Gauge theories with 2, 3, 4 colors and 2 fermions in the two-index symmetric representation

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Outline

- Phenomenology of nonperturbative BSM (what is needed to evade precision electroweak) well developed
- First-principles understanding of potential models is much more uncertain
- Lattice might help. We measure beta function, mass anomalous dimension γ_m using lattice background field methods
- Anticipating the conclusion: we observe $\beta(g^2) = 0$,, small γ_m with $N_c = 2$, 3, 4

My collaborators: Y. Shamir and B. Svetitsky, Tel Aviv

Dramatis Personae

I'll define beta function with respect to inverse coupling

$$\tilde{\beta}(1/g^2) \equiv \frac{d(1/g^2)}{d\log L} = 2\frac{\beta(g^2)}{g^4} = 2\frac{b_1}{16\pi^2} + \ldots. \tag{1}$$

Perturbatively,

$$b_1 = -\frac{11}{3}N_c + \frac{4}{3}N_f T(R) \tag{2}$$

Recall, for $L \to sL$

$$\frac{1}{g^2(s)} = \frac{2b_1}{16\pi^2}\log s + \ldots \tag{3}$$

Mass anomalous dimension defined as

$$\mu\frac{dm(\mu)}{d\mu} = -\gamma_m(g^2)m(\mu). \tag{4}$$

In lowest order in perturbation theory,

$$\gamma_m = \frac{6C_2(R)}{16\pi^2}g^2. \tag{5}$$

$$\langle\bar{\psi}\psi\rangle\big|_{ETC} = \langle\bar{\psi}\psi\rangle\big|_{TC}\exp\left[\int_{\Lambda_{TC}}^{\Lambda_{ETC}}\frac{d\mu}{\mu}\gamma_m\left(g^2(\mu)\right)\right]. \tag{6}$$

Strategies for studying candidate theories on the lattice – the big picture

We compute a running coupling constant (typically via Schrodinger functional)

- Fix bare parameters, vary simulation volume (box size L) at fixed cutoff
- Coupling a derived quantity, running given by its variation in L

Contrast with attempt to do "usual" lattice calculations (spectroscopic observables)

- If "classical TC," expect to see chiral symmetry breaking – if so, compute m_H, f_π, $\langle\bar{\psi}\psi\rangle/f_\pi^3$
- Finite simulation box size L can cause problems if it's too small

"Inside the conformal window," physics is quite different

- Quark mass is the "relevant perturbation." All masses scale as $M(m_q) \sim m_q^{1/ym}$
- You are NEVER in infinite volume ($1/L$ is a relevant perturbation)
- Gauge coupling irrelevant – spectroscopy nearly g^2 - independent as $m_q \to 0$
- $m_q \neq 0$ system just some particle system, with explicit symmetry breaking

Slow running dominates the simulation

- Gauge coupling always runs slowly – b_1 is small to start with

$$\frac{1}{g^2(s)} = \frac{2b_1}{16\pi^2}\log s + \ldots \tag{7}$$

(example: $b_1 = 9$ for $SU(3)$ and $N_f = 3$, $b_1 = 3$ for $SU(3)$ and $N_f = 12$)

- At any set of bare params, maximum s is restricted – $1/g^2(s)$ never changes much
- This amounts to "effective conformality" (issues for spectroscopic observables!)
- Slow running means strong coupling at long distance implies strong coupling at short distance
  - Lattice artifacts can be important
  - Need "improved actions" – but hard to guess, how to create them

The Schrödinger Functional

- Goal: Nonperturbative def'n of α, which heals to PT – used to predict Λ in QCD
- Designed for (and used mostly for) asymptotically free theories
  - $d = 2$ O(N) $\sigma-$ model
  - $d = 4$ pure YM, QCD
- Basically background field method for lattice in box of size L^4
- Boundary conditions for fields depend on parameter η

$$Z = \int_{\eta-boundaries} [d\phi] \exp(-\frac{1}{g^2} S(\phi)) \tag{8}$$

- $\Gamma_{cl} = -\log Z_{cl} = g^{-2} S^{cl}$
- Promote this to $\Gamma = -\log Z = g(L)^{-2} S^{cl}$
- Classically, $\frac{\partial \Gamma}{\partial \eta}|_{\eta=0} = \frac{K}{g^2}$
- $\langle \frac{\partial \Gamma}{\partial \eta}|_{\eta=0} \rangle =$ messy lattice operator, whose expectation value $\equiv \frac{K}{g^2(L)}$

The Schrödinger Functional – Running

Simulate at same bare parameters on volumes L_0 and sL_0, compute the change in the coupling

Interpret as integrated beta function

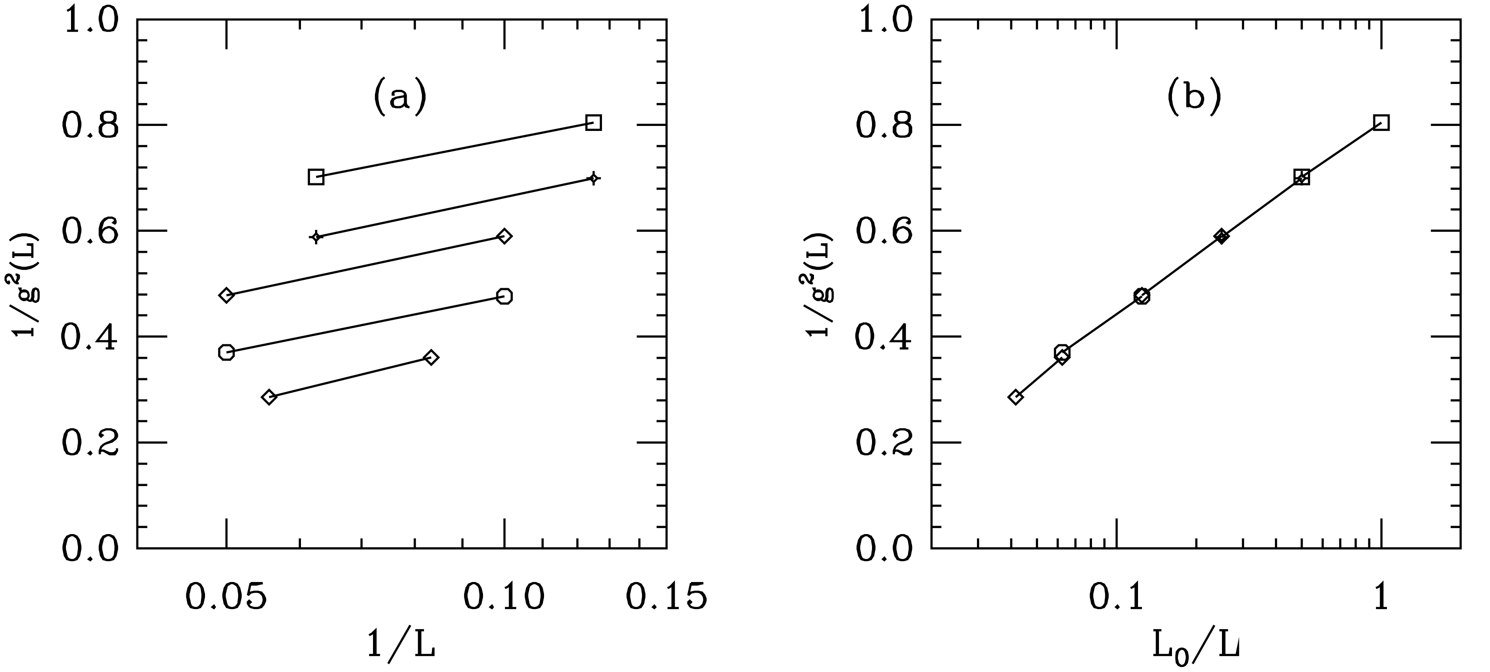
$$\beta(g) = -L\frac{dg^2}{dL}, \tag{9}$$

$$-\int_{L_0}^{sL_0} \frac{dL}{L} = \int_{g^2(L_0)}^{g^2(sL_0)} \frac{dg^2}{\beta(g^2)} \equiv \int_{u}^{\sigma(s,u)} \frac{dv}{\beta(v)}, \tag{10}$$

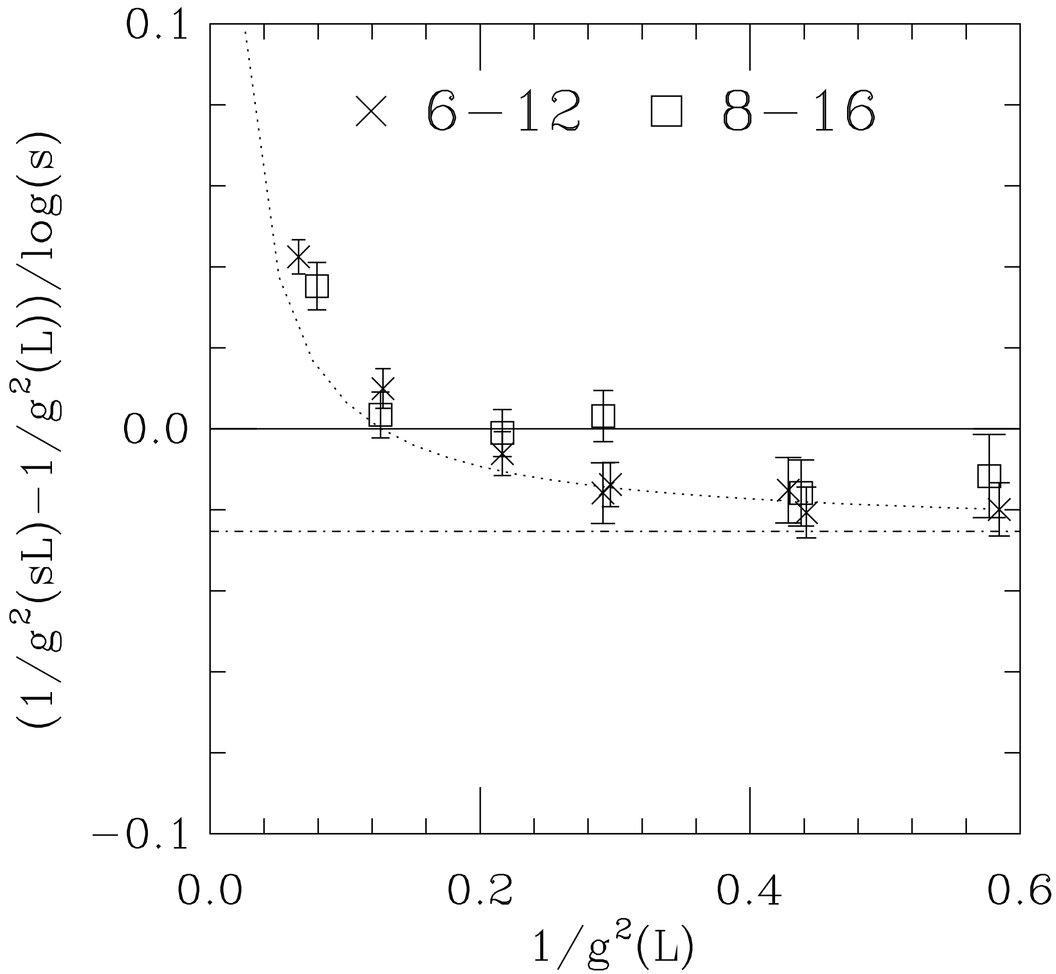
"QCD - like" analysis flowchart

1) Compute $1/g^2(sL) - 1/g^2(L)$ at several L's, interpolate to $a/L \to 0$.
2) "Daisy chain" different $g(L)$'s, $L \to sL \to s^2L \to \dots$ for running over large range of scales
3) Fix overall scale from energy observable at one bare coupling
4) Match to $\overline{MS}$ deep in weak coupling
5) For QCD, predict $\alpha_s(M_Z)$ or $\Lambda = 245$ MeV in terms of a low energy observable

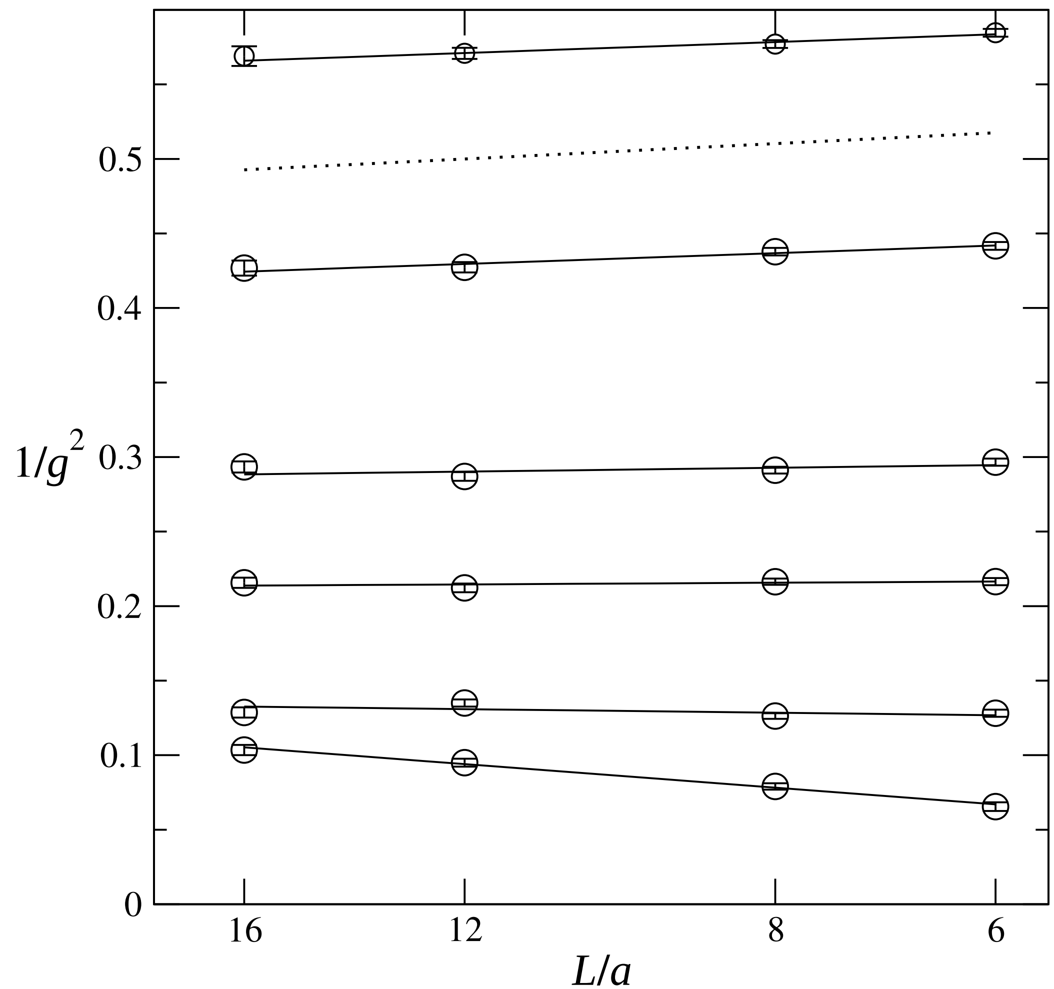
Aside: "Daisy chain"



Quenched QCD (alpha collab): connecting running couplings from many short scales to one long scale



QCD - like analysis, step 1: $SU(2)$ SF running from 6-12 and 8-16 shows slow running –WITH a zero



Caption 1: SF coupling for $SU(2)$ with 2 adjoints – can't daisy chain couplings (without extreme curve fitting)

Caption 2: dotted line is AF slope – constant slope is the beta function – note the sign change!

Slow running is almost no running – I

The "QCD - like" analysis is unnecessarily hard!

Taylor expand the slowly running beta function, call $1/g^2(s) = u(s)$

$$\tilde{\beta}(u) = \tilde{\beta}(u(1)) + B_1(u - u_1) \tag{11}$$

and integrate from scale 1 to scale s

$$u(s) - u_1 = \tilde{\beta}(u_1)\frac{\exp(B_1 \log s) - 1}{B_1} \ . \tag{12}$$

If $B_1 \log s$ is small, rescaled DBF is just the beta function,

$$\frac{u(s) - u(1)}{\log s} = \tilde{\beta}(u(1)) \tag{13}$$

Or: the beta function is the slope the $1/g^2(L)$ vs $\log L$ line

Slow running is almost no running – II

- An IRFP theory has one relevant coupling, m_q, criticality at $m_q \longrightarrow 0$
- g^2 is irrelevant, even location of g^{*2} is RGT dependent

This implies correlation length diverges as

$$\xi \sim m_q^{-1/ym} \tag{14}$$

or

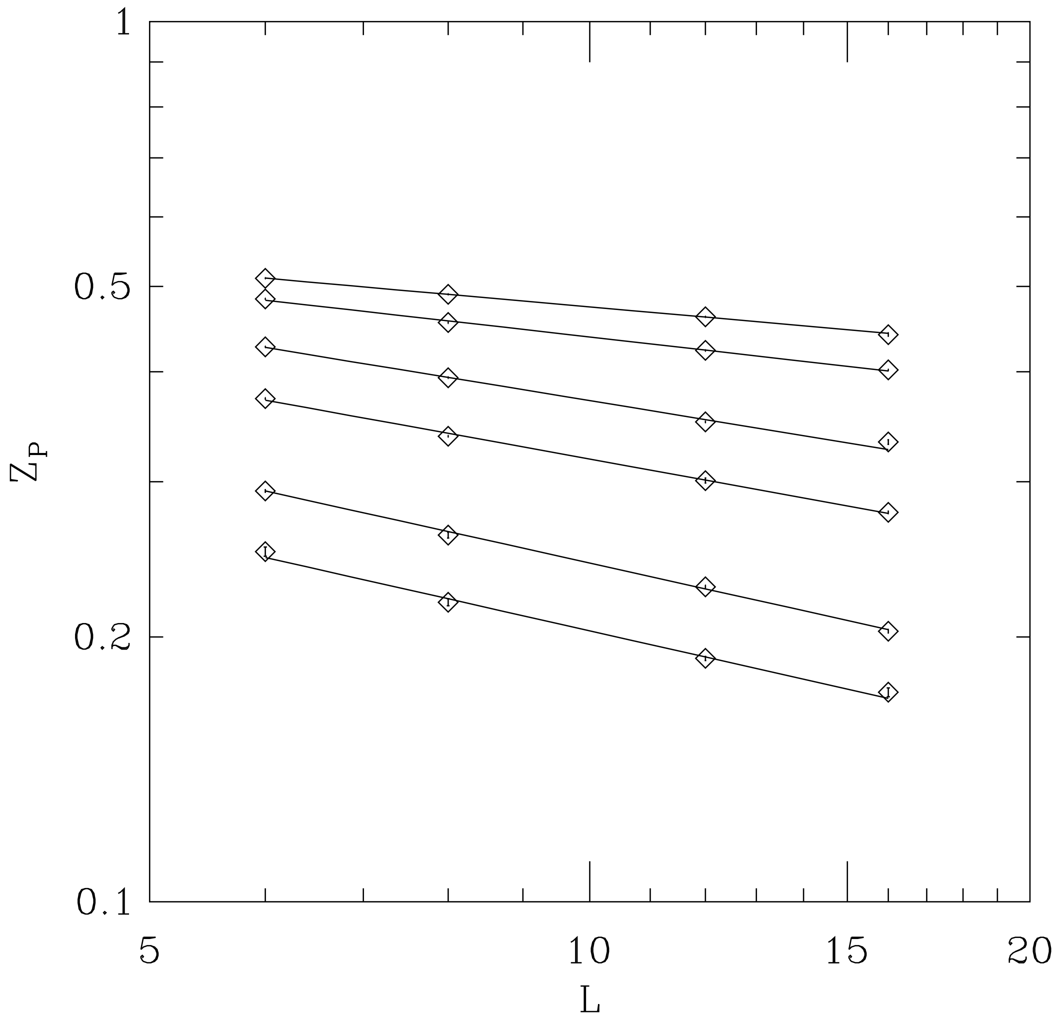
$$M^{ym} \propto m_q \tag{15}$$

- This could be absolutely true (in a real IRFP theory, $g \longrightarrow g^*$ so it's irrelevant)
- This is approximately true if g runs slowly (which, as you just saw, it does)

Evolution over small scales gives power laws,

$$\begin{aligned} \Gamma(sp) &= s^{dn}\Gamma(p) \exp \int_1^s \frac{dt}{t}\gamma(g(t)) \\ &\simeq s^{dn}\Gamma(p) s^{\gamma(g(s))} \end{aligned} \tag{16}$$

Slow running is almost no running – II



- Pseudoscalar renormalization constant for $SU(2)$, $N_f = 2$ adjoints
- log-log slope gives γ_m for each bare parameter set (corresponding to different g^2's)
- This is NOT $\gamma_m(g_*^2)$ since the gauge coupling runs so slowly

A lattice artifact with Wilson type fermions

With Wilson fermions, the notion of "zero quark mass" is a derived concept

- Interactions shift quark mass
- Zero quark mass (still lattice regulated, at scale a) given from Axial Ward Identity,

$$\partial_t \sum_x \langle A_0(x,t) X(0) \rangle = 2m_q \sum_x \langle P(x,t) X(0) \rangle \tag{17}$$

As we went to more and more fermion DoF's, we hit a "wall"

- For a few fermion DoF's, strong coupling limit is "normal"
  - κ_c exists, where $m_q = 0$
  - Confinement, chiral symmetry breaking for all $m_q > 0$
- But with many fermion DoF's (large reps, or large N_f fundamentals)
  - There's a line of first order transitions extending out from $\beta(= 2N/g_0^2) = 0$ to some β_e
  - Along that line the AWI quark mass jumps discontinuously from $+$ to $-$
  - κ_c disappears – m_q never zero – nowhere is the system massless
  - But our calculation needs to be at $m_q = 0$

The cure – change the lattice action

Why not? all lattice actions are pure invention, anyway – universality is what counts

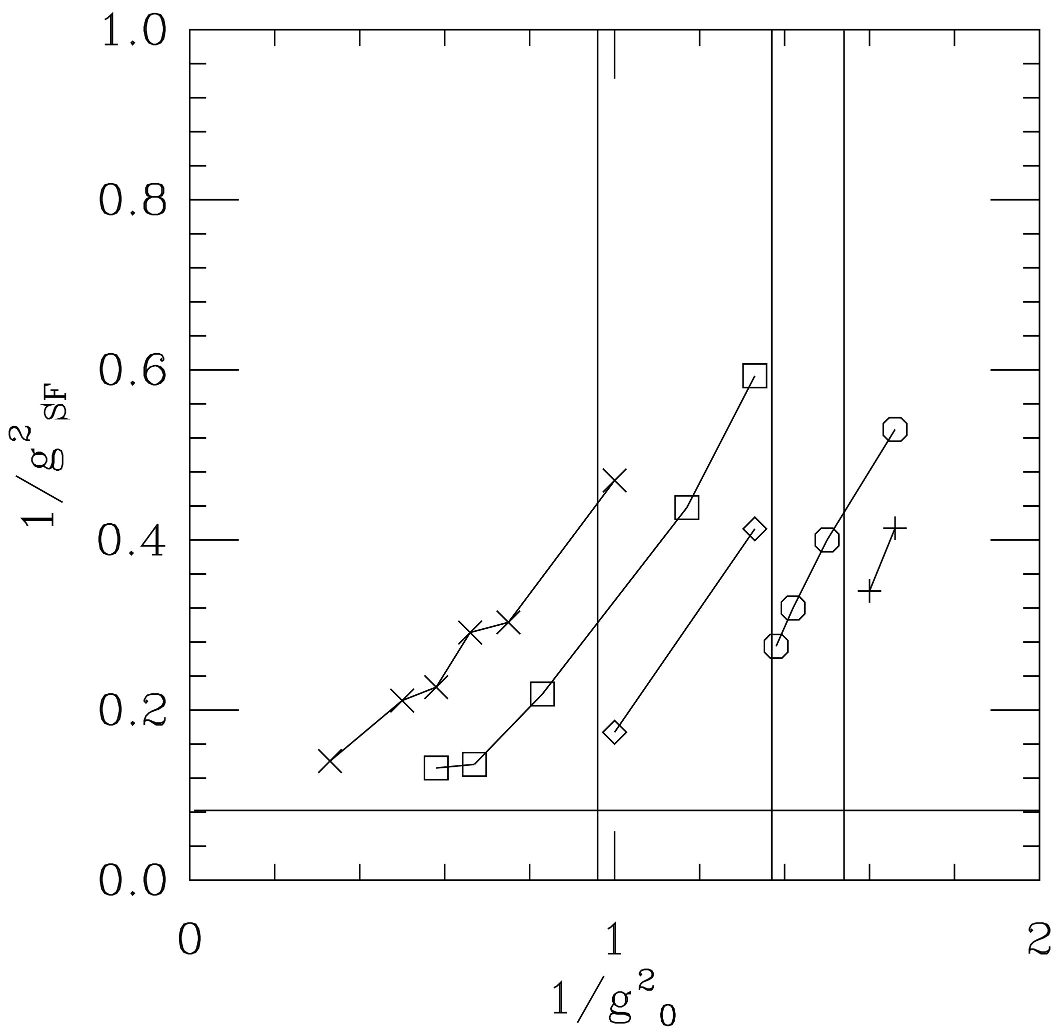
Pause to go technical... non-lattice people, switch off for 2 slides

- From thin link fermions to nHYP link fermions pushes the line to stronger coupling
- This was sufficient to expose the $SU(2)$ IRFP
- For larger N_c we did further improvement

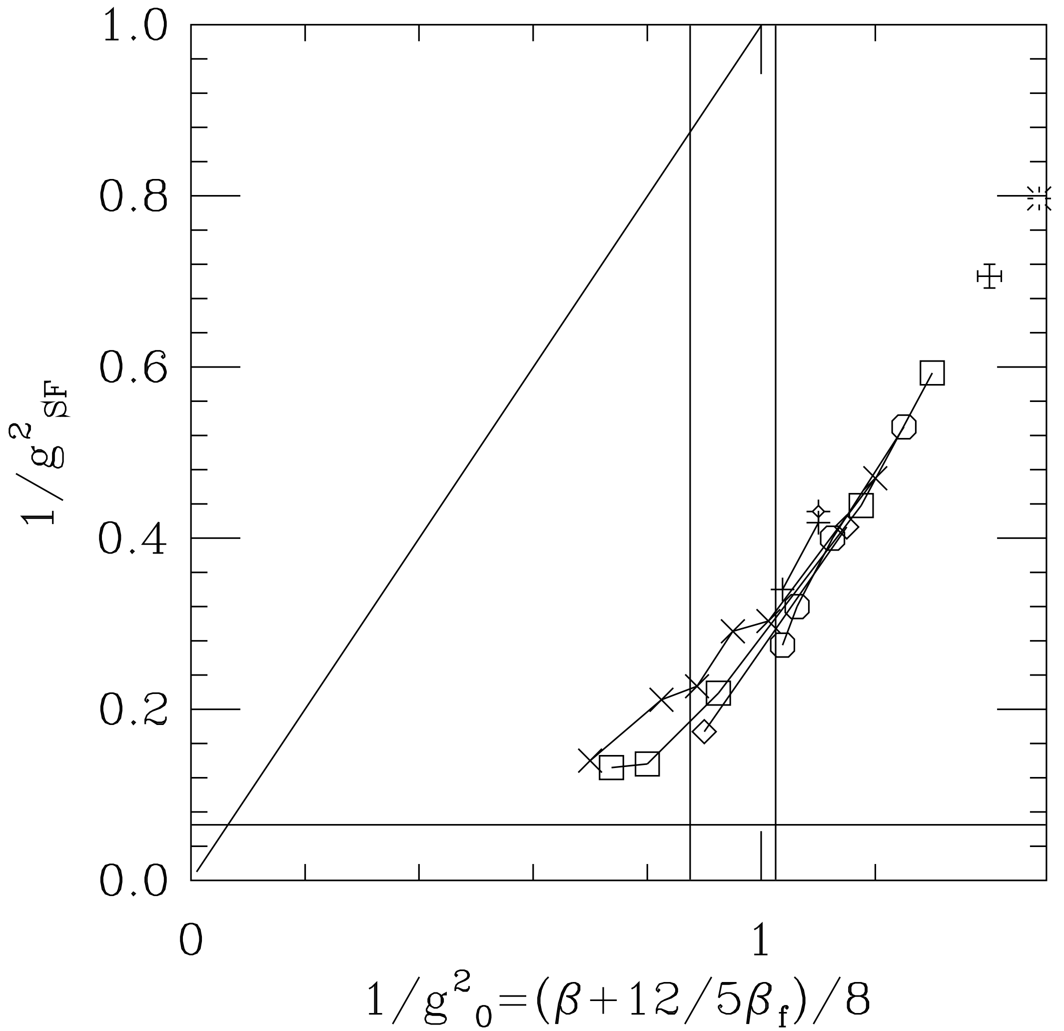
$$S_g = \frac{\beta}{2N_c} \sum \mathrm{Tr} U_p + \frac{\beta_f}{2d_f} \sum \mathrm{Tr} V_p \tag{18}$$

  where V_p is the plaquette made of (fat, fermion representation) links
- Empirically, tune β_f to get to larger g^2_{SF}
- In PT (in addition to a messy gluon propagator)

$$\frac{1}{g_0^2} = \frac{\beta}{2N_c} T(R) + \frac{\beta_f}{2d_f} T(F) \tag{19}$$



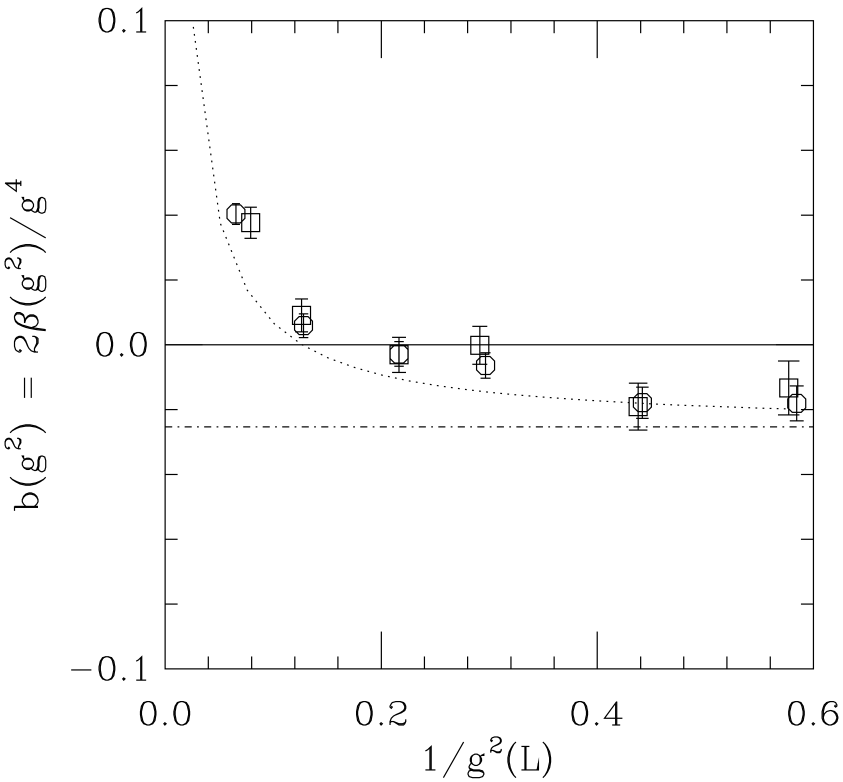
$SU(4)$ SF coupling at $L = 6$ vs $1/g_0^2 = \beta/6$ for different β_f's. Horizontal line: BZ IRFP; vertical lines: location of 1st order transitions where κ_c line ends



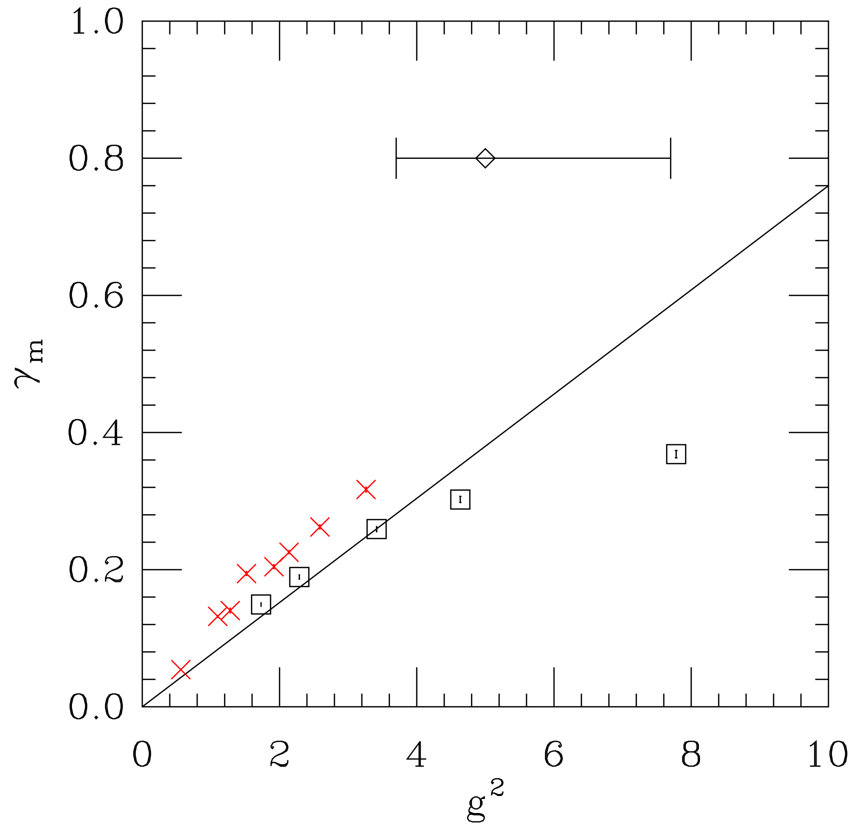
$SU(4)$ SF coupling at $L = 6$ vs $1/g_0^2$ for different β_f's. Horizontal line: BZ IRFP; vertical lines: location of 1st order transitions where κ_c line ends

Also a check of universality wrt lattice action– $g^2_{SF}(L) = g^2 + Cg^4 + \ldots$ means $\frac{1}{g^2_{SF}(L)} = \frac{1}{g^2} - C + \ldots$

$SU(2)$ with $N_f = 2$ adjoints

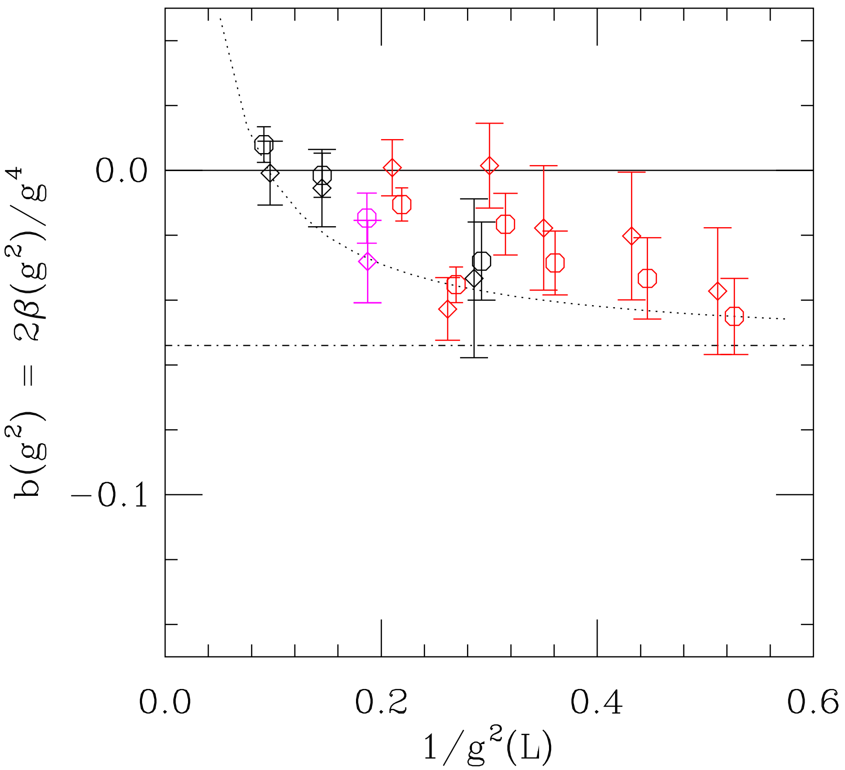


- A clear IRFP at $g^2 \sim 5$, weaker than 2 loops
- Improved action moved the 1st order line away, to expose it
- (Lattice note: gauge action used only the plaquette, fermions had fat links)

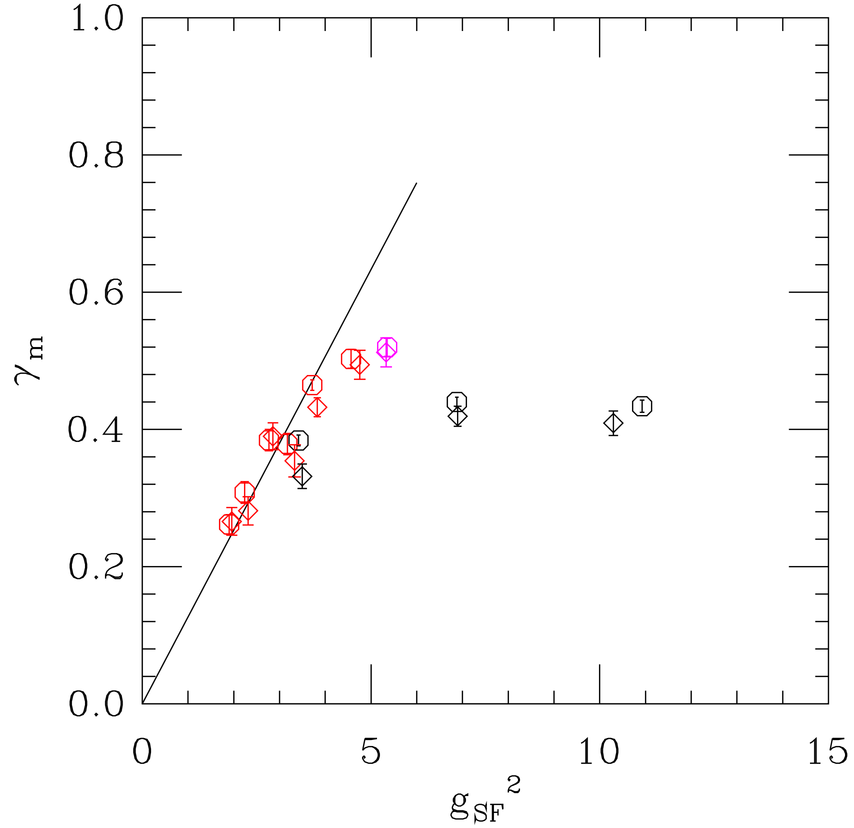


- $SU(2)$ γ_m falls off lowest order result, diamond and lines mark g_*^2 and error
- Squares are our data, crosses, our analysis of Bursa et al
- $\gamma_m = 0.31(6)$ – too small to be exciting
- No ETC here!

$SU(3)$ with $N_f = 2$ sextets

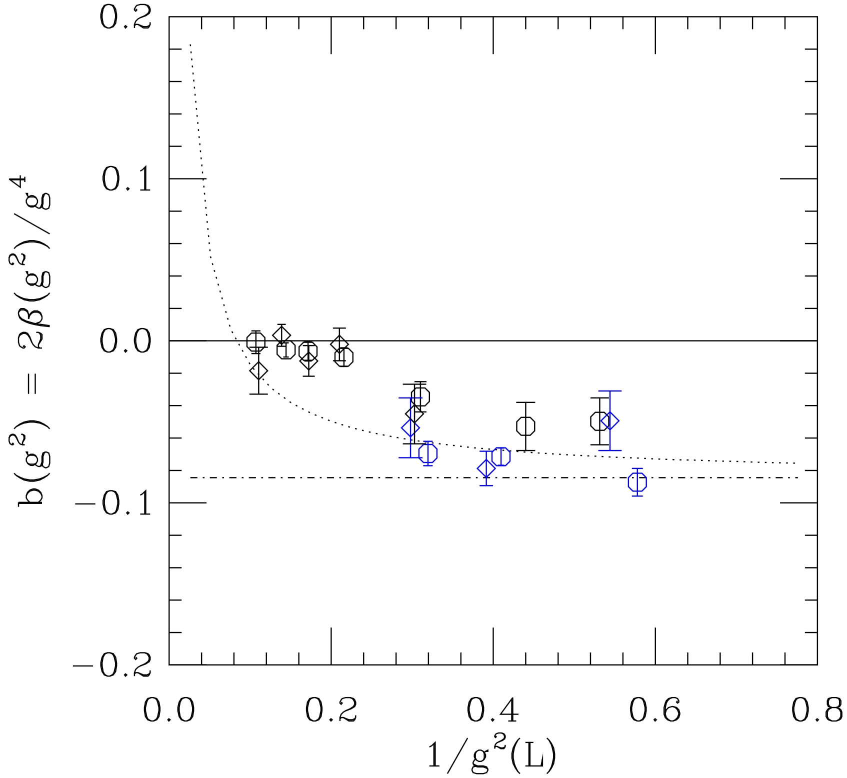


- Our first project (red), no IRFP observed, but we couldn't get to really strong coupling
- New action (black) takes us to the BZ point, $b(g^2)$ zero
- Octagons $L \geq 6$, diamonds $L \geq 8$ for slope of $1/g^2(L)$ vs L
- Old papers show spectroscopy very "conformal" – little dependence on g^2, FSS tests, etc

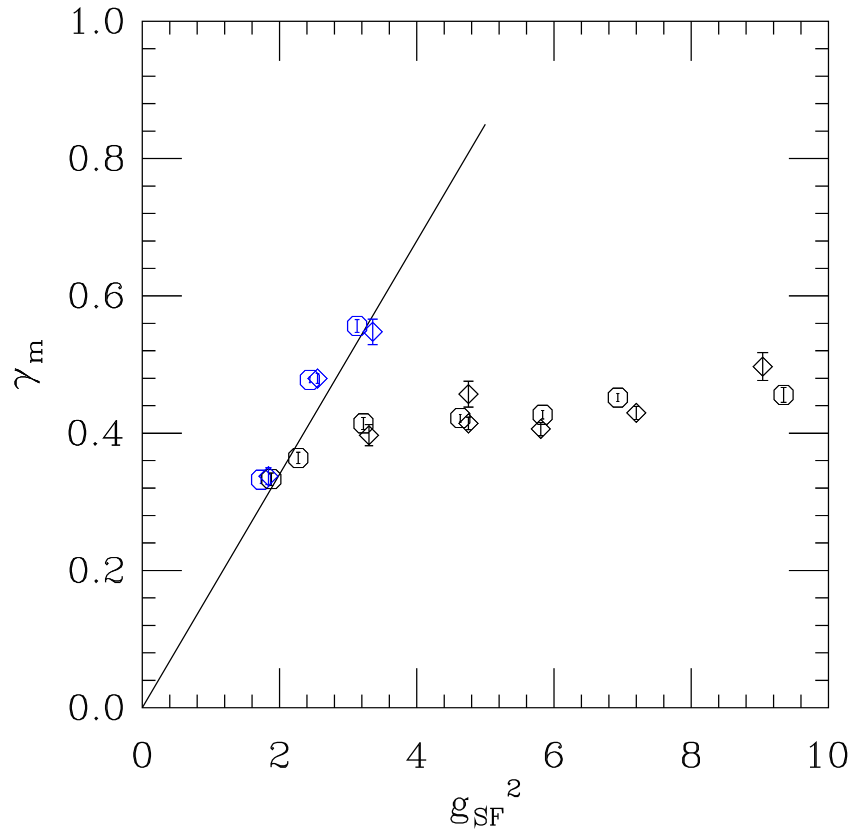


- $SU(3)$ γ_m is never large, always less than 0.5

$SU(4)$ with $N_f = 2$ symmetrics



- In progress, slowly filling in points
- (Glass half empty) My guess is that we won't get cleanly across a zero in the beta function
- (Glass half full:) $b(g) = 0$ at $1/g^2 = 0.1 - 0.2$, the BZ point (again!)



- Universality issue with γ_m (old action, blue points, sees its "wall")
- The action which gets us to strong coupling also gives $\gamma_m < 0.5$

Summary

Results:

- $SU(2)$ with $N_f = 2$ adjoints is conformal, IRFP crossed, $\gamma_m(g_*^2)$ small
- $SU(3)$ with 2-index symmetrics: $b(g) = 0$ at our strongest coupling points, $\gamma_m(g) < 0.5$
- $SU(4)$ with 2-index symmetrics: $b(g) = 0$ at our strongest coupling points, $\gamma_m(g) < 0.5$

Conclusions:

- Our most useful probe of slow walking theories is the system size
- Slow running simplifies large parts of the analysis
- Bug (strong coupling transition) tamed by feature (all lattice actions equally artificial)

Messages:

- To phenomenologists – these theories can't do what you'd like them to, for walking TC
- To "bottom of conformal window" people –these systems all did the same thing, γ_m went flat
- To all theorists – here is a class of "tame" lattice CFT's to play with
- To experimentalists – (I won't write this one down!)