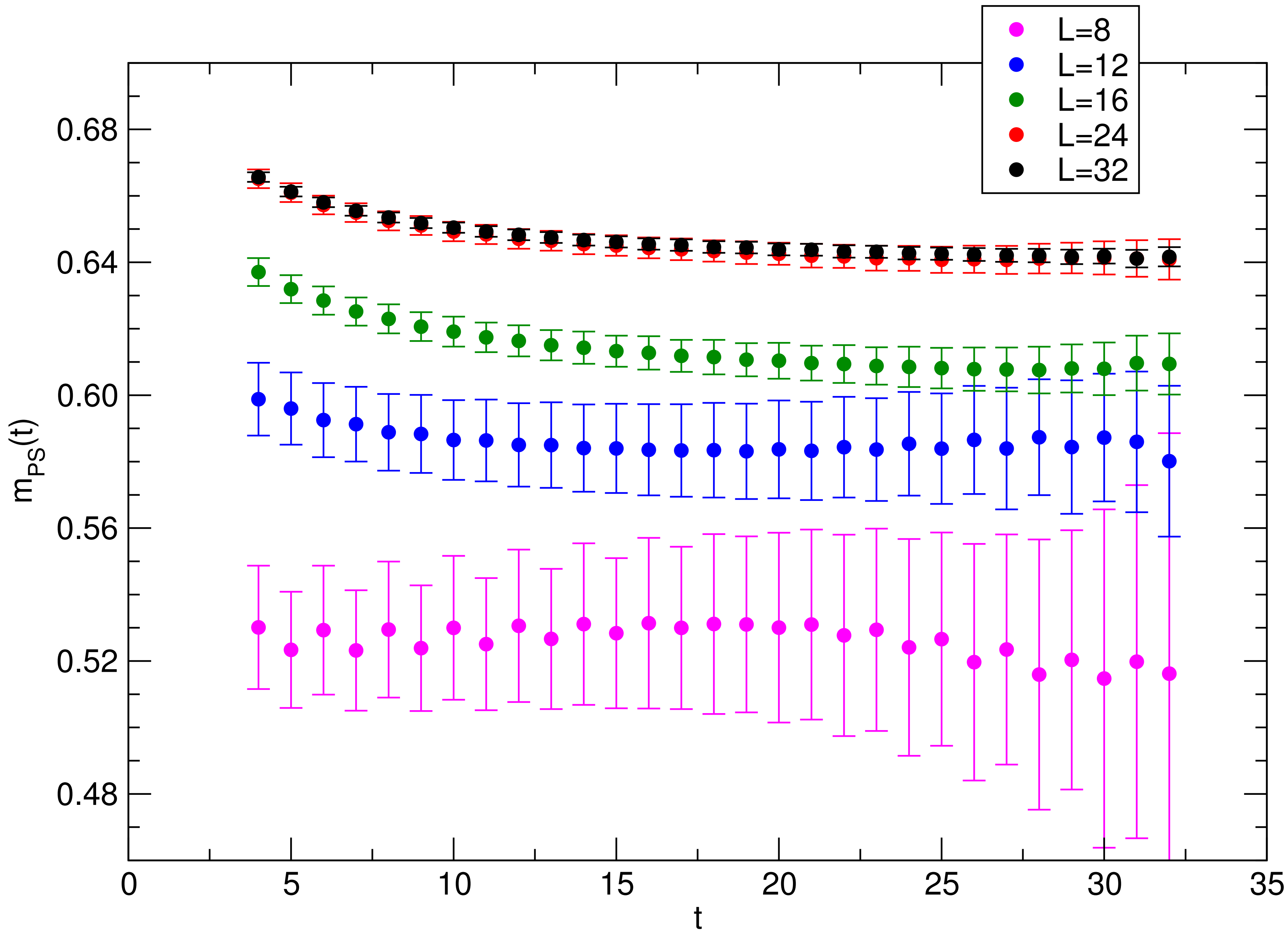
- Lattice simulations yield first principle results on the NP dynamics of strongly interacting theories
- Studies so far have focused on understanding the phase diagram (IRFP)
- Quantitative results on the spectrum and the anomalous dimensions
- Lattice input to phenomenology
- We need to ask the right questions!!!

Spectrum



1.28
1.24
1.20
1.16
1.12
a $m_{PS}(t)$
0
2
4
6
8
10
12
14
16
18
20
t
L 16×8^3
L 24×12^3
L 64×8^3
W 16×8^3
W 24×12^3
W 64×8^3

Spectrum



L=8
L=12
L=16
L=24
L=32
0.68
0.64
0.60
0.56
0.52
0.48
$m_{PS}(t)$
0
5
10
15
20
25
30
35
t