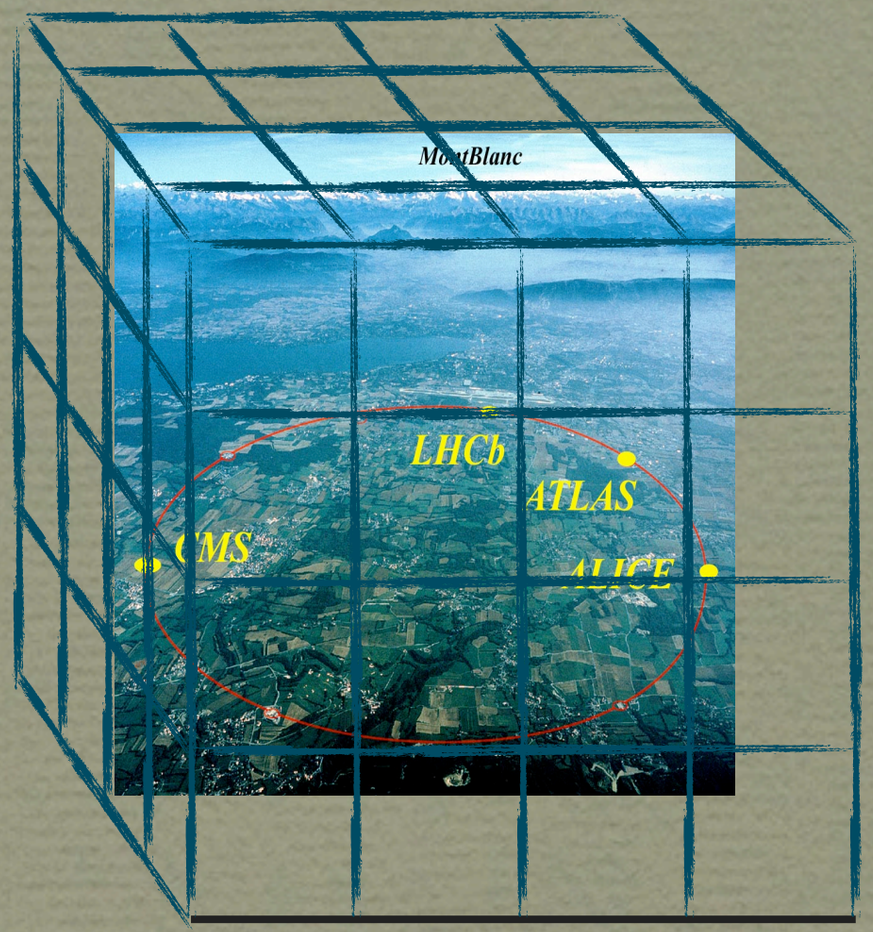
Mass anomalous dimension in ten-flavor QCD

Norikazu Yamada (KEK / GUAS)

with

Masashi Hayakawa, Shunpei Uno
(Nagoya Univ.),
Ken-Ichi Ishikawa, Yusuke Osaki
(Hiroshima Univ.)
Shinji Takeda (Kanazawa Univ.)



Lattice Meets Experiment 2011: Beyond the Standard Model
@Fermilab October 14-15, 2011

Contents

- Introduction
- $\alpha_{\rm SF}(\mu)$
  [Update of Phys. Rev. D83:074509, 2011]
- γ_m

Introduction

- Hierarchies in gauge and Yukawa sectors potentially solvable by Walking Technicolor (WTC)

  Holdom(1981),
  Yamawaki, Bando, Matsumoto(1986),
  Appelquist, Karabali, Wijewardhana(1986),
  Akiba, Yanagida(1986),
  Bando, Morozumi, So, Yamawaki(1987)

  c.f. TC : Weinberg(1979), Susskind(1979),
  ETC: Eichten and Lane ('80), Dimopoulos and Susskind ('79)

One of the Keys

$$m_{\rm SM} \sim \frac{1}{M_{\rm ETC}^2} C(\mu) \langle 0 | \bar{\psi}\psi(\mu) | 0 \rangle$$

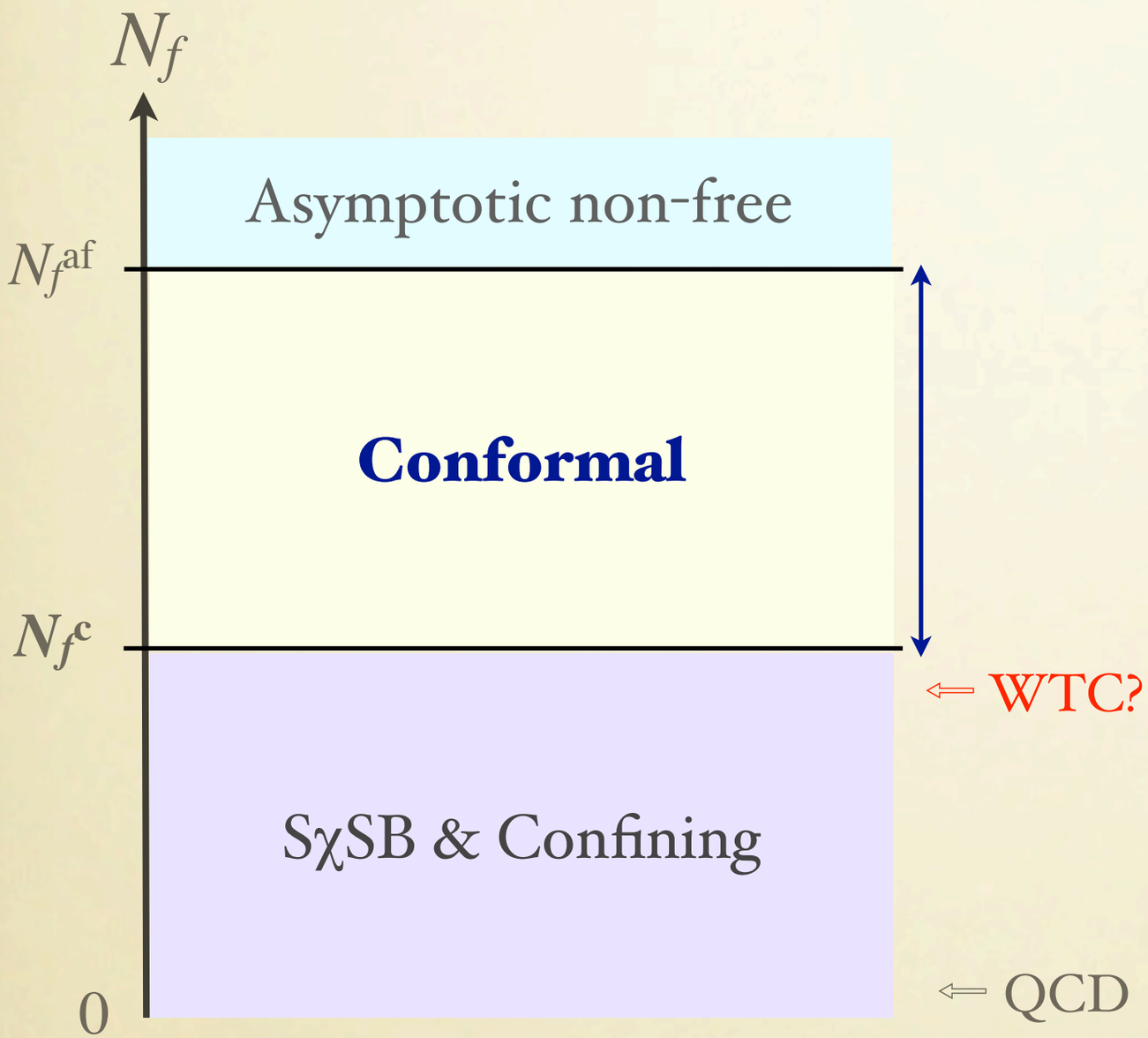
$$C(\mu) = \exp\left(\int_{\mu}^{M_{ETC}} d\mu' \frac{\gamma_m(\mu')}{\mu'} \right)$$

$$\sim \left(\frac{M_{\rm ETC}}{M_{\rm TC}} \right)^{\gamma_m} \gg 1$$

γ_m must be large when $\beta \sim 0$!

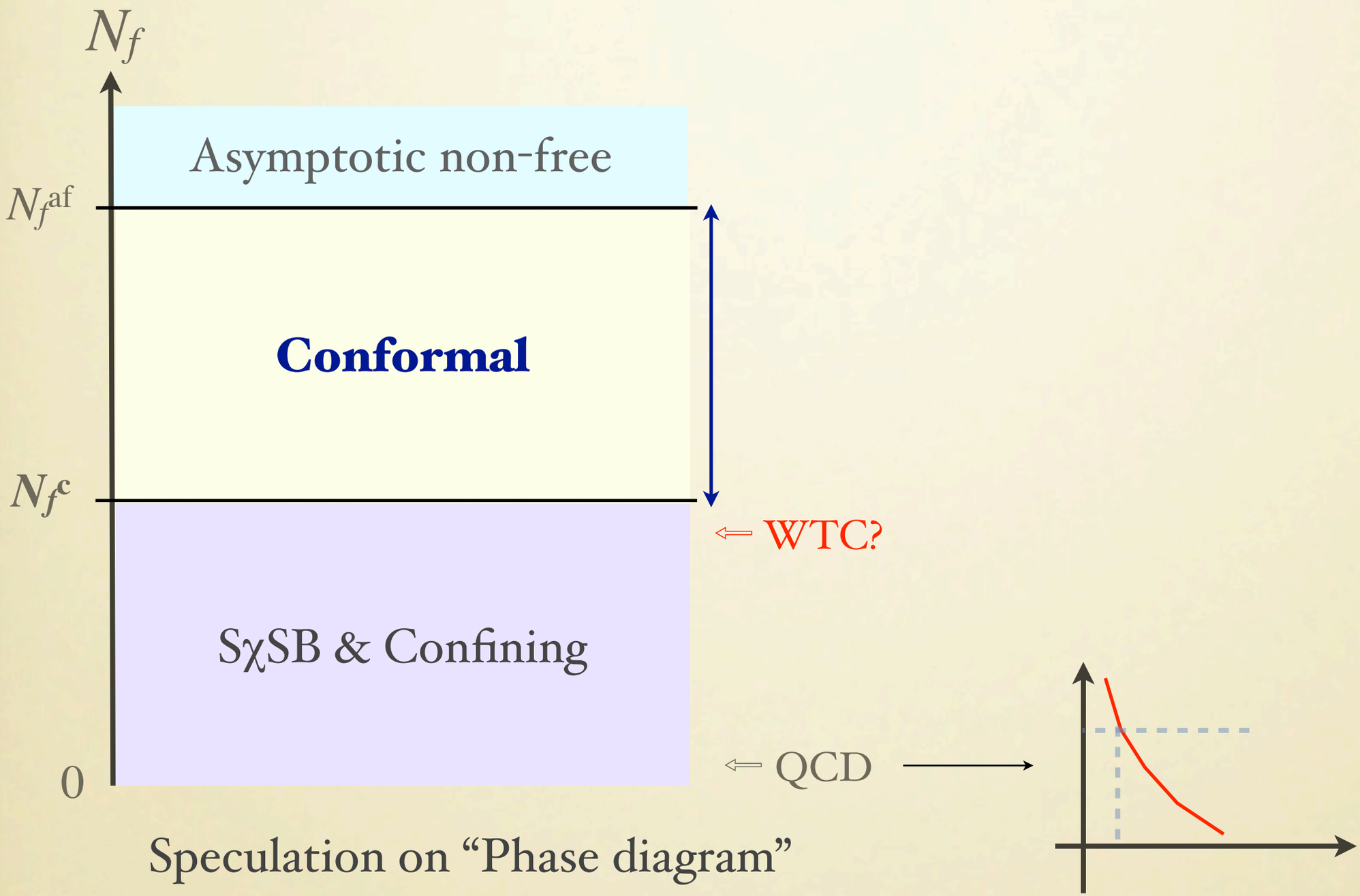
Find such gauge theories.

Conformal Window



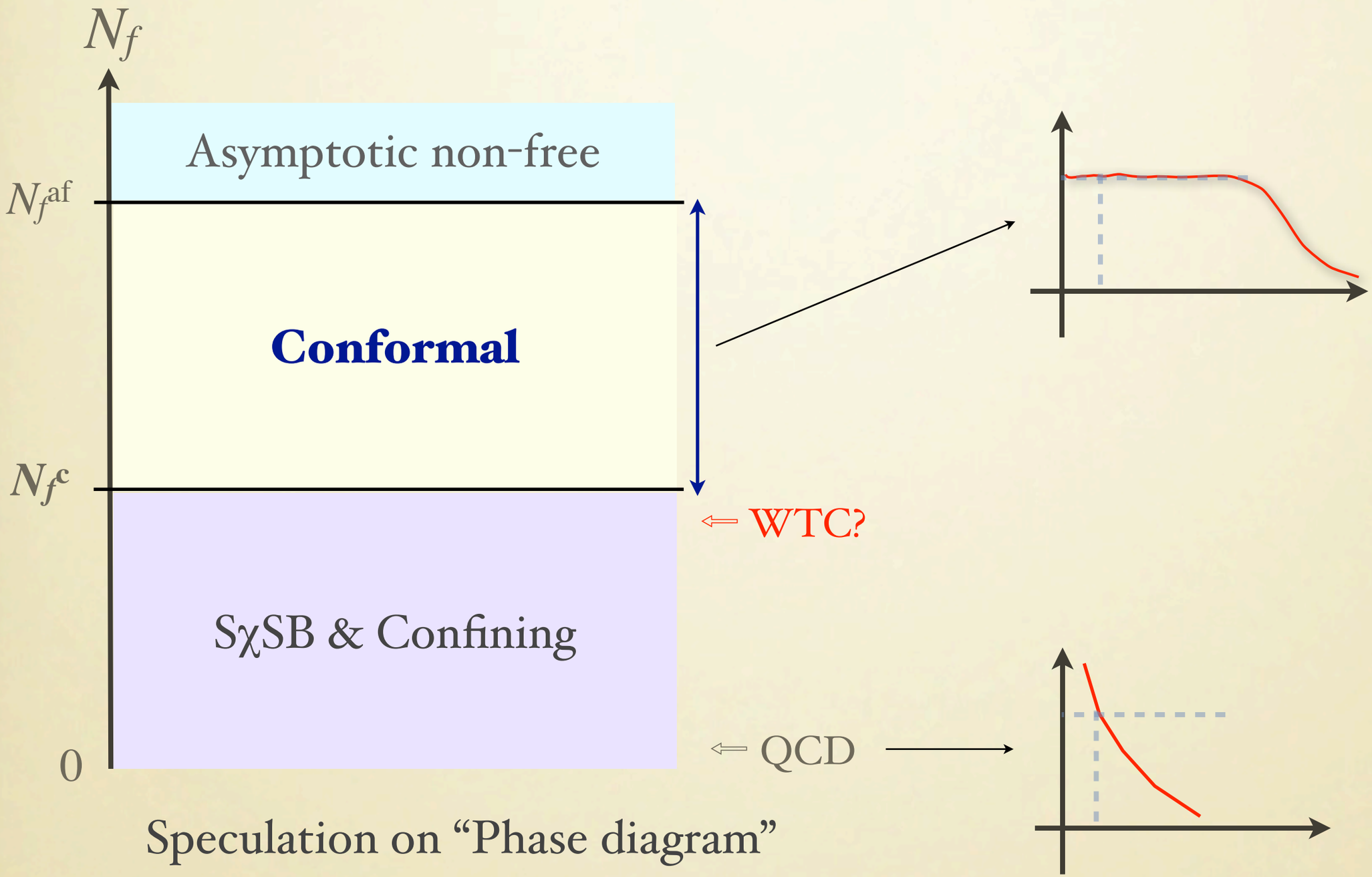
Speculation on "Phase diagram"

Conformal Window



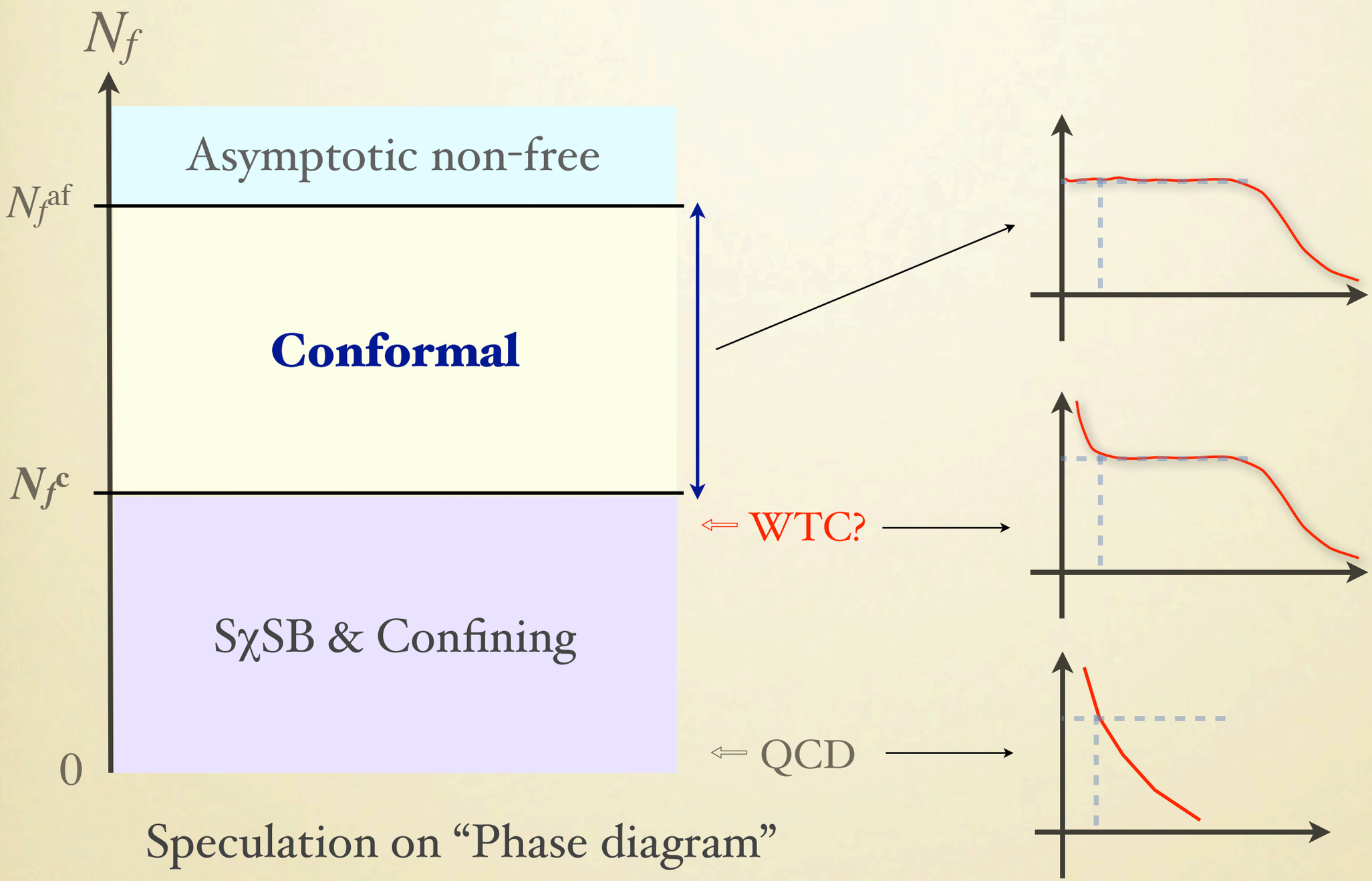
Speculation on "Phase diagram"

Conformal Window



Speculation on "Phase diagram"

Conformal Window



Speculation on "Phase diagram"

Find the location of N_f^c in various gauge theories.

Perturbative prediction

SU(3) gauge theory with N_f fundamental Dirac fermions:

Perturbative IRFP (g_{FP}^2)

N_f	4	6	8	10	12	14	16
2-loop universal				27.74	9.47	3.49	0.52
3-loop SF	43.36	23.75	15.52	9.45	5.18	2.43	0.47
3-loop $\overline{\text{MS}}$		159.92	18.40	9.60	5.46	2.70	0.50
4-loop $\overline{\text{MS}}$			19.47	10.24	5.91	2.81	0.50

Anomalous dimension of mass in SF scheme:

$$\gamma_m^{\text{SF}} = \frac{8}{(4\,\pi)^2}\, g^2\, \left\{1 + (0.1251 + 0.0046\, N_f)\; g^2\right\}$$

With g_{FP}^2 for 3-loop β-function in SF scheme,

$$\gamma_{\text{FP}}^{\text{SF}} = \begin{cases} 2.76183 & \text{for } N_f = 8 \\ 1.25265 & \text{for } N_f = 10 \\ 0.50772 & \text{for } N_f = 12 \end{cases} \sim O(1)$$

Lattice setup

- Schrödinger Functional method
- Wilson gauge + Wilson fermions
- Box size : $L/a = 6, 8, 12, 16$
- Bare coupling : $\beta = 4.4 \sim 96.0$
- # of trajectories for each simulation : 10 k ~ 500 k
- Standard HMC algorithm (implemented in cuda)

- Wilson fermion explicitly breaks χ-symmetry
  $\Rightarrow$ kappa has been tuned to "m_q=0" in all (β, L/a).
- Step scaling factor is fixed to s=2. Two data s=12/6 and 16/8 available.
- Continuum limit using two data assuming linear scaling violation

$\Rightarrow$ Validity is checked.

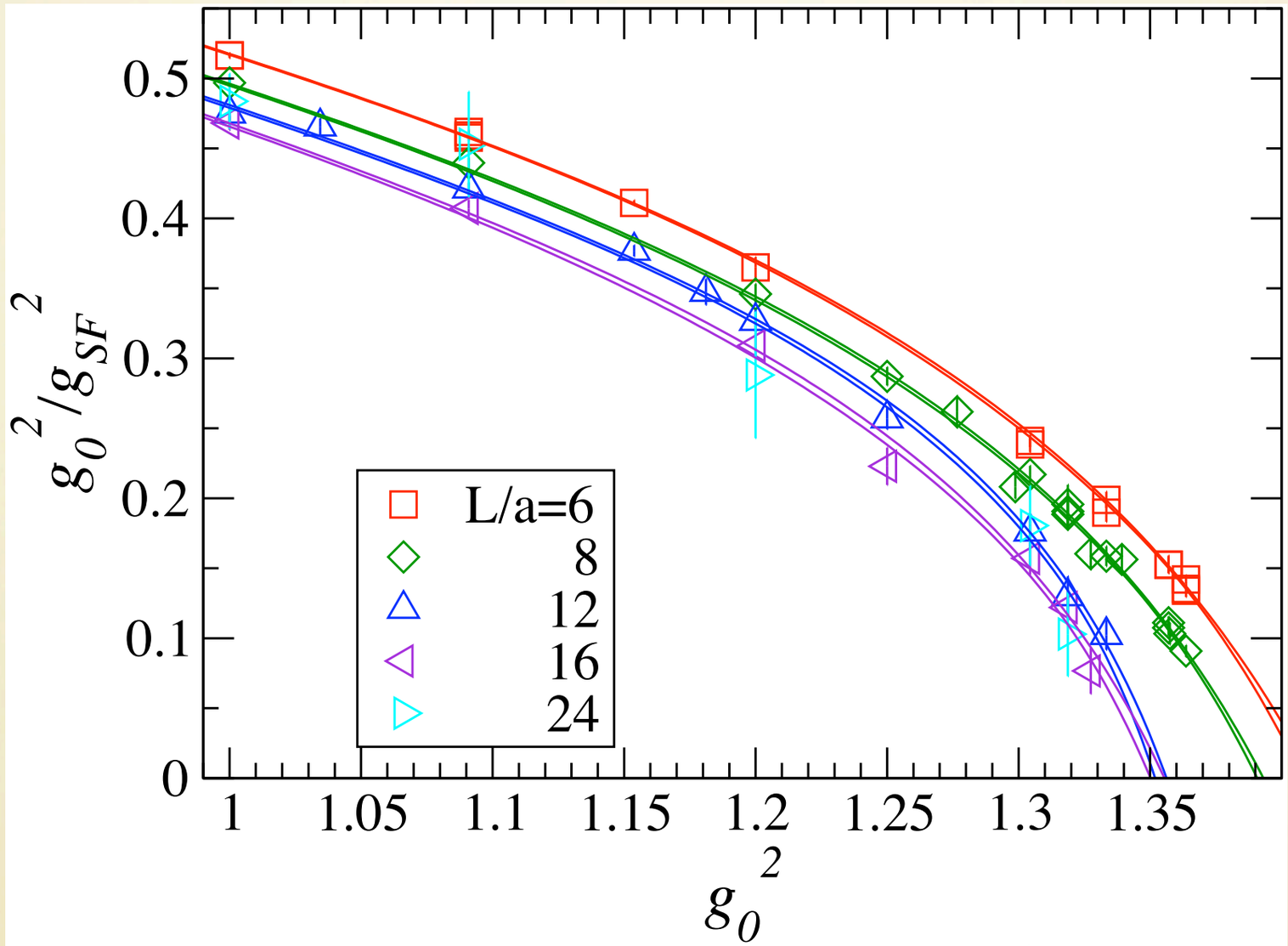
Machines

- Supercomputer@KEK (SR11K, BG/L)
- GPGPU & CPU servers@KEK
- INSAM GPU cluster@Hiroshima
- GPGPU, GCOE cluster system@Nagoya
- B-factory computer system@KEK
- φ@KMI, Nagoya
- ...

$\alpha(\mu)$ in 10-flavor QCD

Update of Phys. Rev. D83:074509, 2011 [arXiv:1011.2577 [hep-lat]]

$g_{SF}^2(L)$ vs g_0^2



Fitted to $\dfrac{g_0^2}{{g_{L/a}^{\rm fit}}^2} = \dfrac{1 - a_{L/a,1}\, g_0^4}{1 + p_{1,L/a}\, g_0^2 + \sum_{n=2}^{N} a_{L/a,n} \times g_0^{2\,n}}.$

N is taken to be 4 or 5 depending on L/a.

Discrete Beta function

Y. Shamir, B. Svetitsky, and T. DeGrand, Phys. Rev. D 78, 031502 (2008).
T. DeGrand, Y. Shamir, and B. Svetitsky, Phys. Rev. D 82, 054503 (2010).

In continuum,

$$B(u,s) = \frac{1}{\sigma(u,s)} - \frac{1}{u},$$

$$u = g_R^2(L),$$

$$\sigma(u,s) = g_R^2(s\,L)$$

On the lattice,

$$B_0(u,s,l) = \frac{1}{\Sigma_0(u,s,l)} - \frac{1}{u},$$

$$l = L/a,$$

$u = g_R^2(g_0^2, l)$: input,

$\Sigma_0(g_0^2, s\,l)$: output

We take Schrödinger Functional scheme for g^2_R.

One-loop improvement

Unimproved DBF: $B_0(u,s,l) = \dfrac{1}{\Sigma_0(u,s,l)} - \dfrac{1}{u}.$

One-loop improvement of DBF is performed.

$$\begin{aligned}\delta_0(u,s,L/a) &= \frac{\Sigma_0(u,s,L/a) - \sigma(u,s)}{\sigma(u,s)} \\ &= \delta^{(1)}(s,L/a)u + \delta^{(2)}(s,L/a)u^2 + \delta^{(3)}(s,L/a)u^3 + \cdots\end{aligned}$$

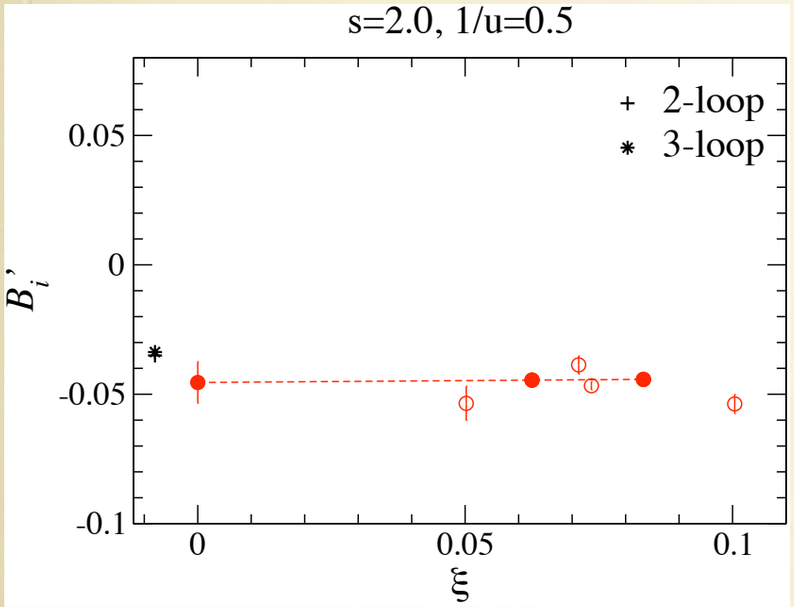
$$\delta^{(1)}(s,l) = \Big(p_{1,s\cdot l} - b_1 \ln(s\cdot l)\Big) - \Big(p_{1,l} - b_1 \ln(l)\Big) = p_{1,s\cdot l} - p_{1,l} - b_1 \ln(s)$$

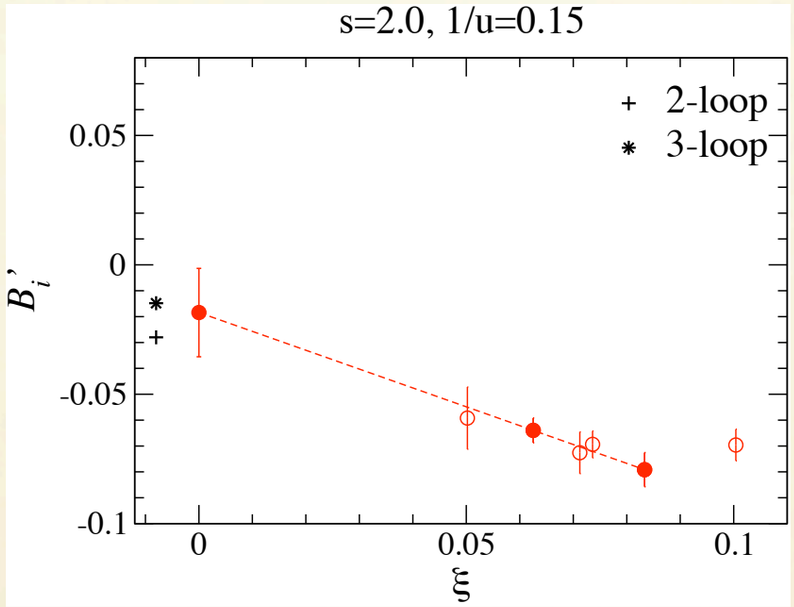
Improved DBF:

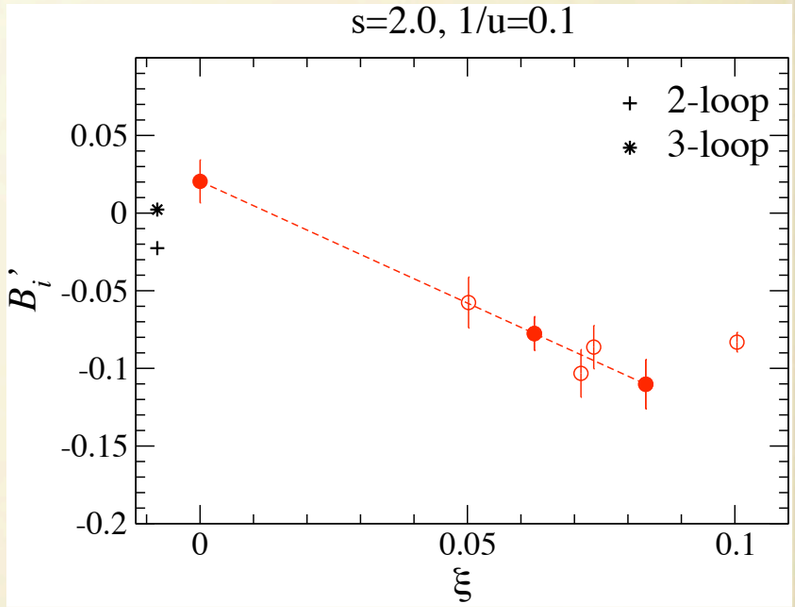
$$\begin{aligned}\Sigma_1(u,s,l) &= \Sigma_0(u,s,l)/(1+\delta^{(1)}(s,l)\,u) \\ B_1(u,s,l) &= \frac{1}{\Sigma_1(u,s,l)} - \frac{1}{u}.\end{aligned}$$

Continuum limit

coupling: weak → **strong**







continuum limit: negative | consistent with zero | positive

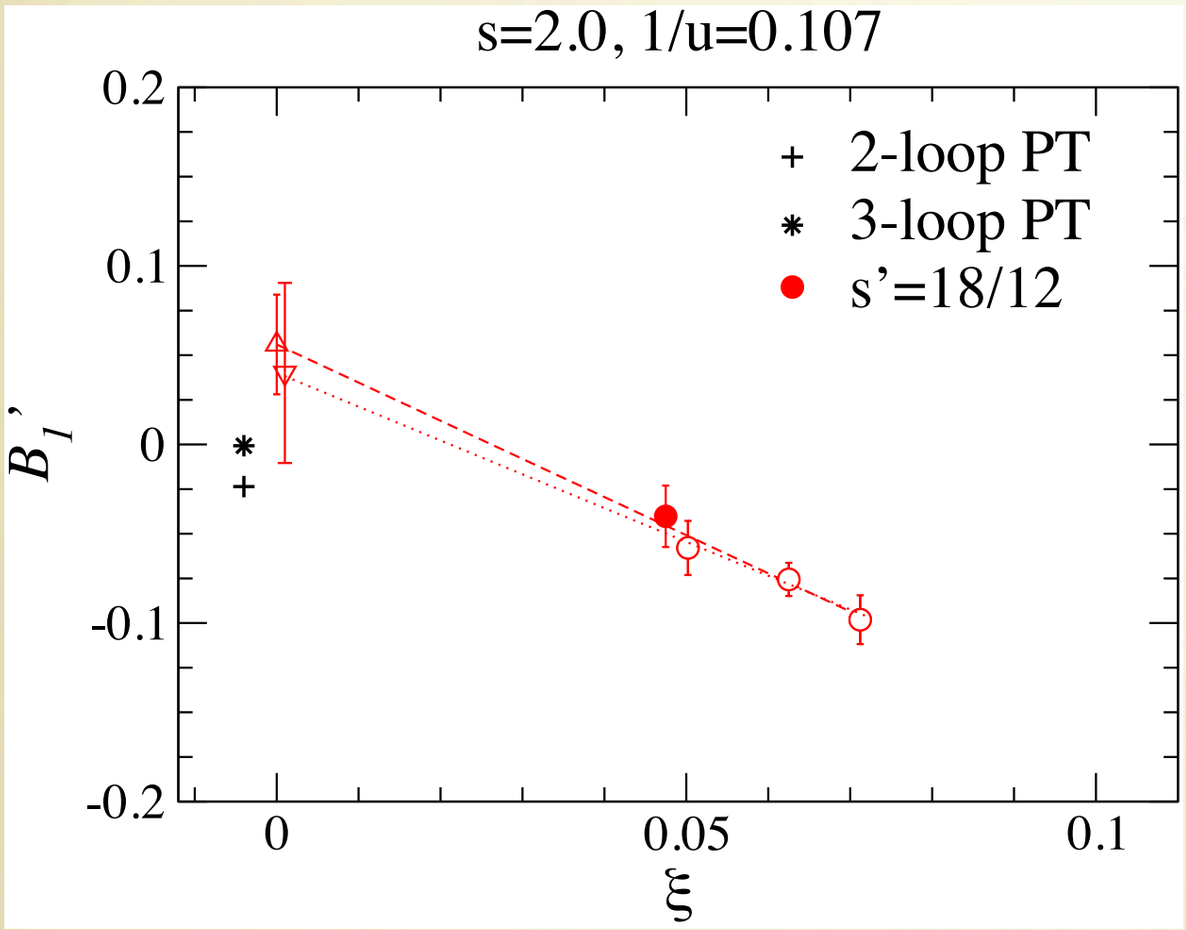
- ✓ The filled symbols (s=2) used in the extrapolation.
- ✓ Open symbols are the rescaled DBF.

$$B_i'(u,s,l,s') = \frac{\ln(s)}{\ln(s')} B_i(u,s',l) \qquad \xi(s,l,s') = \frac{\ln(s)}{\ln(s')}\left(\frac{1}{l} - \frac{1}{s'l}\right)$$

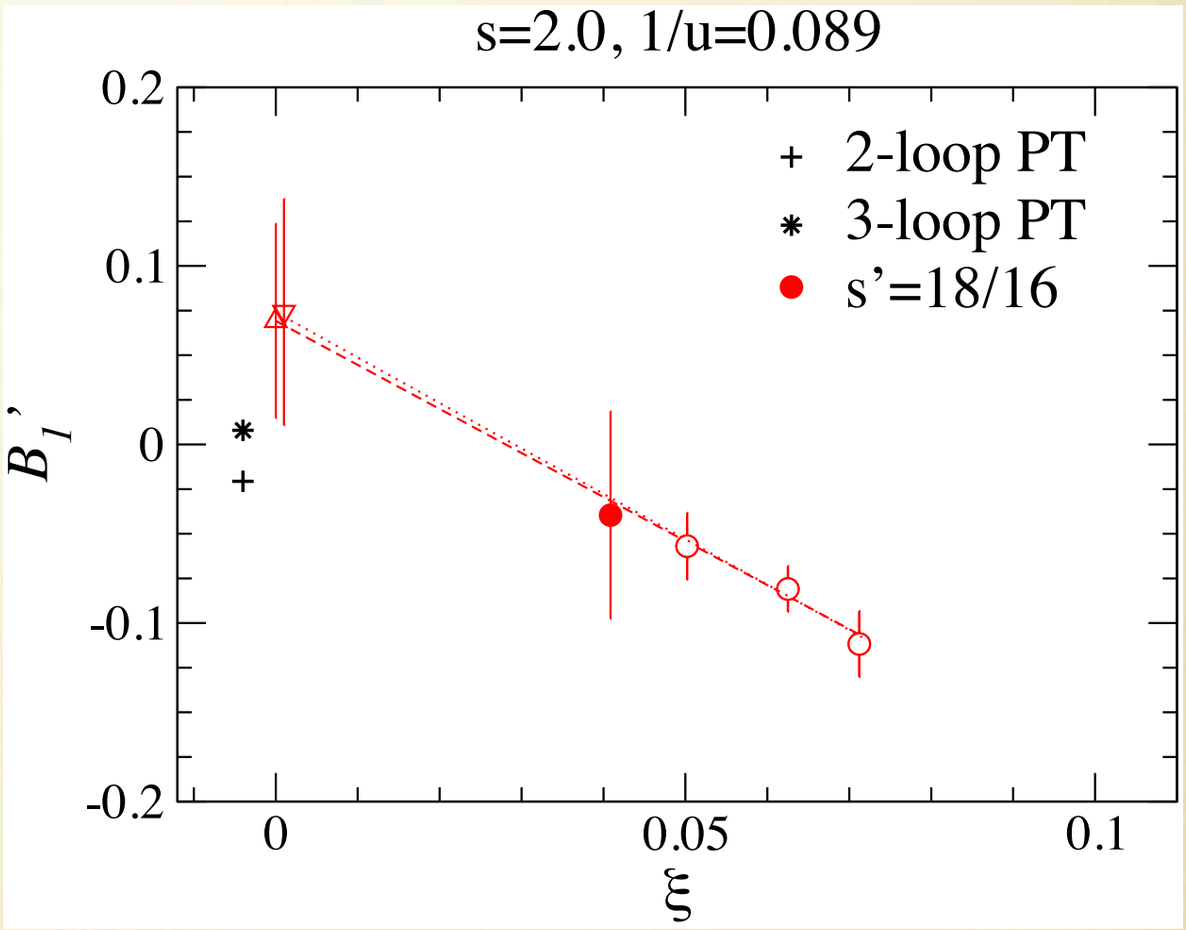
$$B_i'(u,s,l,s') = B(u,s) + \xi(s,l,s')\, e_i(u) - \delta B(u,s,s') + O(l^{-2})$$

- ✓ If linear ansatz is valid and the continuum DBF is small, they should be on the fit line. ➡ Self-consistent.

Cross check



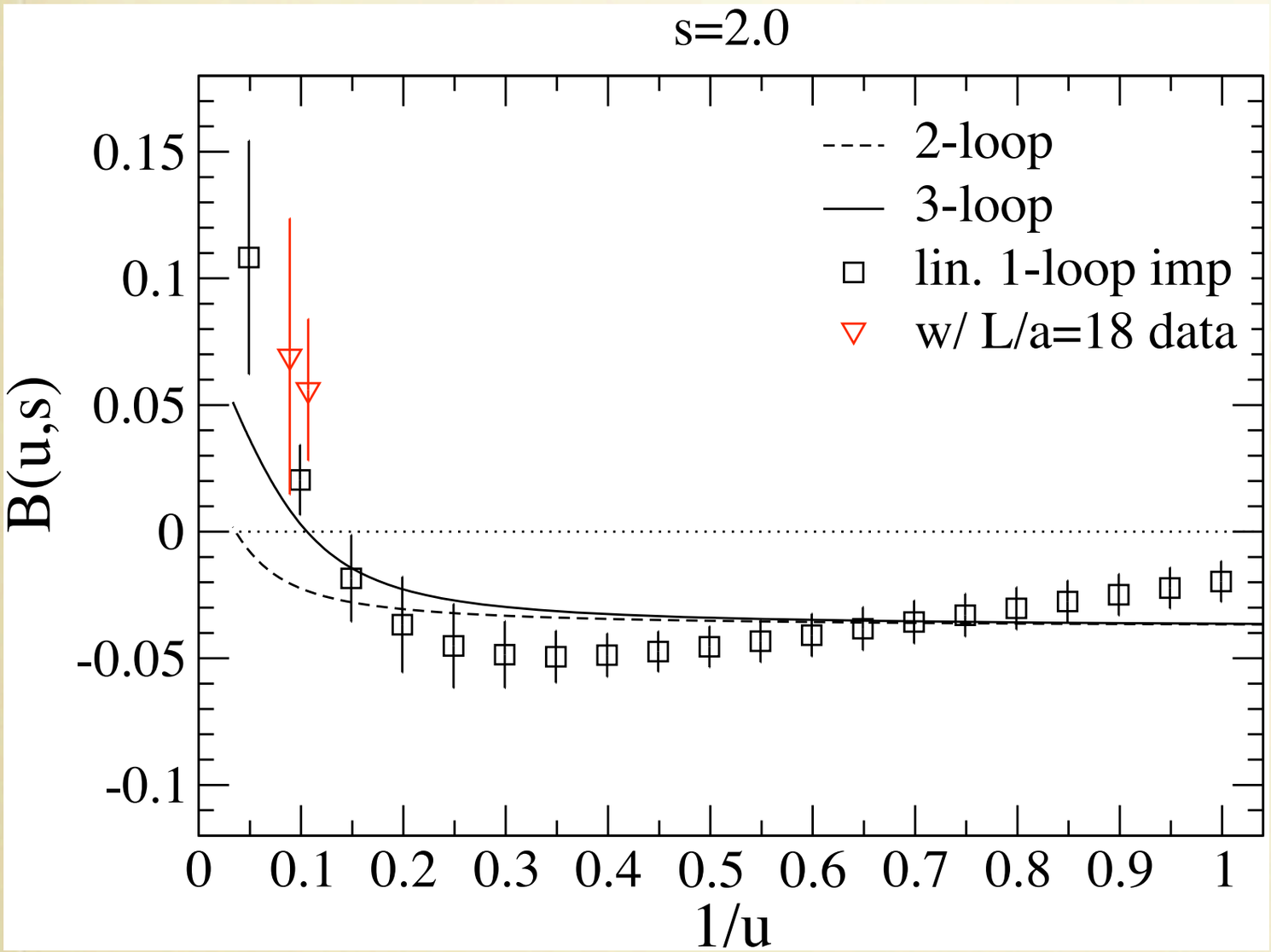
positive



positive

- ✓ Both open and filled symbols used in the extrapolation.
- ✓ When $u > 10$, DBF > 0.

DBF in the continuum



Suggesting IRFP at $g_{FP}^2 = 6.7 \sim 10.0$.

For SU(3),

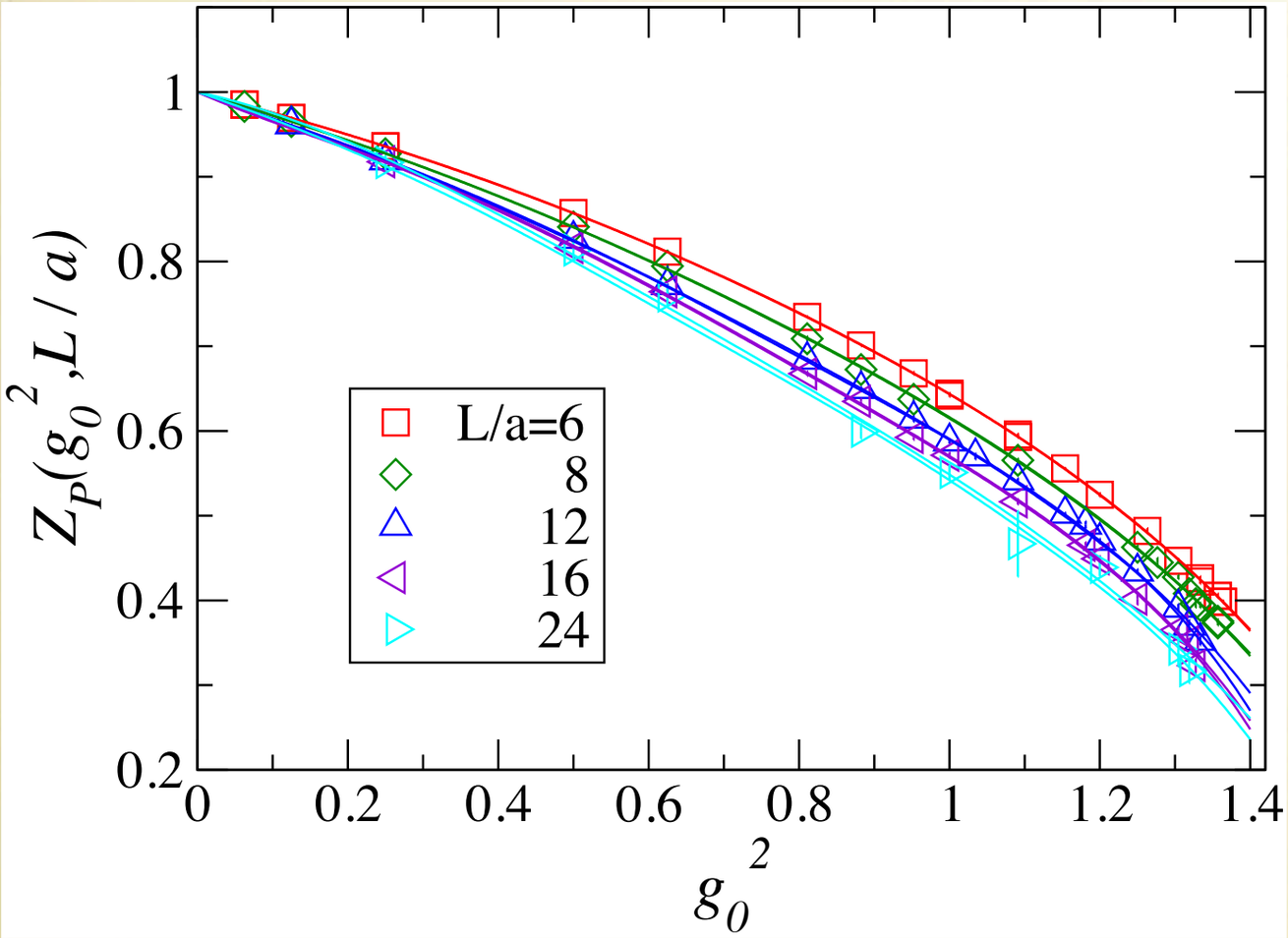
$8? < N_f^c < 10$

Comment on "positive value in the continuum limit": When the cont. limit > 0, the extrapolation may not make sense unless nontrivial UV fixed point exists.

γ_m in 10-flavor QCD

γ_m in 10-flavor QCD

$$Z_m(L) = \frac{1}{Z_S(L)} = \frac{1}{Z_P(L)} \qquad \sigma_P(u,s) = \frac{Z_P(sL)}{Z_P(L)} = \exp\left(-\int_u^{\sigma(u,s)} du' \frac{\tau(u')}{\beta(u')}\right)$$



SF scheme used to calculate Z_P. Fitted to

$$Z_P^{\rm lat}(g_0^2, l) = 1 + \sum_{i=1}^{N} z_i (g_0^2)^i.$$

$$\Sigma_P(u, s, a/L) = \frac{Z_P(g_0^2,\, s\, L/a)}{Z_P(g_0^2,\, L/a)} \quad \text{at } m_q^{\rm PCAC} = 0, \;\; g_{\rm SF}^2(L) = u.$$

1-loop improvement of Σ_P

PACS-CS collaboration, **JHEP 1008 (2010) 101**

$$\Sigma_P^{(0)}(u,s,a/L) = \frac{Z_P(g_0^2,\, s\,L/a)}{Z_P(g_0^2,\, L/a)} \qquad \text{at } m_q^{\rm PCAC} = 0,\;\; g_{\rm SF}^2(L) = u.$$

$$\frac{\Sigma_P^{(0)}(u,s,l) - \sigma_P^{\rm PT}(u,s)}{\sigma_P^{\rm PT}(u,s)} = \delta_P^{(1)} u + O(u^2)$$

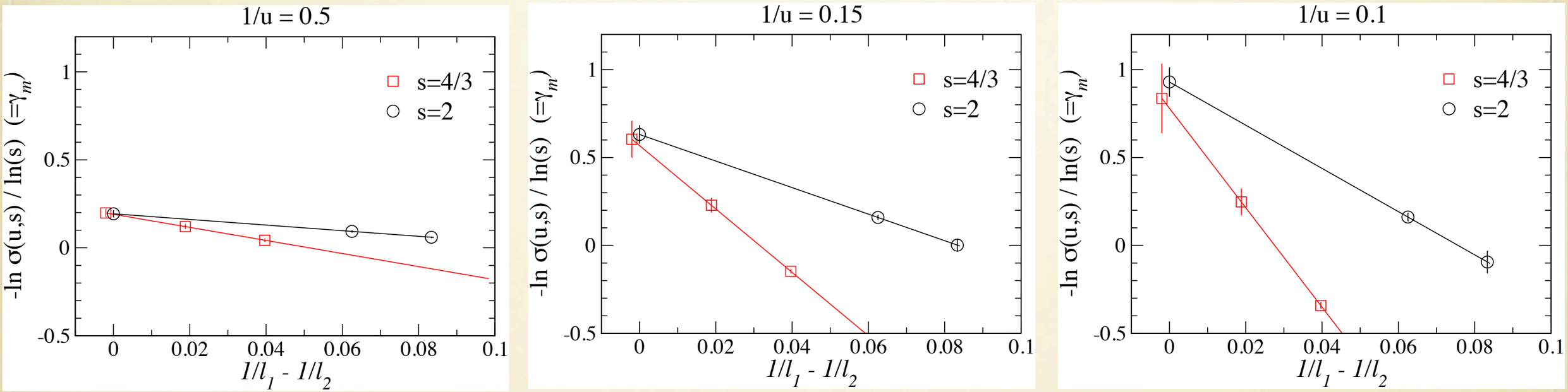
$$\Sigma_P^{(1)}(u,s,l) = \frac{\Sigma_P^{(0)}(u,s,l)}{1+\delta_P^{(1)}(s,l)u}.$$

$$\sigma_P(u,s) \;\; = \;\; \lim_{a\to 0}\; \Sigma_P^{(1)}(u,s,l)$$

At the fixed point, $\gamma_m = -\dfrac{\ln \sigma_P(u,s)}{\ln s}$

Continuum limit

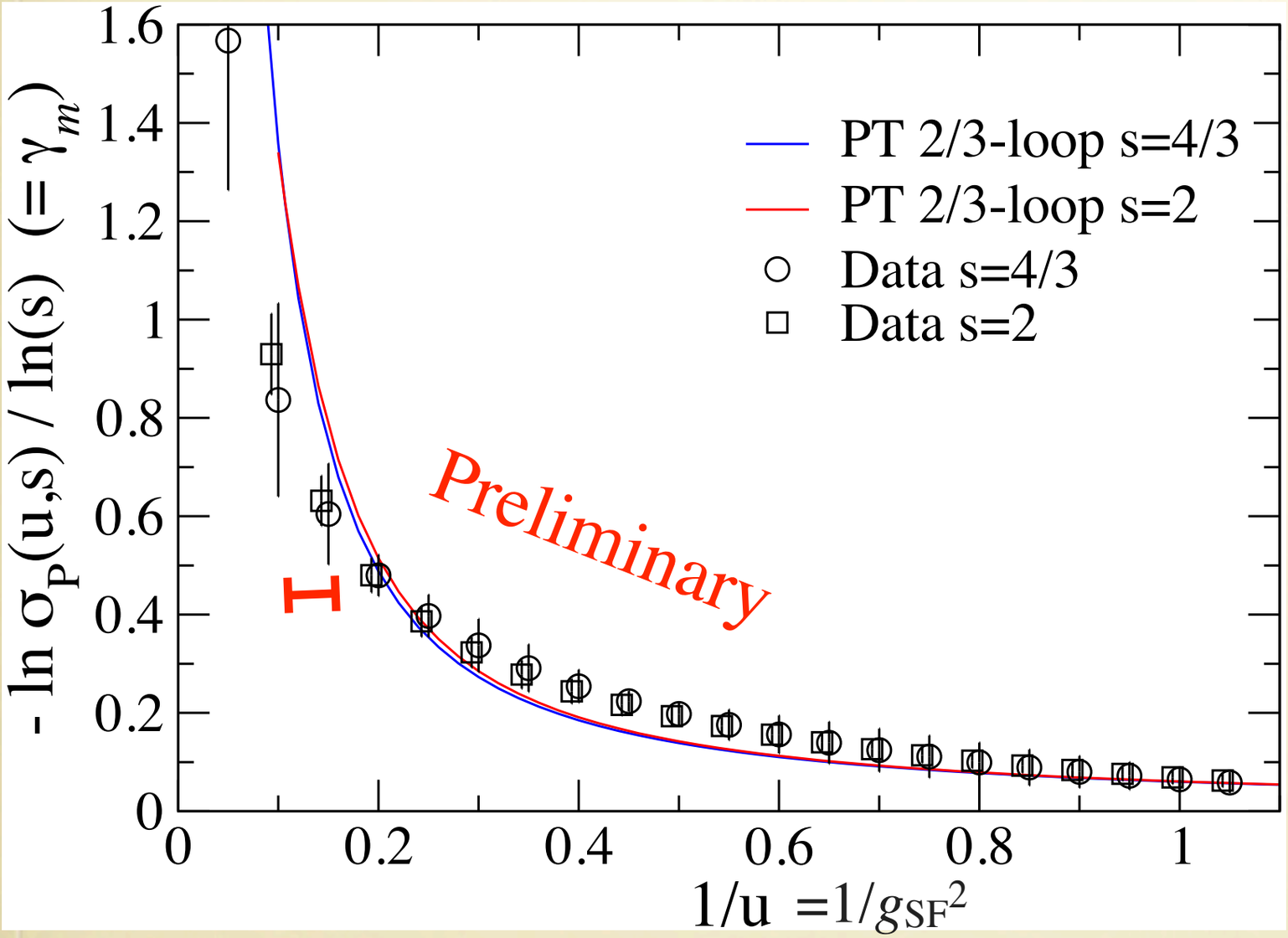
coupling: weak → **strong**



$$\frac{-\ln \sigma_P(u,s)}{\ln s} = \lim_{a\to 0} \frac{-\ln \Sigma_P(u,s,a)}{\ln s} \quad [= \gamma_m(u)]$$

- $l_1 = a/L_1$, $l_2 = s\, l_1$
- Scaling violation gets worse as one goes to the strong coupling.
- But the linear extrapolations with different s agree with each other.

Continuum limit



- With $6.7 < g_{FP}^2 < 10.0$,

  $0.50 < \gamma_m < 1.03$

  (statistical error only)
- $\gamma_m \sim O(1)$!

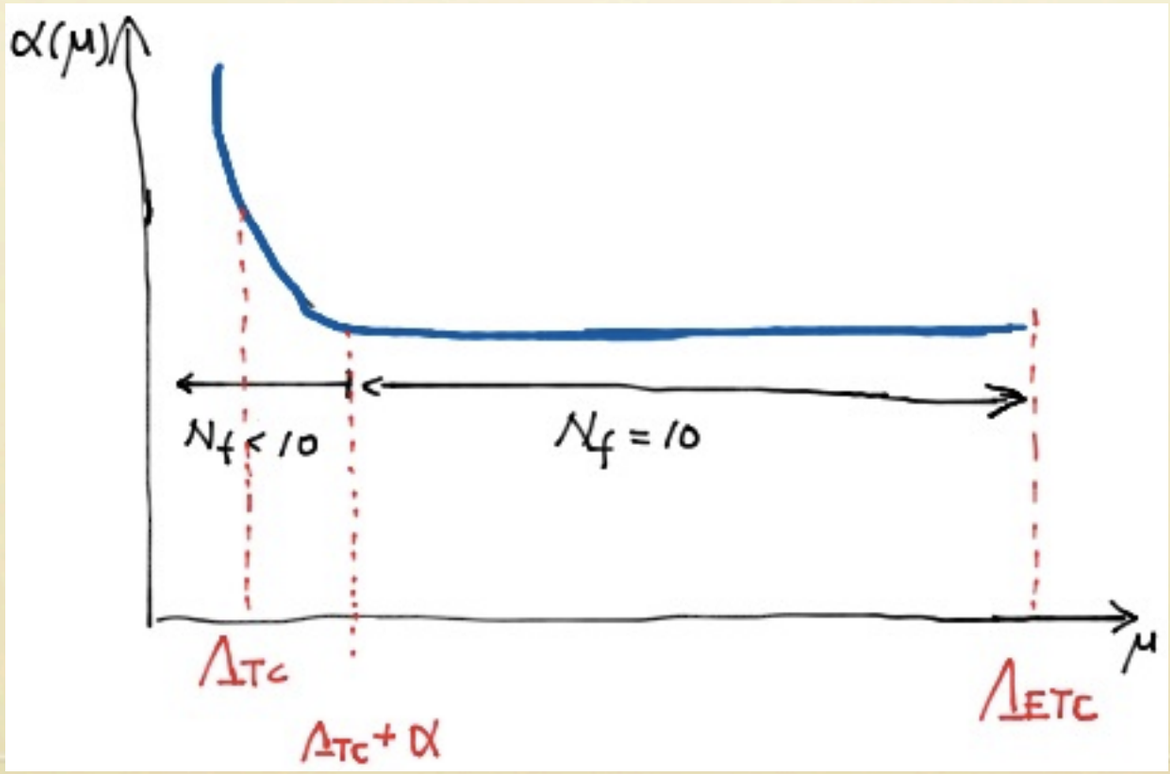
Summary

✓ Evidence for an IRFP in $6.7 < g^2_{FP} < 10.0$.

$\Rightarrow 8(?) < N_f^c < 10$

✓ $0.50 < \gamma_m < 1.03$ from preliminary analysis.

g_{FP} & γ_m consistent with T. Ryttov and R. Shrock, PRD83, 056011 (2011)

✓ Pinning down γ_m requires precise value of g^2_{FP}.

✓ If $\gamma_m \sim 1$ has been established, 10-flavor QCD may be used as follows: