Lattice Methods in BSM Theory Space

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What this talk is not: reviewing work from lattice groups

Good progress is being made

Will be discussed by lattice groups in talks of the next two days

What this talk is: overview of lattice methods commonly faced in lattice BSM work

Lattice specific: cut-off, volume, fermion mass

Familiar to on-lattice workers

Talk is mostly for off-lattice workers
while they are thinking about making proposals for us

Outline

- Composite Higgs Mechanism at the LHC
lattice BSM goals in Theory Space
world-wide lattice BSM effort
lattice resources (GPU technology)

- Below the Conformal Window
lattice specific: cut-off, volume, fermion mass
RG flow and lattice continuum physics
BSM specific χPT
m=0 chiral limit and finite volume issues

- Inside the conformal window
RG flow and lattice continuum physics
finite size scaling
running coupling and tunneling
Nf=16 case study

- Outlook
from workshop discussions: new input into lattice projects?

Large Hadron Collider - CERN

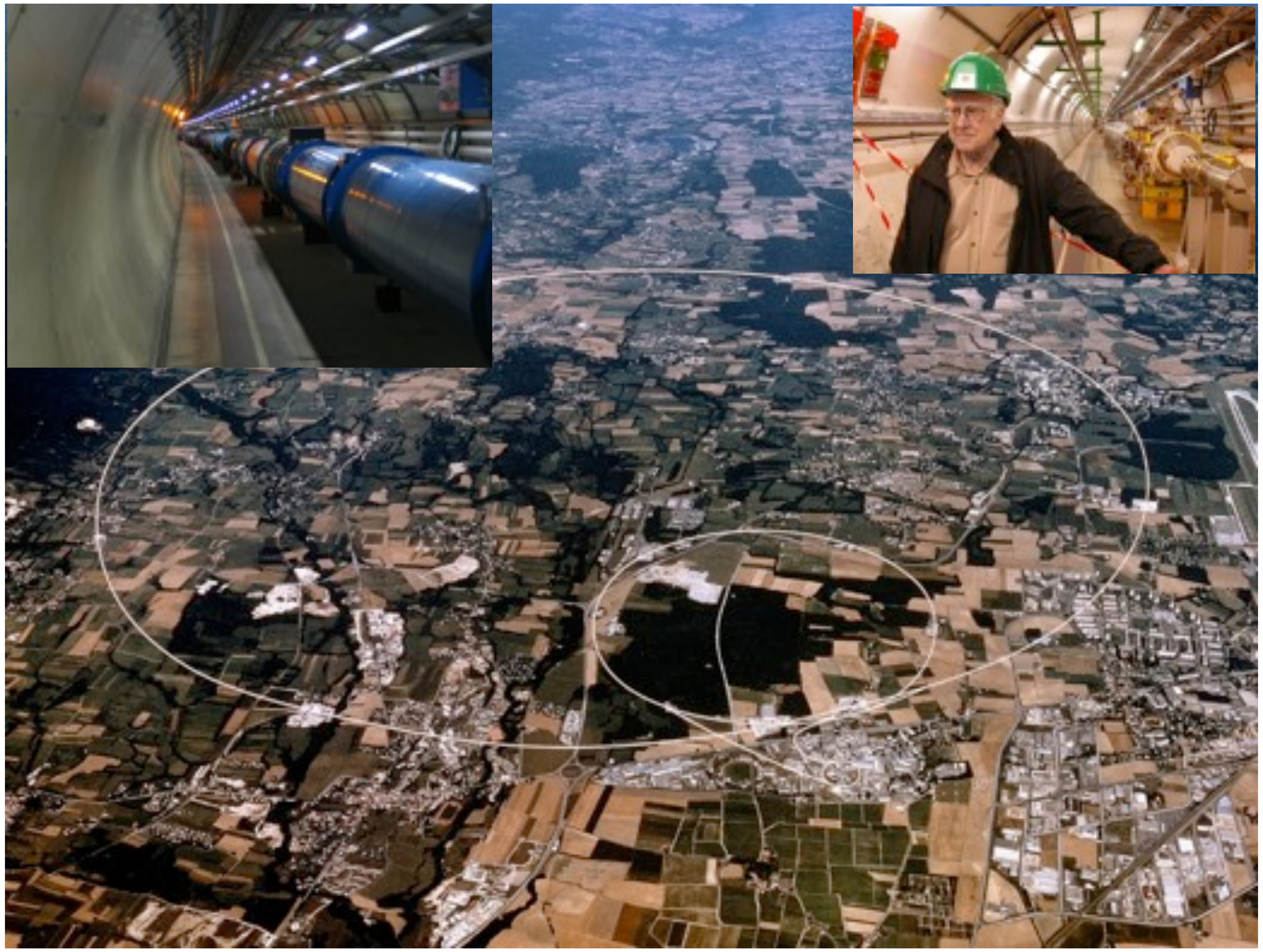
primary mission:

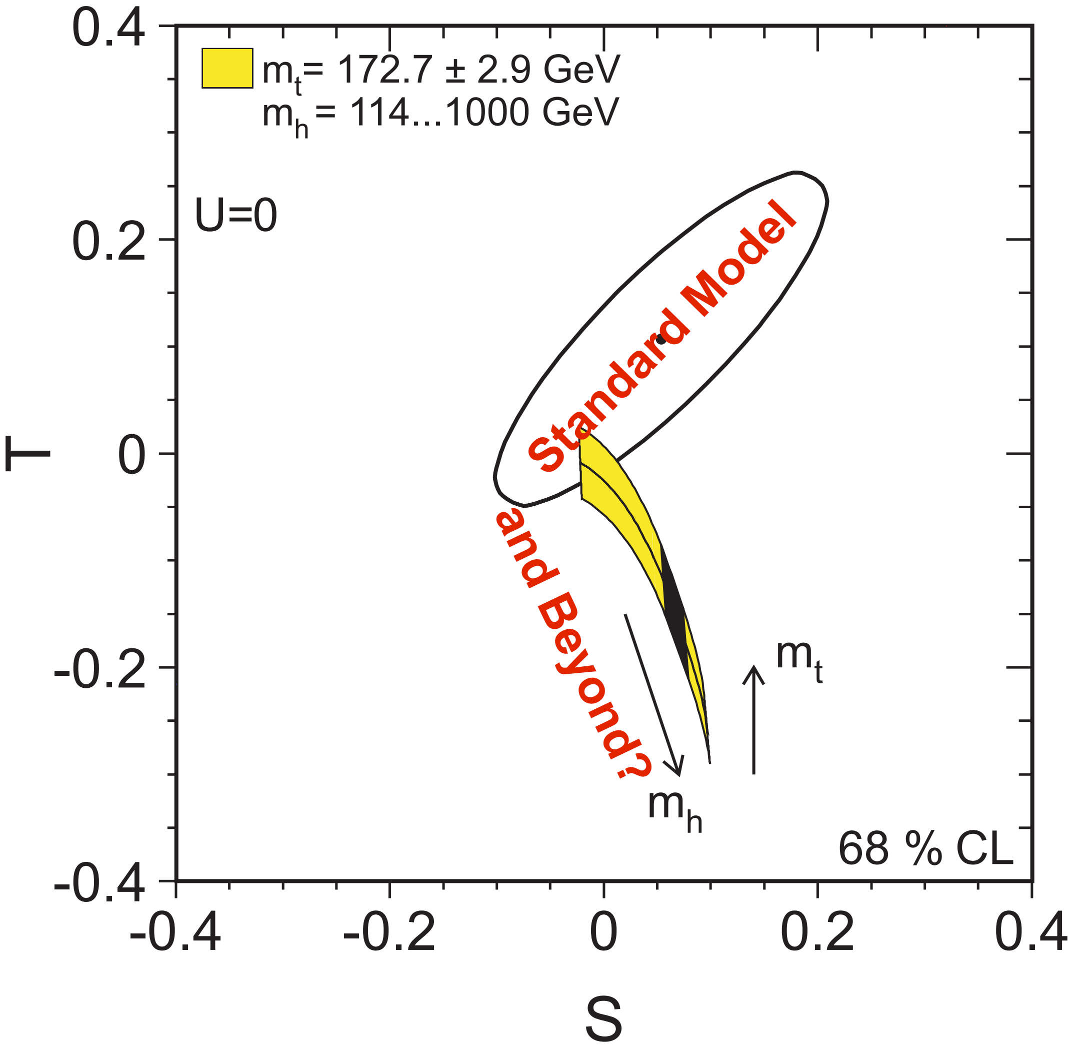
- Search for Higgs particle

- Origin of Electroweak symmetry breaking

- Is there a Standard Model Higgs particle?
- If not, what generates the masses of the weak bosons and fermions?
- New strong dynamics?
- Composite Higgs mechanism?

Primary focus of lattice BSM effort and of this talk





Two logical choices to accommodate heavy Higgs (or no Higgs) scenario:

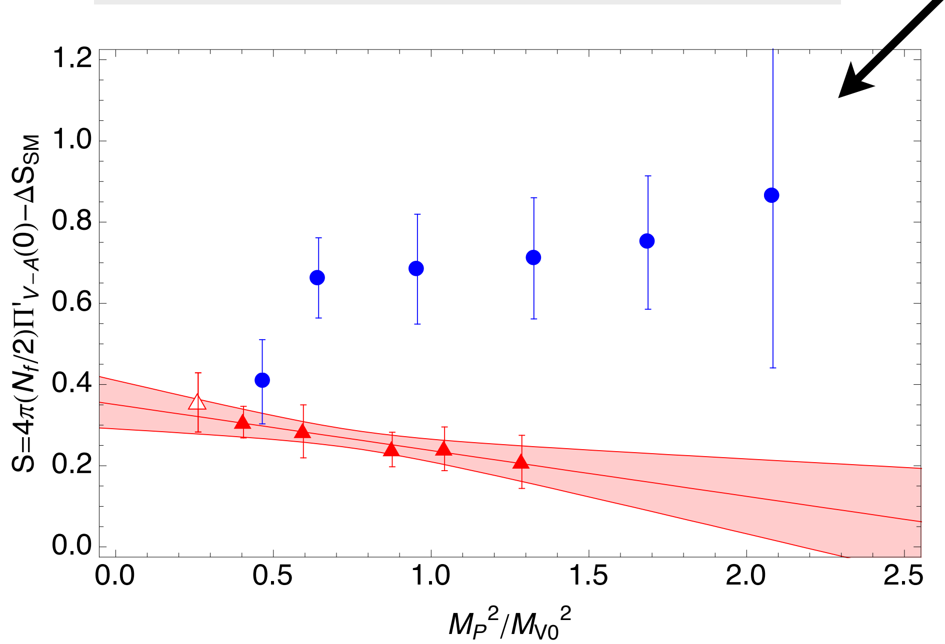
- **use some effective theory with TeV scale higher dimensional operators**

- **new microscopic theory on TeV scale**

Composite Higgs mechanism - Technicolor 2.0 ?

- **The paradigm is interesting again**
- **Requires non-perturbative lattice studies**
- **It is difficult, but there will be real results**

Nf=2 and Nf=6 S-parameters
Lattice Strong Dynamics Collaboration



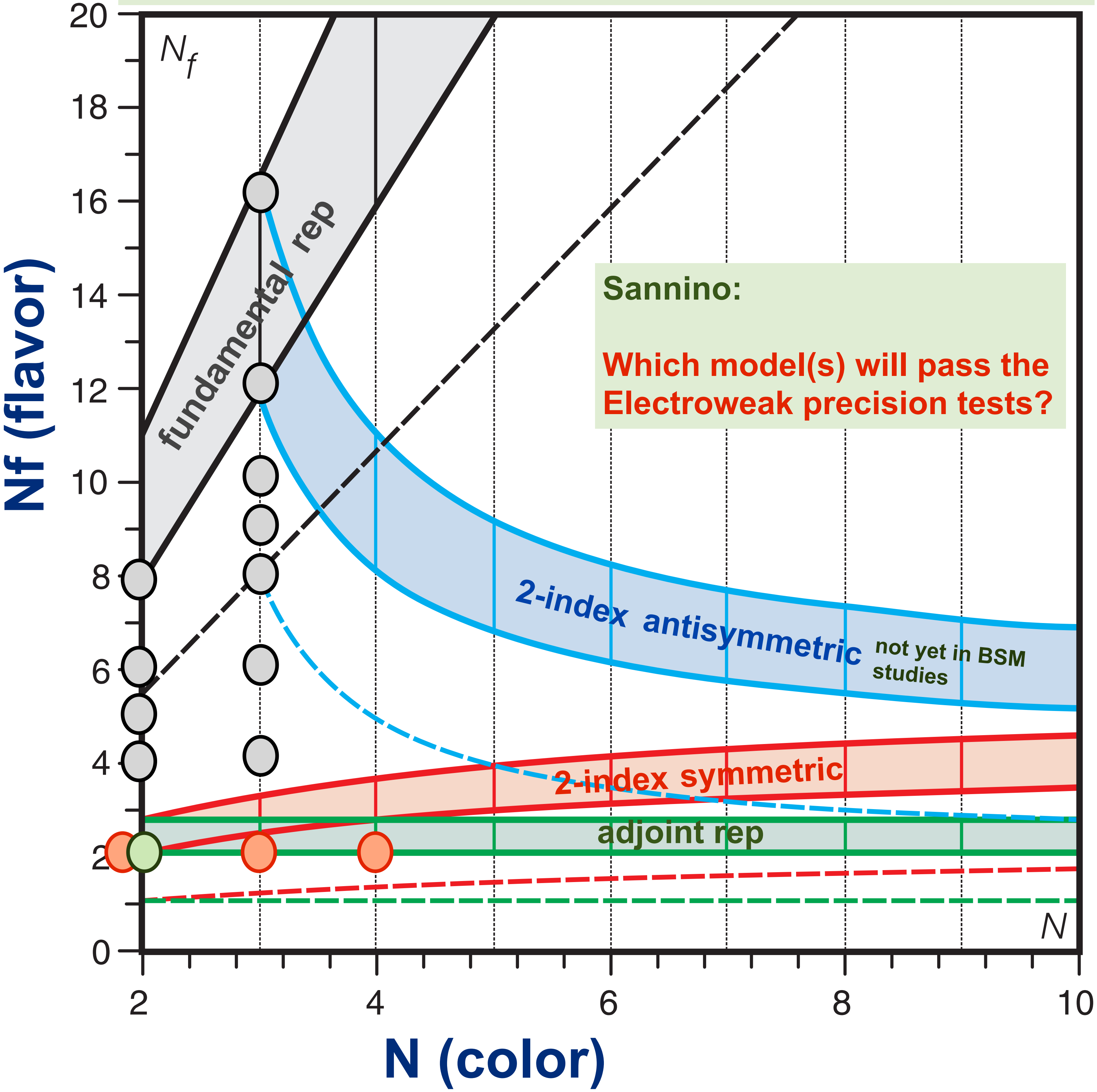
Parity Doubling and the S Parameter Below the Conformal Window.
LSD Collaboration (Thomas Appelquist , Ron Babich, Richard C. Brower, Michael Cheng, Michael A. Clark, Saul D. Cohen, George T. Fleming, Joe Kiskis, Meifeng Lin, Ethan T. Neil, James C. Osborn, Claudio Rebbi, David Schaich, Pavlos Vranas) **Phys.Rev.Lett.**

e-Print: **arXiv:1009.5967 [hep-ph]**

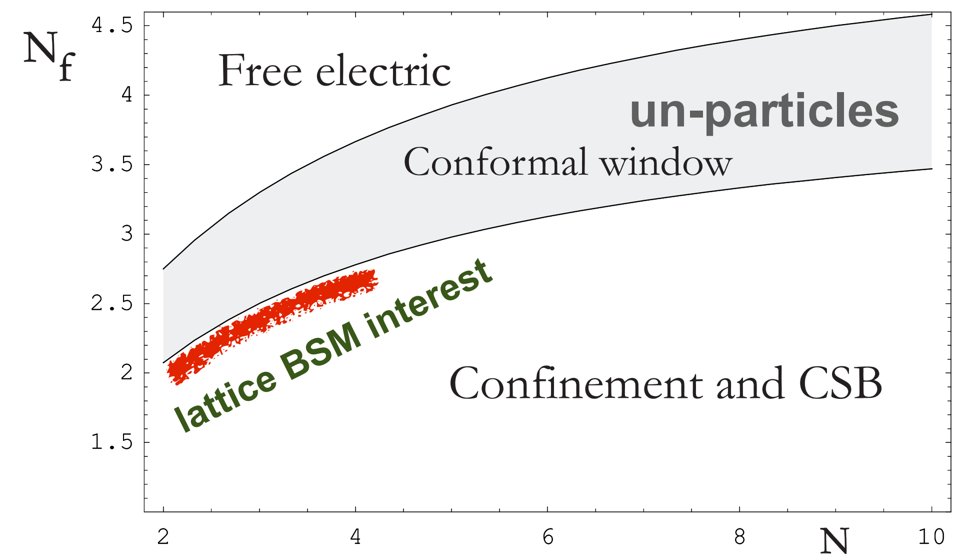
more at this workshop (David Schaich)

theory space and conformal window: critically important for composite Higgs and TC/ETC

space of color, flavor, and fermion representation



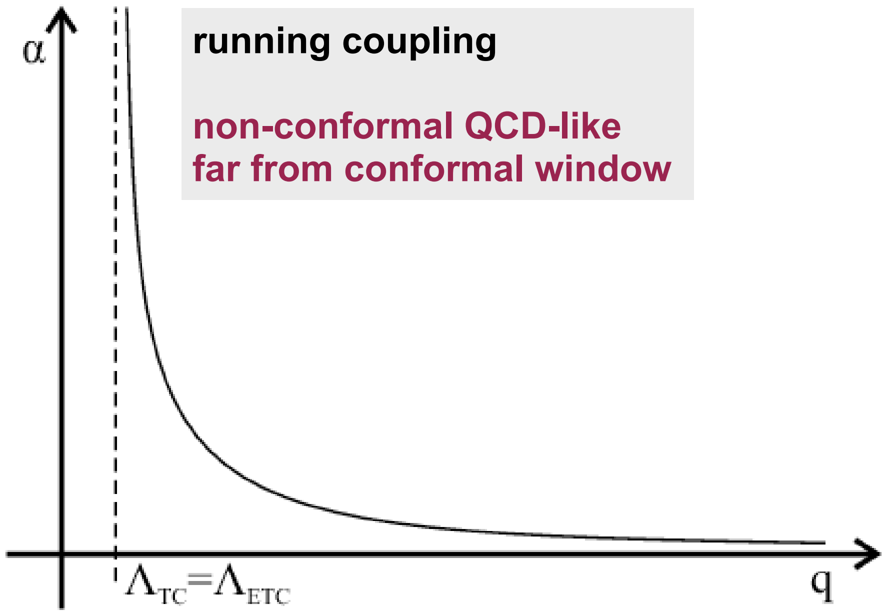
for each rep BSM interest is below conformal window but close to it:

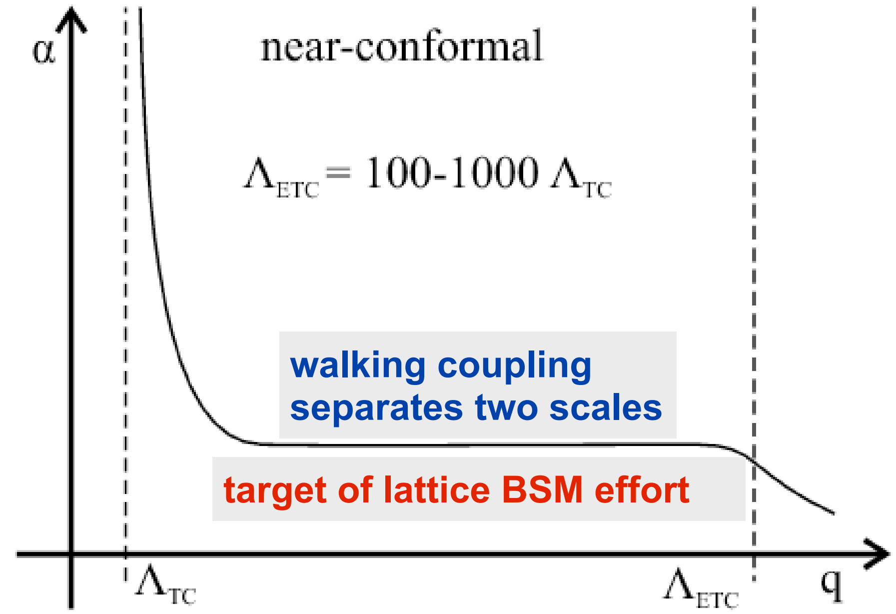


lattice results of last 3 years in
3 reps including new projects
just starting

it is stimulating to have controversial results close to the conformal window: these are the interesting candidate models

Composite Higgs mechanism? (Technicolor and Extended Technicolor in the past)



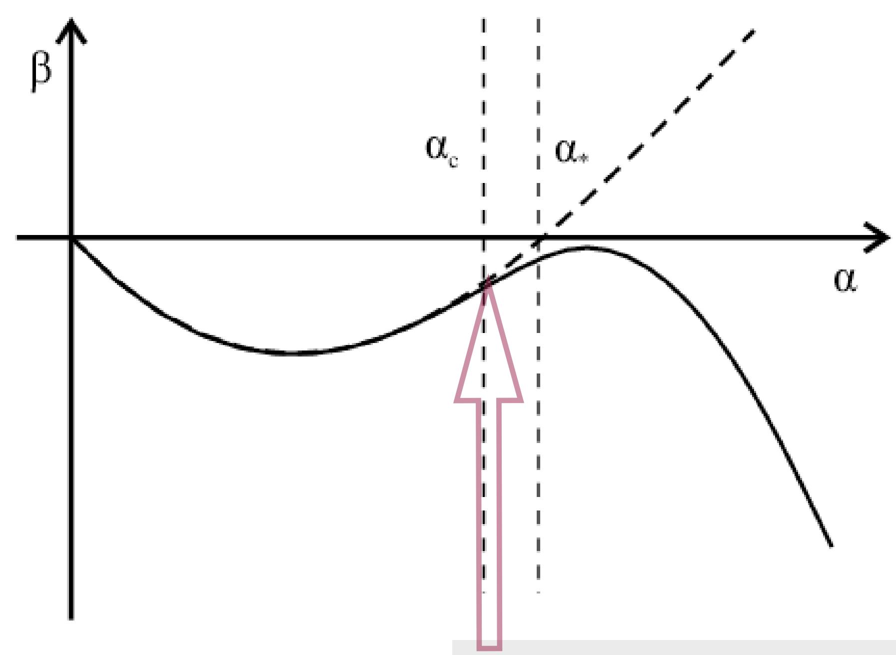


Extended Technicolor paradigm:

- requires walking gauge coupling chiral SB on $\Lambda_{TC} \sim TeV$ scale
- fermion mass generation from scale at $\Lambda_{ETC} \sim 100 - 1000\Lambda_{TC}$
- can solve problem of flavor changing currents
- composite Higgs mechanism
- broken Dilaton ➔ unusual composite Higgs particle in BSM ?
- can avoid conflict with EW precision constraints
- *candidate models require non-perturbative lattice studies*

original textbook Technicolor paradigm:

- one massless fermion doublet $\begin{bmatrix} u \\ d \end{bmatrix}$ chiral SB
- three Goldstone pions
- become longitudinal components of weak bosons
- composite Higgs mechanism scale of Higgs condensate ~ F=250 GeV $\Lambda_{TC} \sim TeV$
- flavor changing currents and fermion mass generation would be problems
- conflicts with EW precision constraints



Chiral symmetry breaking turns conformal FP into walking

This is what lattice studies in BSM theory space potentially could deliver

This is all difficult and not QCD-like!

We do not know the answer, but:

If we knew what we were doing, it wouldn't be called research

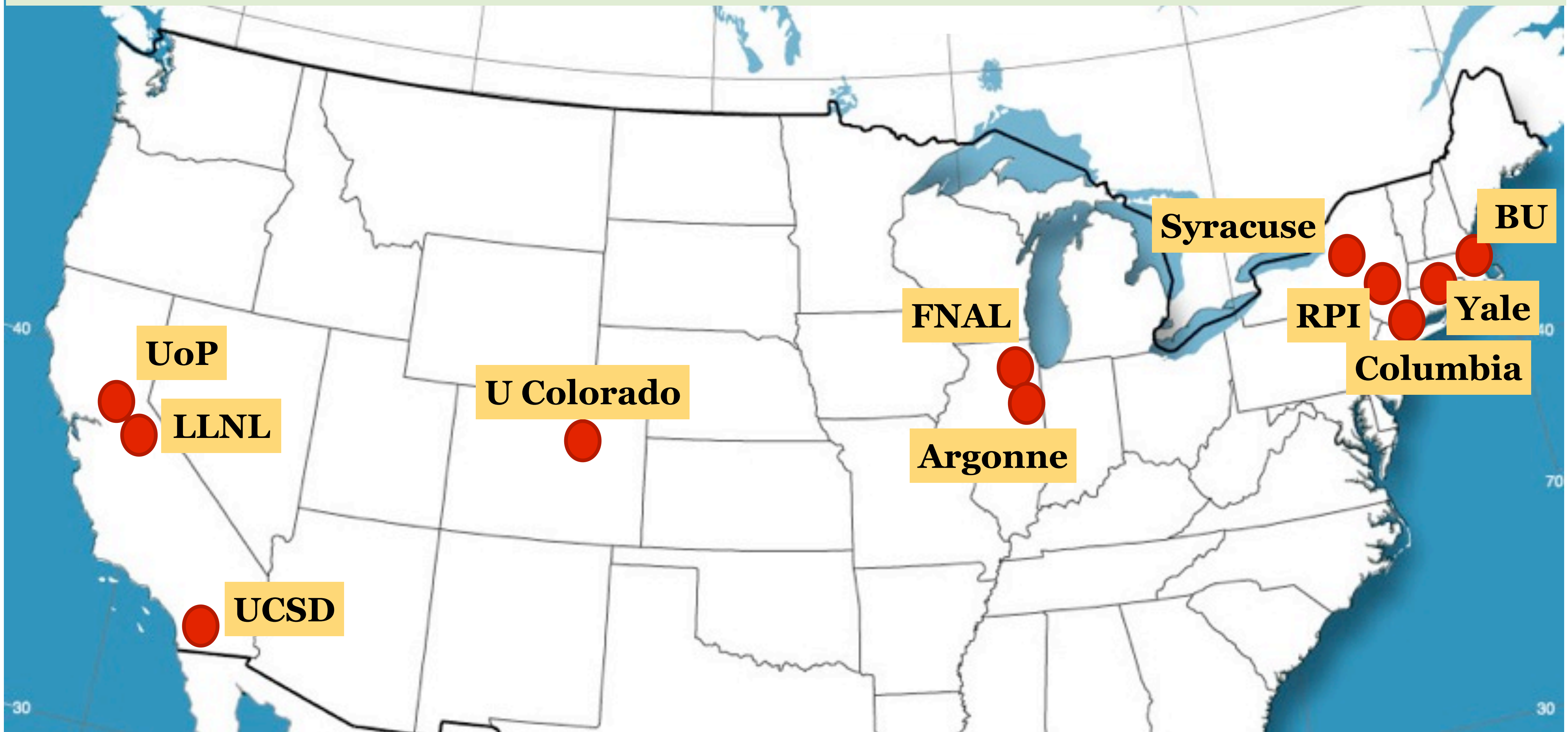
A. Einstein

It is a world-wide effort:

US BSM project sites using USQCD hardware & software support

(three years ago map was almost empty)

Kudos to the Yale group for stimulation and letting the genie out !



Lattice BSM groups study the composite Higgs mechanism
TC scale - perhaps stretched to ETC scale by walking coupling?

fermion mass generation has to be built on the top of it
- some new theory on ETC scale

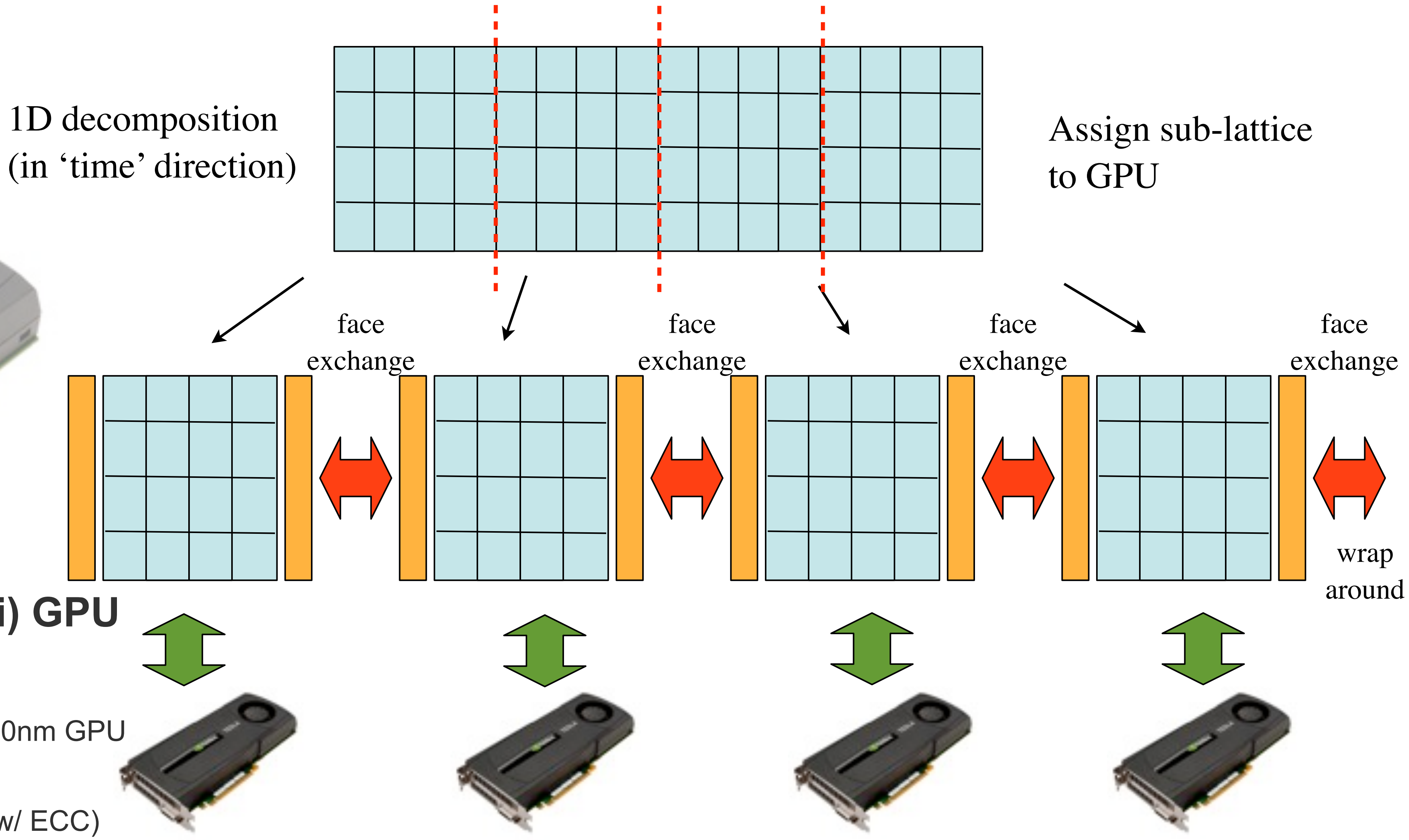
example of growing resources: **Lattice BSM GPU computing**
Technicolor video games

We have new computing technology for lattice BSM effort

Lattice Higgs Collaboration

Wuppertal technology

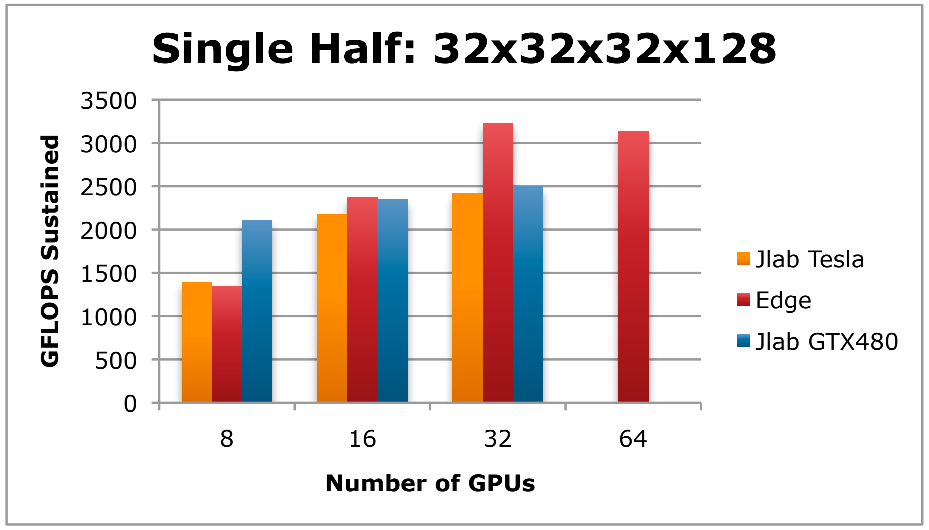
QUDA Parallelization USQCD



NVIDIA Tesla C2050 (Fermi) GPU

Silicon Mechanics Part Number: 19417
Manufacturer Part Number: 900-21030-2200-000

- NVIDIA® Tesla™ third-generation 40nm GPU
- 448 CUDA cores
- 3 GB GDDR5 Memory (2.625 GB w/ ECC)
- Dual Precision 515 GFlops
- Single Precision 1003 GFlops
- PCIe 2.0 x16 full-length, dual slot



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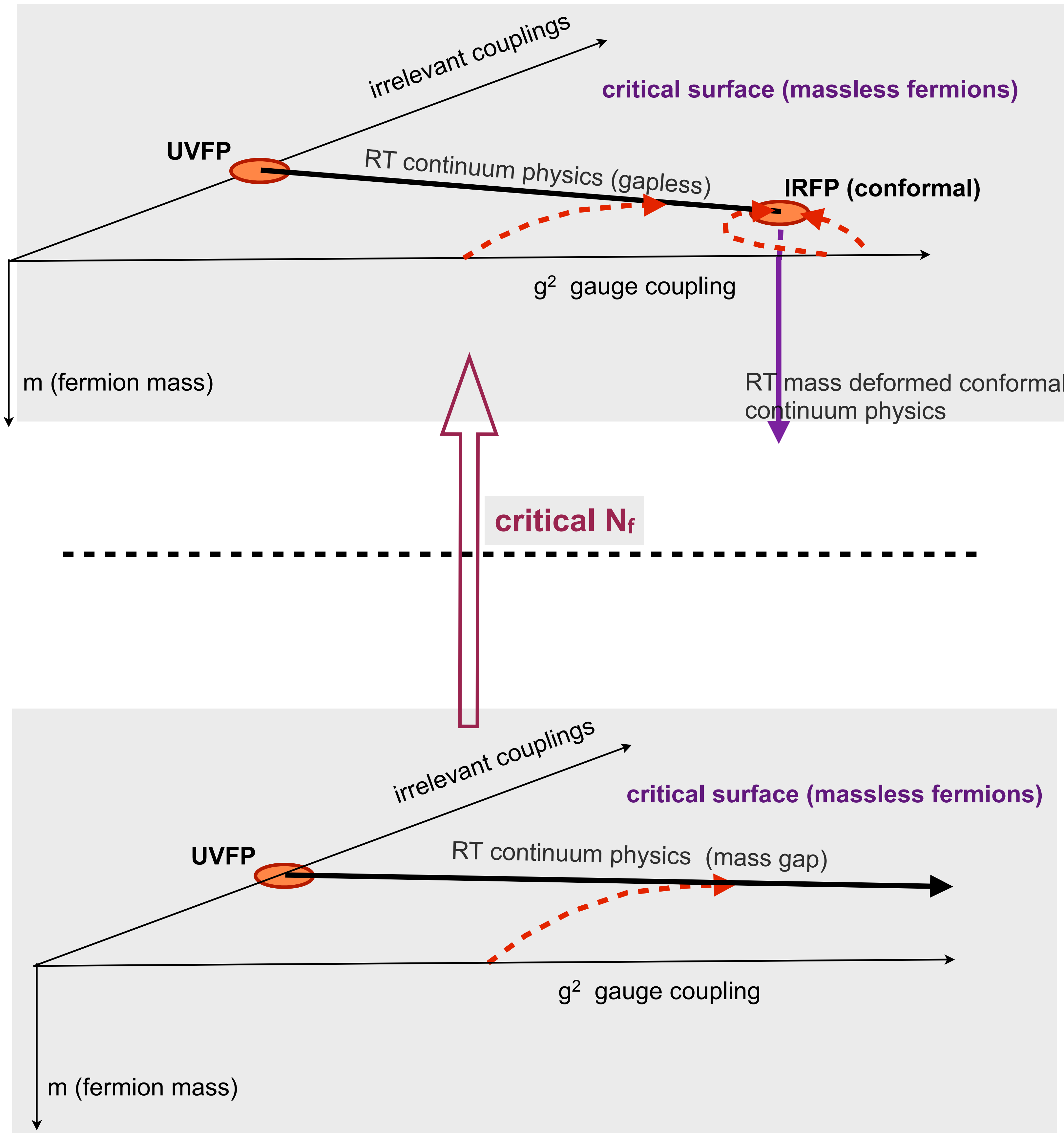
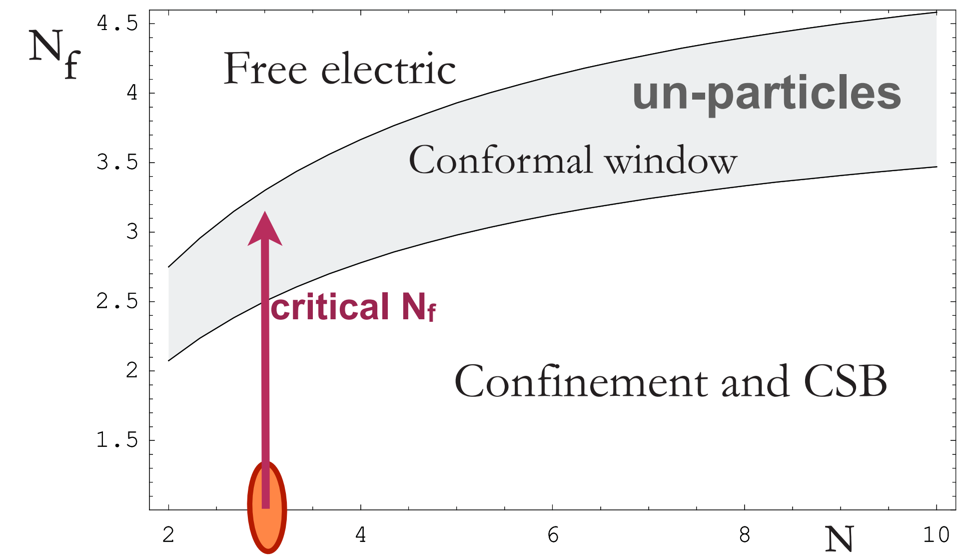
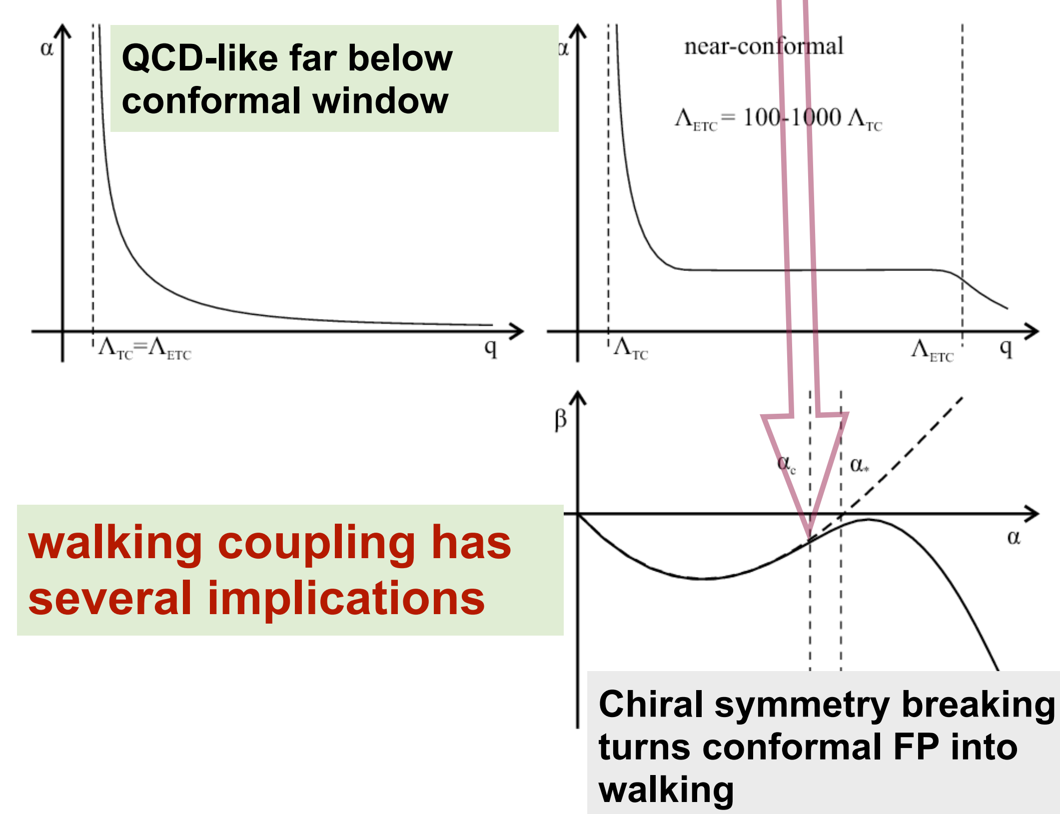


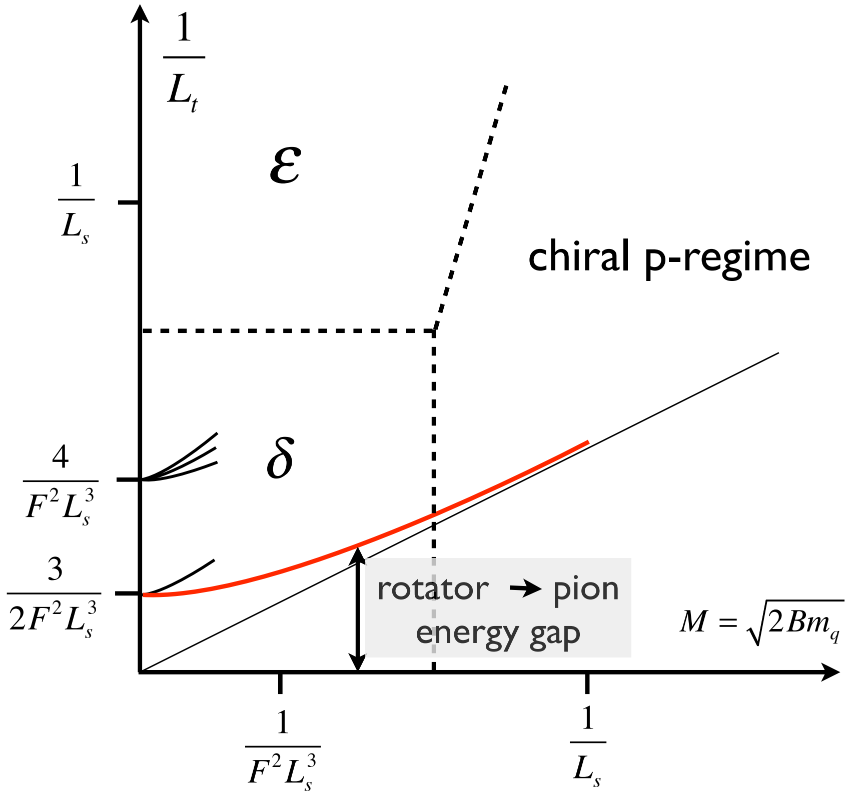
inside the conformal window:



with increasing N_f walking scenario expected to arise:



Chiral regimes to identify in theory space below conformal window:



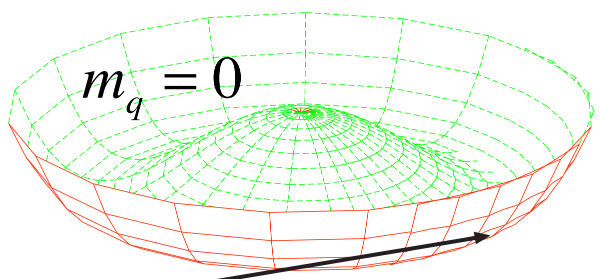
Goldstone dynamics is different in each regime

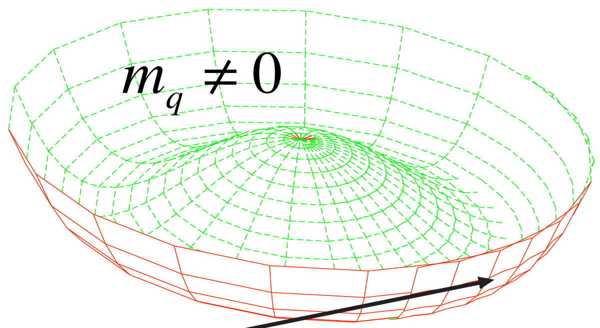
We study δ and ε-regimes (RMT)

and p-regime (probing chiral loops)

complement each other

interpretation of rotator levels in $m_q \to 0$ limit:

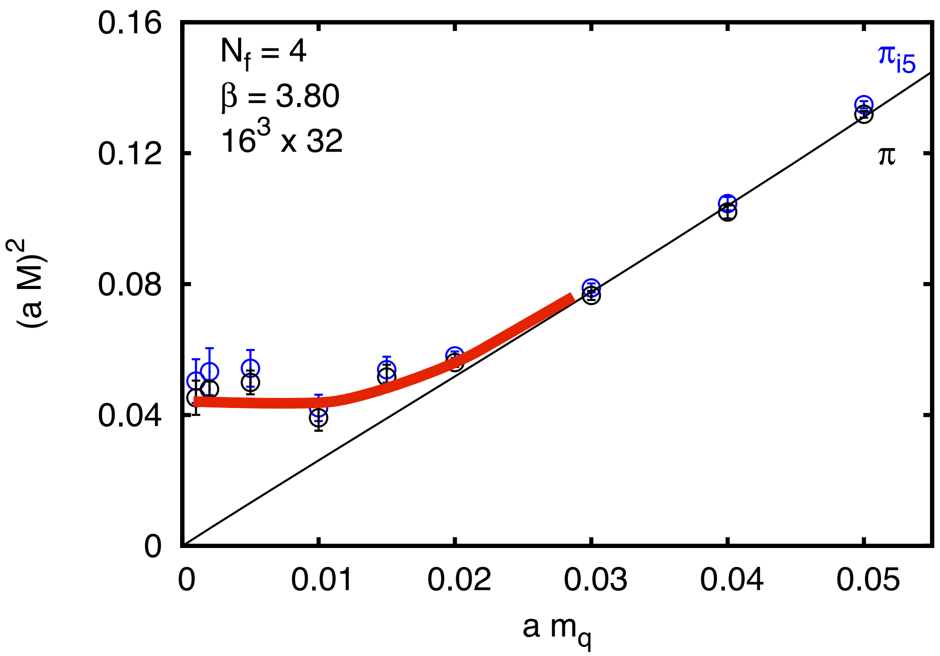


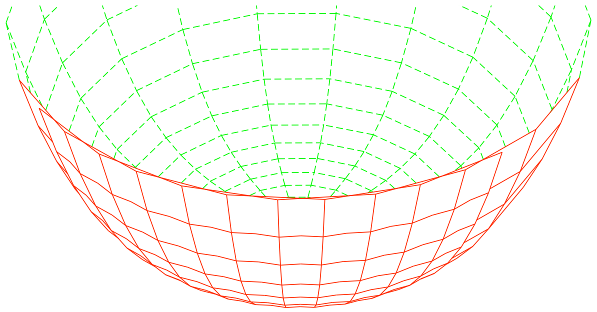


Veff: chiral condensate in flavor space

arbitrary orientation of condensate

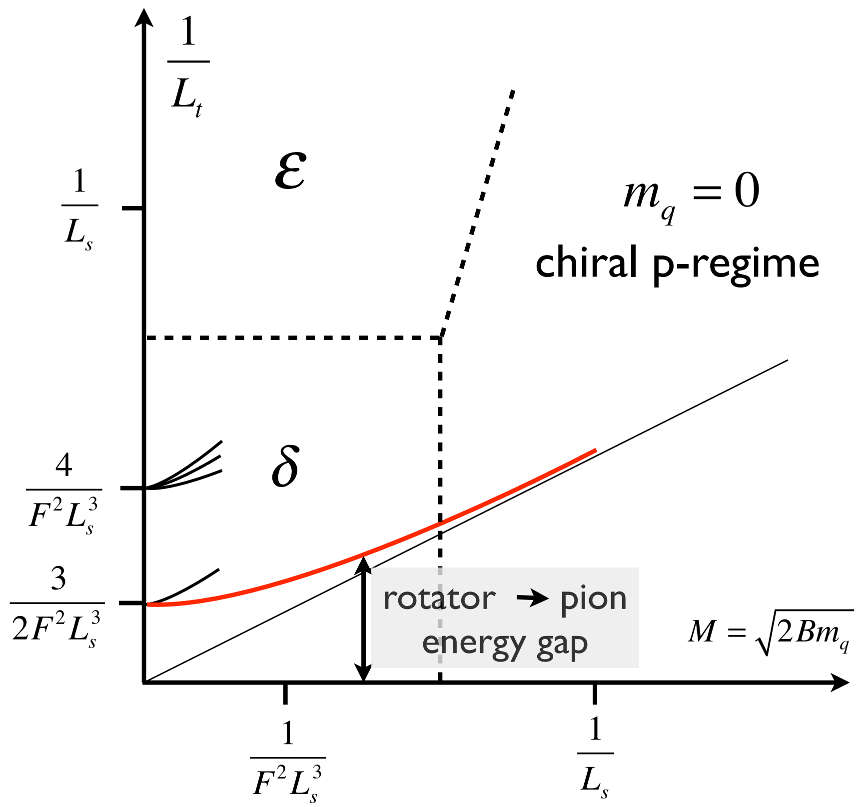
tilted condensate





Not to misidentify rotator gaps as evidence of chirally symmetric phase !

One-loop chiral expansion in p-regime:



Note 1/N$_f$ scaling of pion mass!

warning: 2-loop ~ (N$_f$)^2 (Bijnens)

$$M_\pi^2 = M^2\left[1 - \frac{M^2}{8\pi^2 N_f F^2} ln\left(\frac{\Lambda_3}{M}\right)\right] \quad + O((N_f)^2) \qquad M^2 = 2Bm$$

$$F_\pi = F\left[1 + \frac{N_f M^2}{16\pi^2 F^2} ln\left(\frac{\Lambda_4}{M}\right)\right] \quad + O((N_f)^2)$$

$$M_\pi(L_s,\eta) = M_\pi\left[1 + \frac{1}{2N_f}\frac{M^2}{16\pi^2F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right] \qquad \lambda = ML_s$$

$$F_\pi(L_s,\eta) = F_\pi\left[1 - \frac{N_f}{2}\frac{M^2}{16\pi^2F^2}\cdot\widetilde{g}_1(\lambda,\eta)\right] \qquad \tilde{g}_1(\lambda,\eta) \approx 24K_1(\lambda)/\lambda \text{ for } \eta = \frac{L_t}{L_s} \gg 1$$

Chiral expansion parameter is $N_f \frac{M^2}{16\pi^2F^2}$ with $\ll$ 1 condition

$N_f = 8$ fundamental rep in USQCD BSM project

set $N_f \frac{M^2}{16\pi^2F^2}$ =0.3, with $a \cdot m_\rho = 0.25$ (to keep cut-off under control), and $m_\rho / F \approx 10$ (as expected), $a \cdot M_\pi \approx 0.01$ is needed

The $M_\pi \cdot L_s \approx 10$ condition (to control FSS) will require $L_s \approx 100$! Same scale as largest QCD projects!

$N_f = 2$ higher reps (like sextet) are more favorable for chiral expansion

Condition of reaching the chiral expansion regime can also be estimated from rotator spectrum $\Rightarrow$

Condition of reaching the chiral expansion regime can also be estimated from rotator spectrum $\Rightarrow$

$E_l = \frac{1}{2\theta} l(l+2)$ with $l = 0,1,2,\ldots$ rotator spectrum for $SU(2)_f \times SU(2)_f$

with $\theta = F^2 L_s^3 (1 + \frac{C(N_f = 2)}{F^2 L_s^2} + O(1/F^4 L_s^4))$ (P. Hasenfratz and F. Niedermayer)

(there is in E_l an overall factor $\frac{N_f^2 - 1}{N_f}$ for arbitrary N_f)

$C(N_f = 2) = 0.45$, C will grow with $\sim N_f$, (P.Hasenfratz, $O(N_f)$ model)

there are similar considerations in the ε-regime

The rotator spectrum has the expansion parameter $\sim C\frac{N_f/2}{F^2 L_s^2}$ with $\ll$ 1 condition

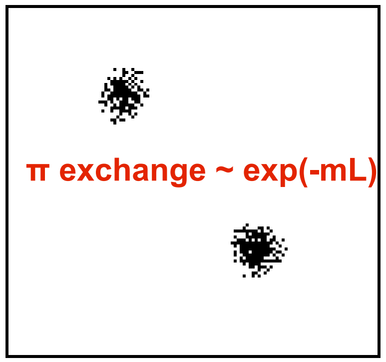
with $C\frac{N_f/2}{F^2 L_s^2} = 0.3$ $FL_s \approx 2.5$ for $N_f = 8$ (USQCD project)

with $a \cdot m_\rho = 0.25$ (to keep cut-off under control), and $m_\rho / F \approx 10$ (as expected), $L_s \approx 100$ is needed!

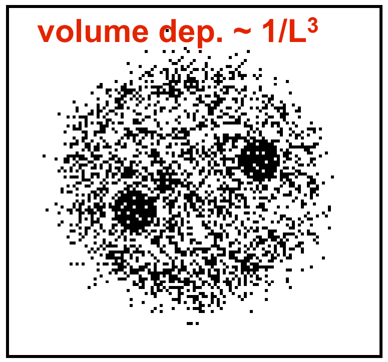
When expansion breaks down in δ – regime, same is expected in the p-regime

Deceptions of finite size behavior:

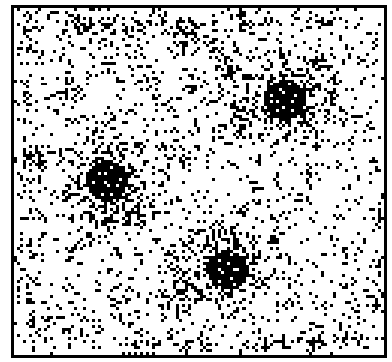
large volume
hadrons point-like



squeezed wavefunction

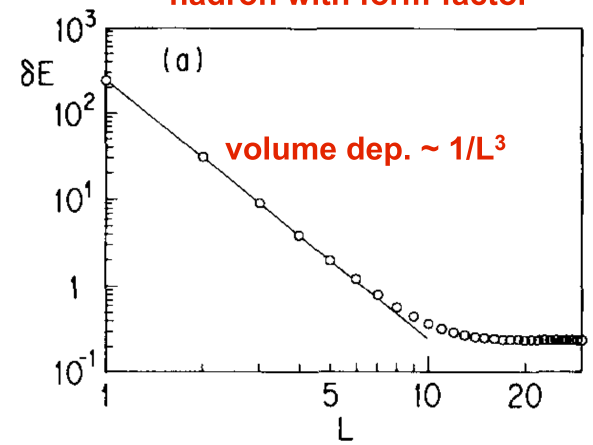


crossover to femto world

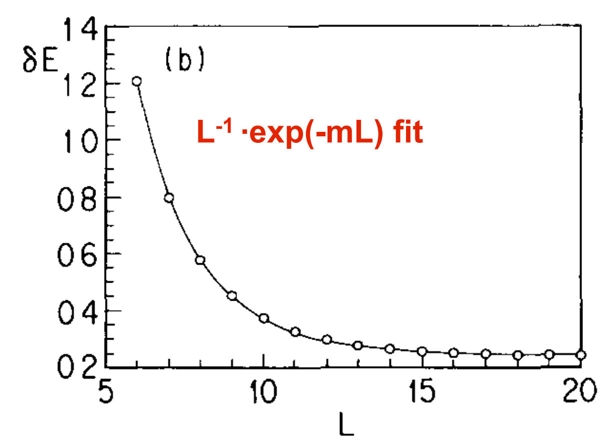


$$\hat{V}(\vec{k}) = \frac{F(\vec{k})^2}{\vec{k}^2 + m^2}$$ extended hadron with form factor $F(\vec{k})$

hadron with form-factor



$$F(k) = \frac{1}{1 + c \cdot \vec{k}^2}$$



$$F(k) = \frac{1}{1 + c \cdot \vec{k}^2}$$

$$\delta E = \sum_{\vec{n}} V(\vec{n}L)$$ hadrom self energy from interaction with images

$$\delta E = \frac{1}{L^3} \sum_{\vec{n}} \hat{V}(\vec{n}\frac{2\pi}{L})$$ Poisson resummation, $\hat{V}(\vec{k})$ is the Fourier transform

$$\hat{V}(\vec{k}) = \frac{1}{\vec{k}^2 + m^2} \quad \Rightarrow \quad \mathrm{V(r)} = \frac{e^{-mr}}{r}$$ for large r in point-like approximation

$$\delta E \approx V(0) + 6V(L) \qquad \delta E \approx \frac{e^{-mL}}{L}$$ point-like interaction for large L (non-relativistic)

Lüscher made it relativistic using field theory

Leutwyler put in the chiral vertices, hence the $\tilde{g}(mL)$ form in chiral PT

the size where the $1/L^3$ correction to the masses disappears and the exponential behavior sets in depends on the behavior of the hadron form factor

the characteristic inverse power vs. exponential behavior can frustrate at limited lattice sizes the analysis of chiral vs. conformal hypotheses

the size where the 1/L3 correction to the masses disappears and the

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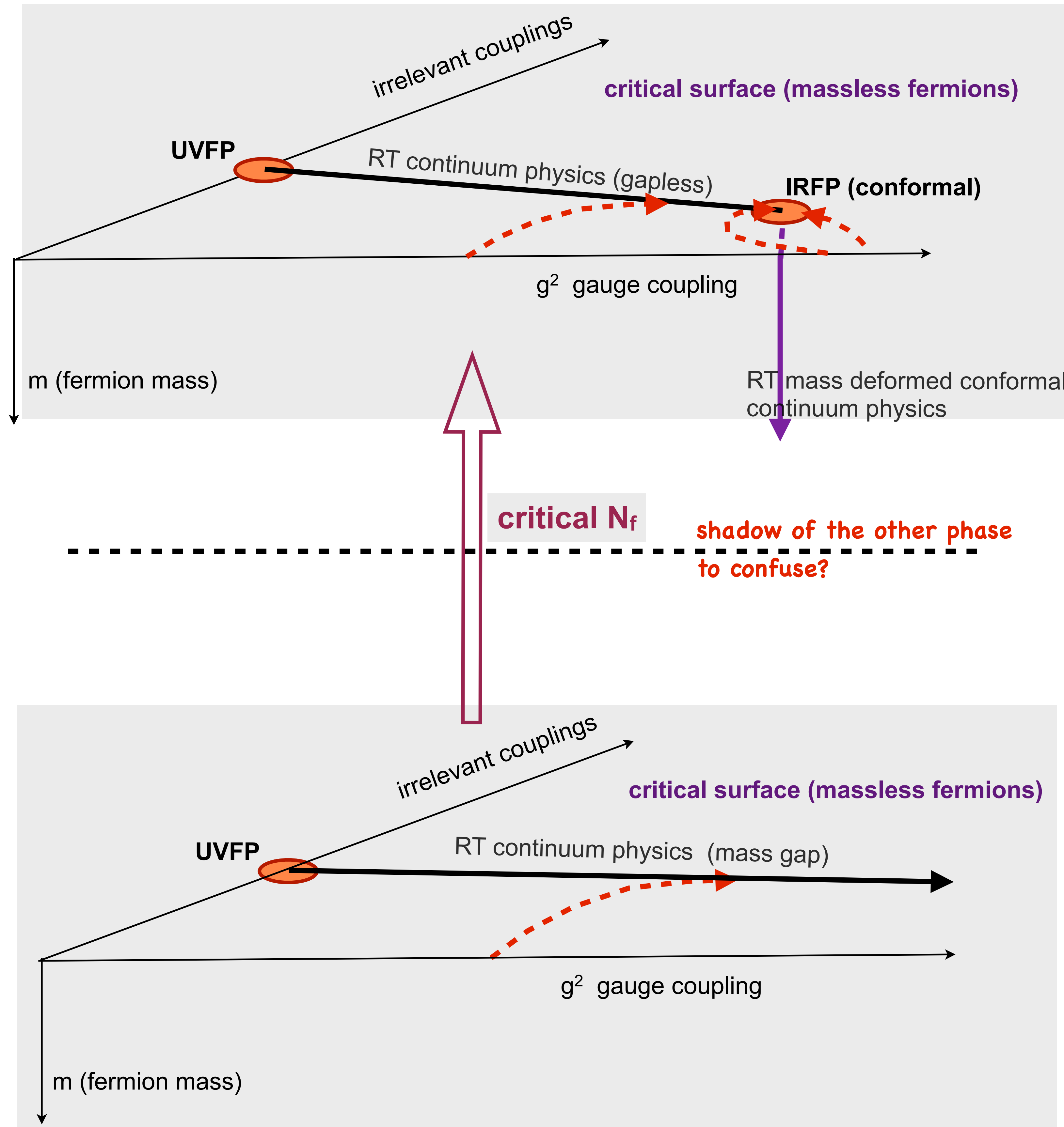
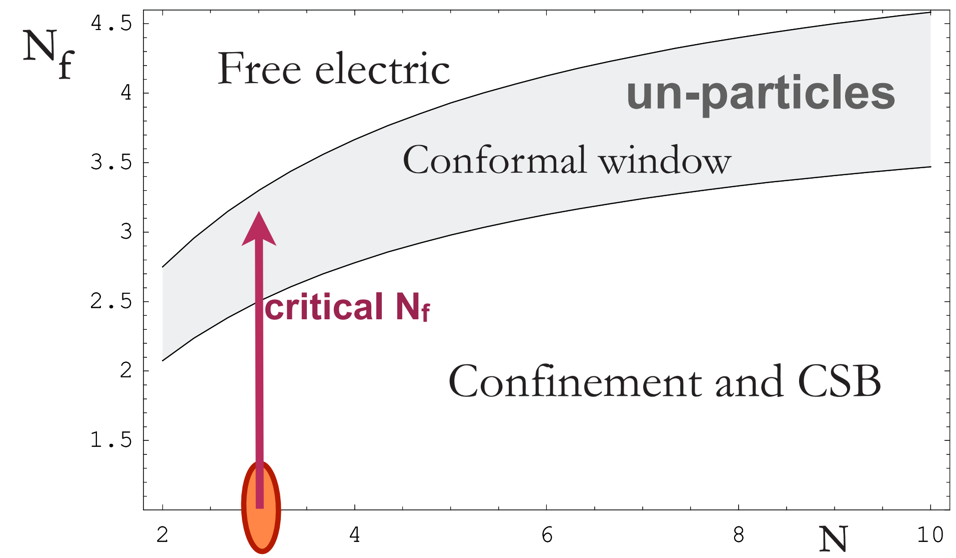
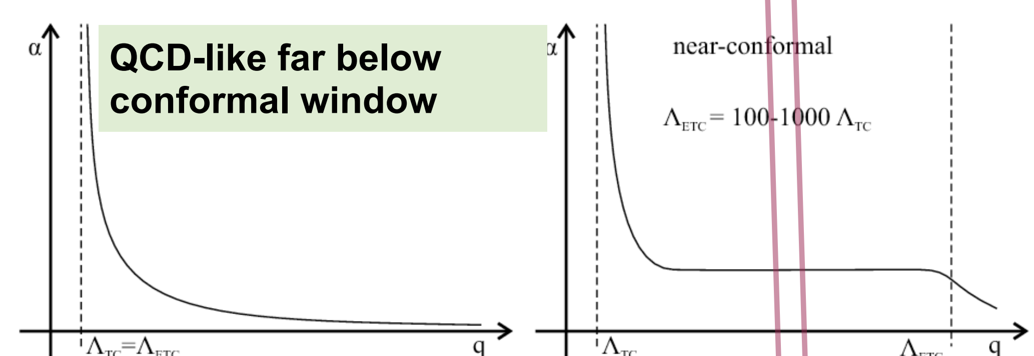
cut-off control in non-perturbative lattice calculations from RG flow

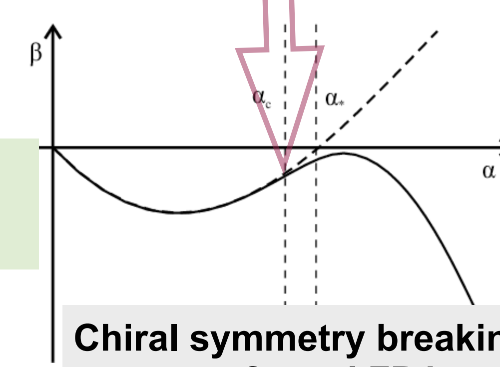


inside the conformal window:



with increasing N$_f$ walking scenario expected to arise:

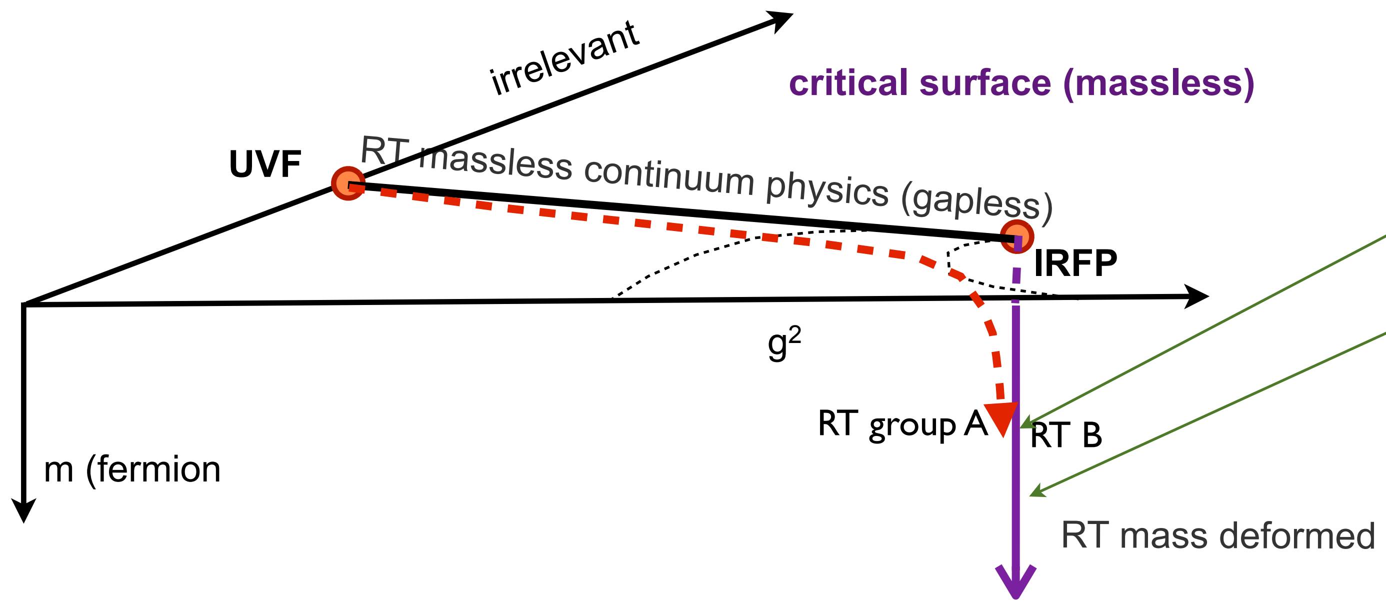




walking coupling has several implications

Chiral symmetry breaking turns conformal FP into walking

conformal scaling and scaling violations



if model had conformal IRFP

two interchangeable RT descriptions?

continuum mass deformed conformal theory is on RT coming out of IRFP

I worked out as an example all the details of 3D scalar theory (Ising model) with IRFP

textbook material

Debbio and collaborators
early conform apps

free energy on RT:

$$f(u_1,u_2,\ldots) = g(u_1,u_2,\ldots) \;+\; b^{-d} f_s(b^{y_1}u_1, b^{y_2}u_2\ldots)$$

analytic singular

$y_1 > 0$ only relevant exponent in our case
$u_1 = t \sim m$ identified, $y_1 = y_m$ in Technicolor notation

y_2 controls scaling violations, leading correction term

analytic function which can have terms like $\sim m^k$ are typically sub-leading

Fisher and Brezin worked out most of what we know!

RG scaling of 2-point function:

$$G^{(2)}(r,m,u_2,\ldots) = b^{-2d} G(r/b, b^{y_m} m, b^{y_2} u_2,\ldots)$$

from $G^{(2)}(r,m,u_2,\ldots) \sim e^{-Mr}$ asymptotics with $M \sim m^{1/y_m}$ scaling follows

leading correction to the scaling term should be $\sim m^{\omega}$ where $\omega = \beta'(g^*)$

analysis would change with second relevant operator at IRFP!

- analytic terms exists, but no reason to be leading conformal scaling correction

- correlators of composite operators require inhomogeneous RG!

similarly, in conformal finite size scaling analysis:

$$\xi / L = f_1(x) + L^{-\omega} f_2(x) \;\; \text{with} \;\; x = Lm^{1/y_m} \longrightarrow$$

This directly transcribes to hadron masses and F_π

correlation length measured in L units

finite size scaling correction terms require very accurate data

Chiral hypothesis   incomplete analysis on each side   Conformal hypothesis

chiral logs not reached yet in important models!
(like N_f=8, or N_f=12)

$$(M_\pi^2)_{NLO} = (M_\pi^2)_{LO} + (\delta M_\pi^2)_{1-loop} + (\delta M_\pi^2)_{m^2} + (\delta M_\pi^2)_{a^2 m} + (\delta M_\pi^2)_{a^4}$$

$\sim m^2$ kept $\sim a^2 m$ cutoff term in B $\sim a^4$ see LO a^2 term

$$(M_\pi^2)_{LO} = 2B \cdot m + a^2 \Delta_B$$

would require more data

$$(\delta M_\pi^2)_{1-loop} = [(M_\pi^2)_{LO} + a^2]^2 \ln(M_\pi^2)_{LO}$$

$$\boxed{M_\pi^2 = c_1 m + c_2 m^2 \ + \text{logs}}$$

fitted function for all Goldstones

$$\boxed{M_{nuc} = c_0 + c_1 m \ + \ \text{logs}}$$

nucleon states, rho, a1, higgs, ...

$$(F_\pi)_{LO} = F, \quad (\delta F_\pi)_{1-loop} = [(M_\pi^2)_{LO} + a^2] \ln(M_\pi^2)_{LO}$$

chiral log regime was not reached in fermion mass range

$$(\delta F_\pi)_{m^2} \sim m, \quad (\delta F_\pi)_{a^2 m} = a^2$$

kept   cutoff term in F

$$\boxed{F_\pi = F + c_1 m + \text{logs}}$$

fitted function

$$\boxed{\langle \bar\psi\psi \rangle = \langle \bar\psi\psi \rangle_0 + c_1 m + c_2 m^2 + \text{logs}}$$

chiral condensate

$$\boxed{M_\pi = c_\pi \cdot m^{1/y_m}, \qquad y_m = 1 + \gamma}$$

leading conformal scaling
functional form for all hadron masses

$$\boxed{F_\pi = c_F \cdot m^{1/y_m}, \qquad y_m = 1 + \gamma}$$

same critical exponent

$$\boxed{\langle \bar\psi\psi \rangle = c_\gamma \cdot m^{(3-\gamma)/y_m} + c_1 m}$$

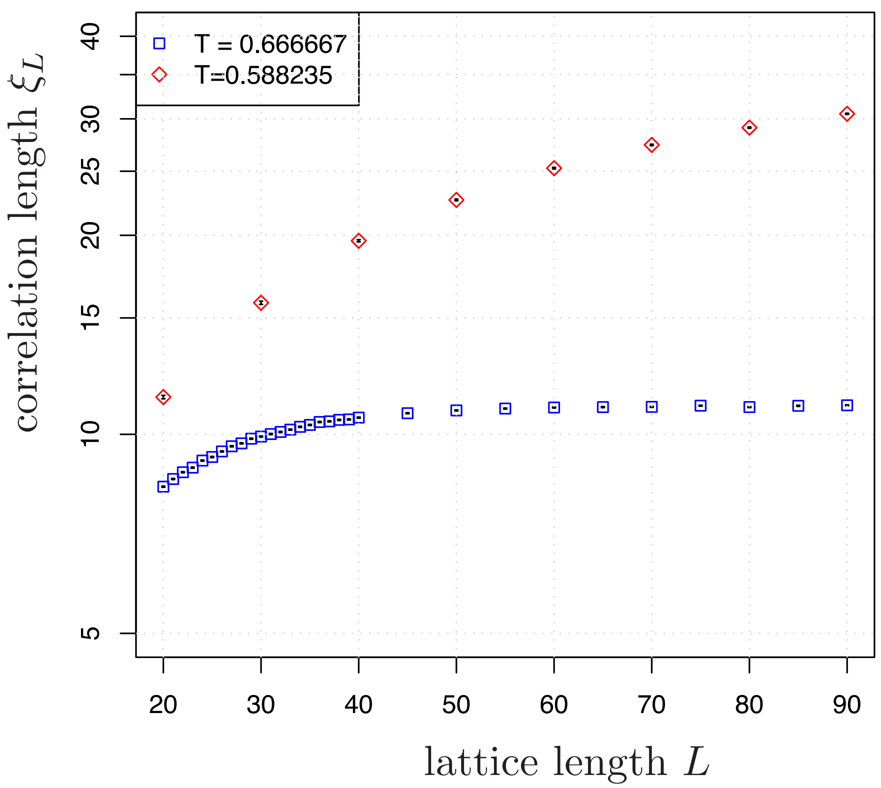
Debbio and Zwicky

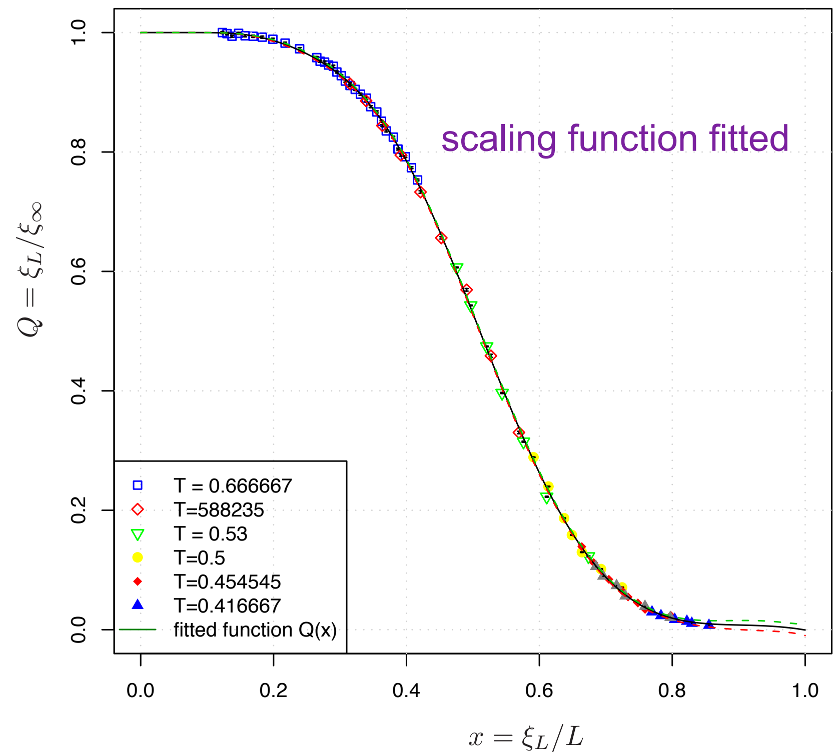
Asymptotic infinite volume limit has not been reached yet in important candidate models for conformal window

infinite volume conformal scaling violation analysis ?

conformal finite size scaling analysis and its scaling violations ?

but FSS works! 2d O(3) model UVFP (at T=1/β=0)





$$H = -\beta \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad m = 1/\xi = C_m \cdot \Lambda_L$$

$$\Lambda_L = 2\pi\beta \cdot \exp(-2\pi\beta)[1 + a_1/\beta + \ldots]$$

from Bethe ansatz:

$$m/\Lambda_{MS} = 8/e \qquad \Lambda_{MS}/\Lambda_L = 2^{5/2} e^{\pi/2}$$

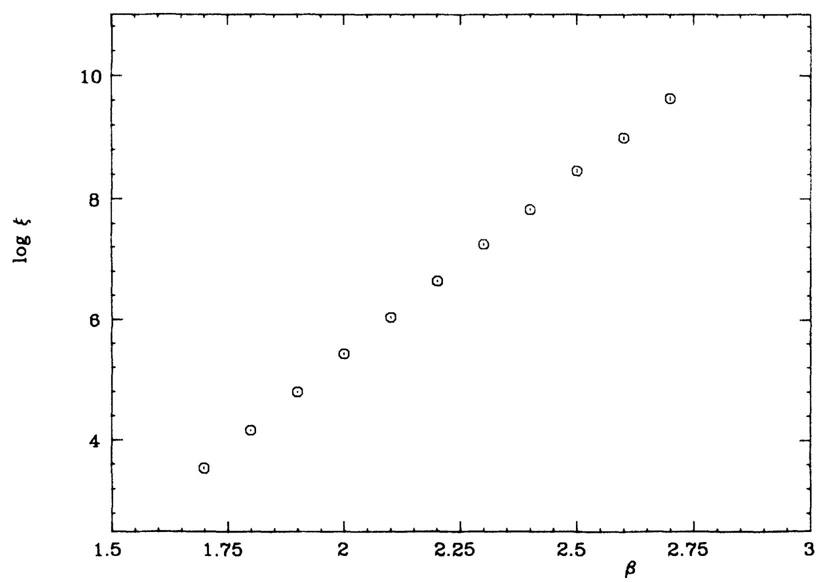
from FSS:

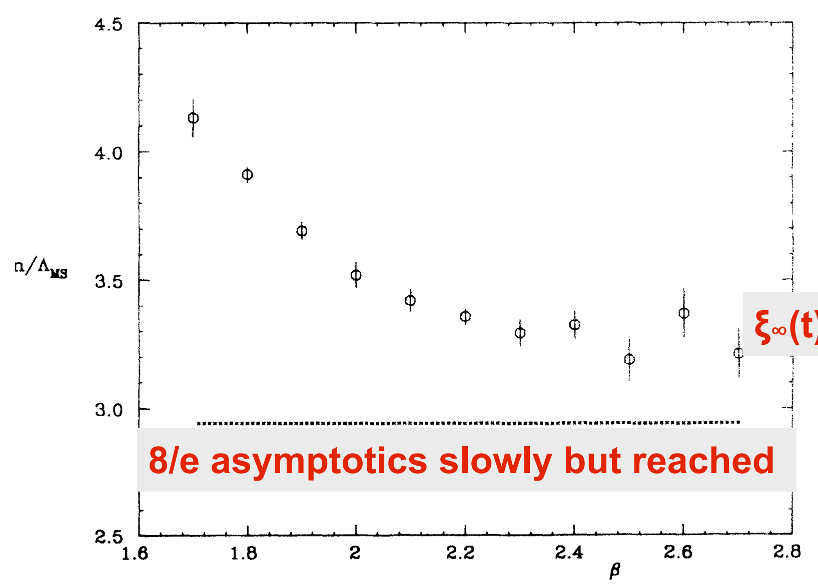
$$P_L(t) = P_\infty(t) \cdot Q_P(x(t)), \quad x(t) = \xi_L(t)/L$$

for any bulk physical quantity P(t)

$Q_P(x(t))$ does not depend on t explicitly!

applied to $P_\infty(t)=\xi_\infty(t)$



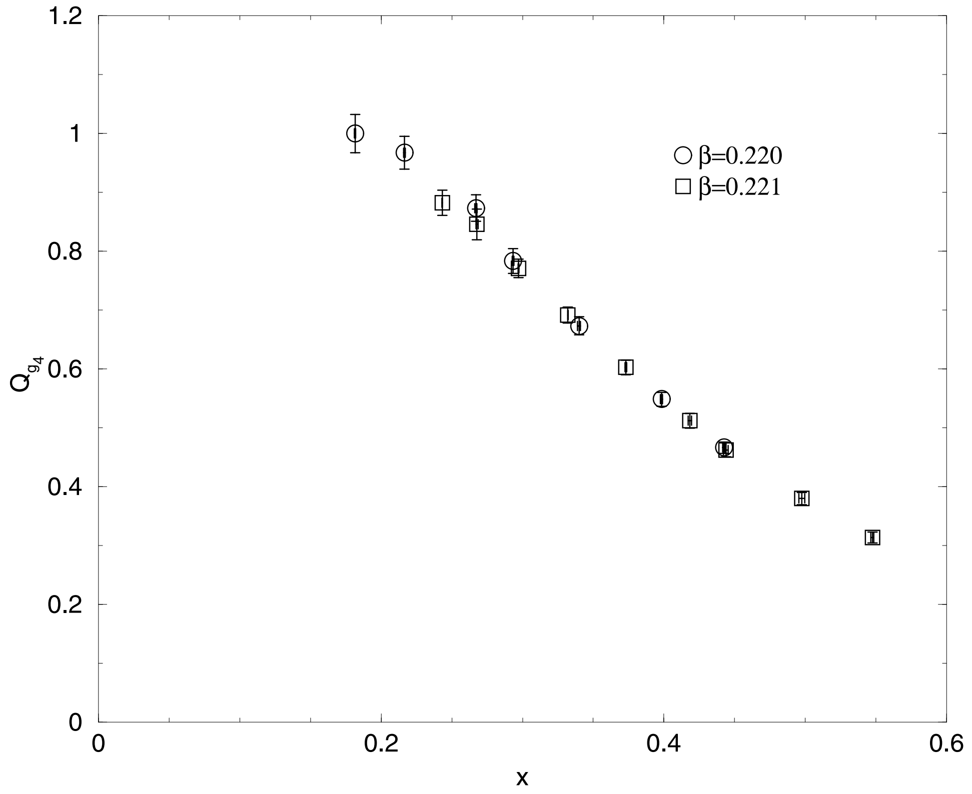


$\xi_\infty(t)$ would be ~ 22,000 at β=2.7 !

8/e asymptotics slowly but reached

FSS is enormously powerful

FSS works again! 3d Ising model IRFP $(g_4)^*$ conformal



applied again from FSS:

$$P_L(t) = P_\infty(t) \cdot Q_P(x(t)), \quad x(t) = \xi_L(t)/L$$

applied to $P_\infty(t)=g_4(t)$ renormalized coupling

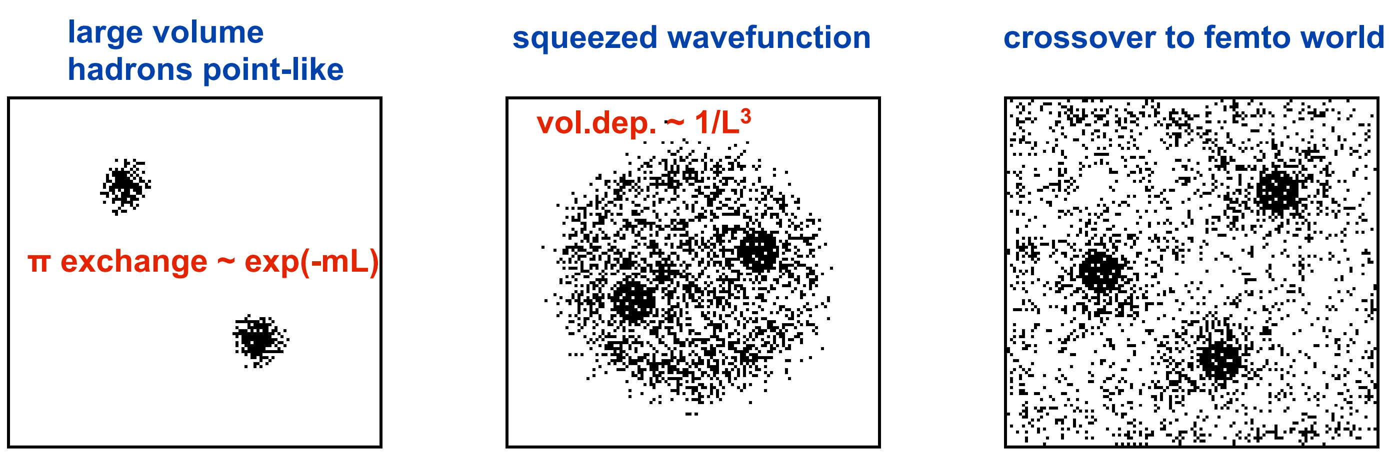
$$g_4(t) = -\frac{\chi_4(t)}{\xi^3 \cdot \chi_2(t)^2}$$

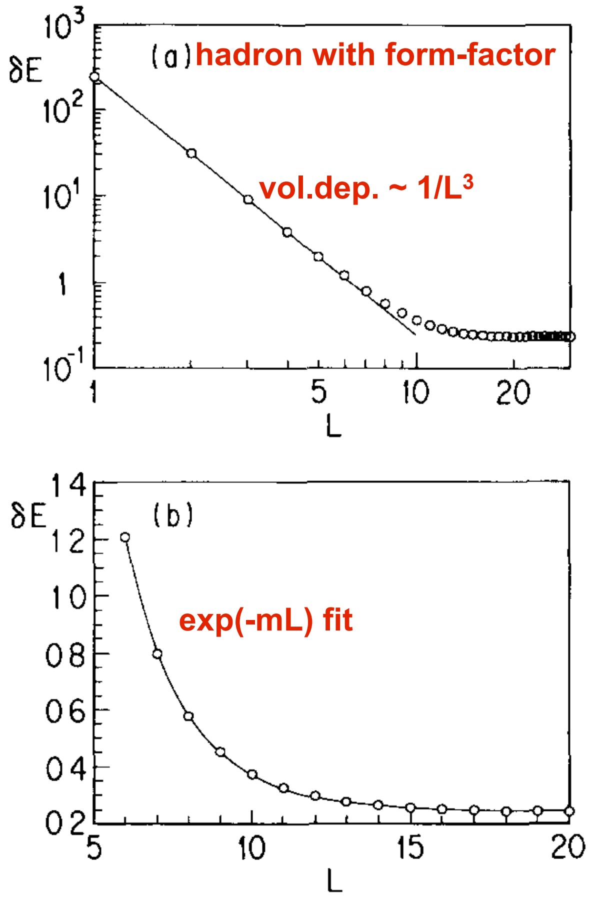
we are working on similar FSS methods in Nf=12 model under the conformal hypothesis

caviats:

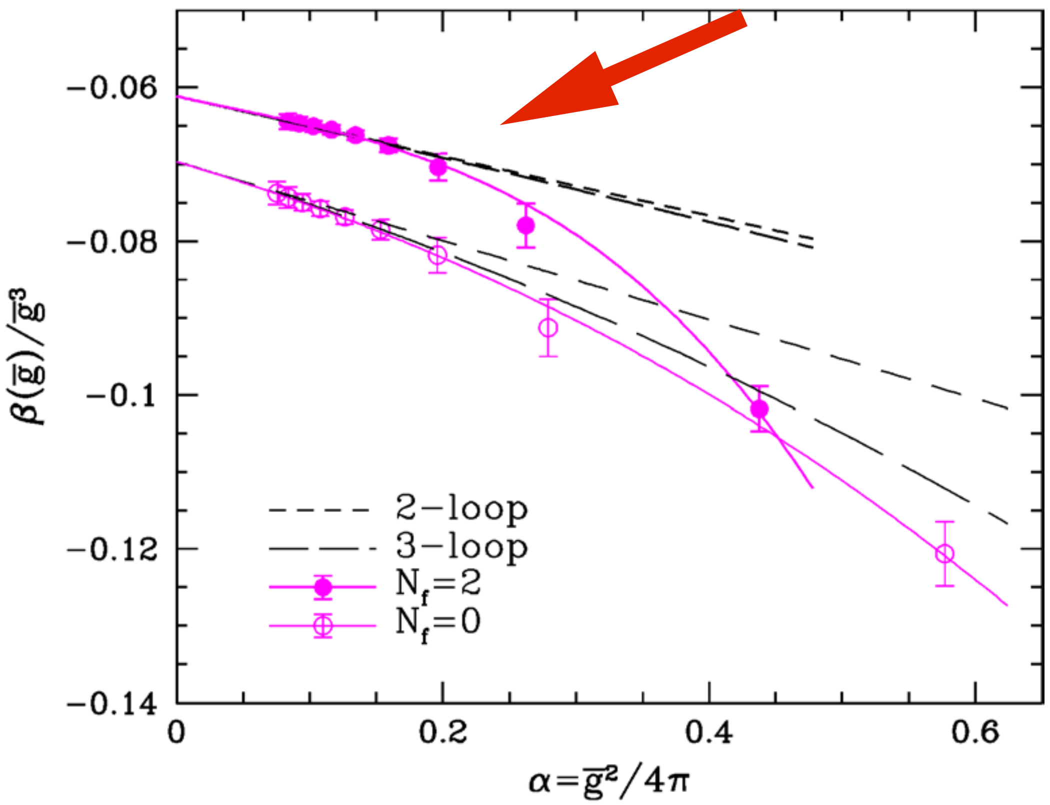
composite operators and composite states make a similar analysis more difficult

can the two phases (chiral and conformal) get confused in FSS?





running coupling and tunneling



Schrödinger Functional N_f=0 and N_f=2
massless fermions
Alpha collaboration

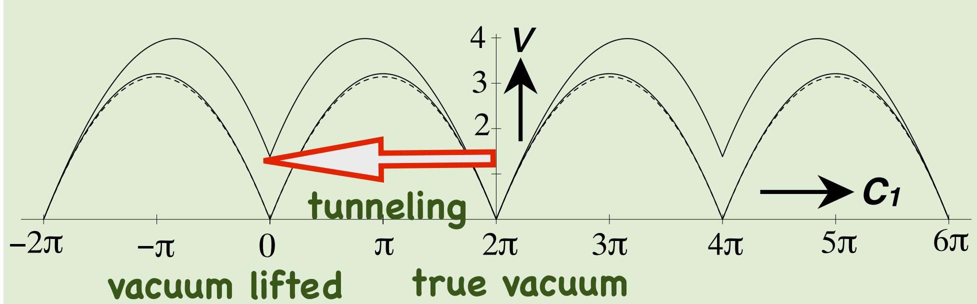
around g^2 ~ 2.5 the N_f=2 β-function breaks away from perturbative form where 2-loop and 3-loop still run closely together

g^2 ~ 2.5 is the onset of tunneling
(most likely to a metastable local minimum)

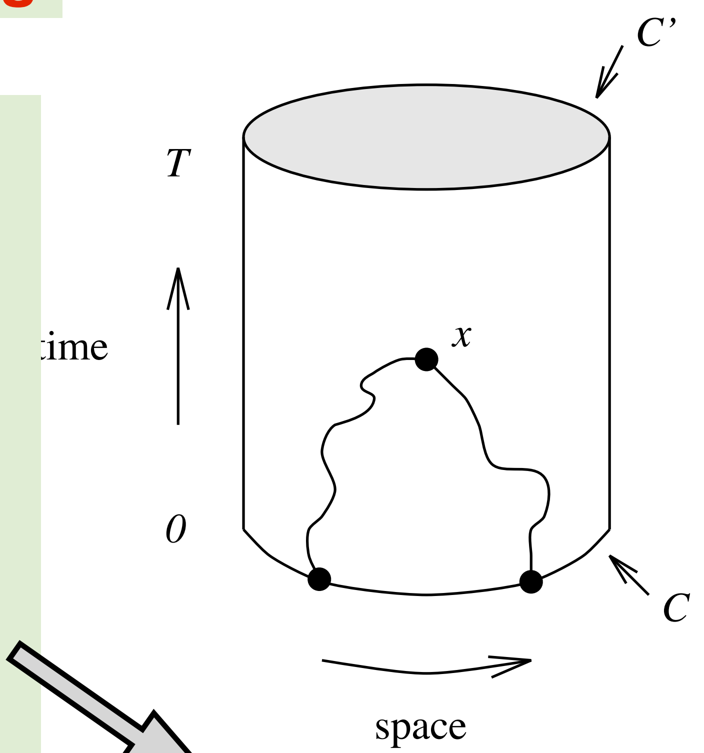
running becomes non-perturbative in very small box where L_{max} < 0.4 fm

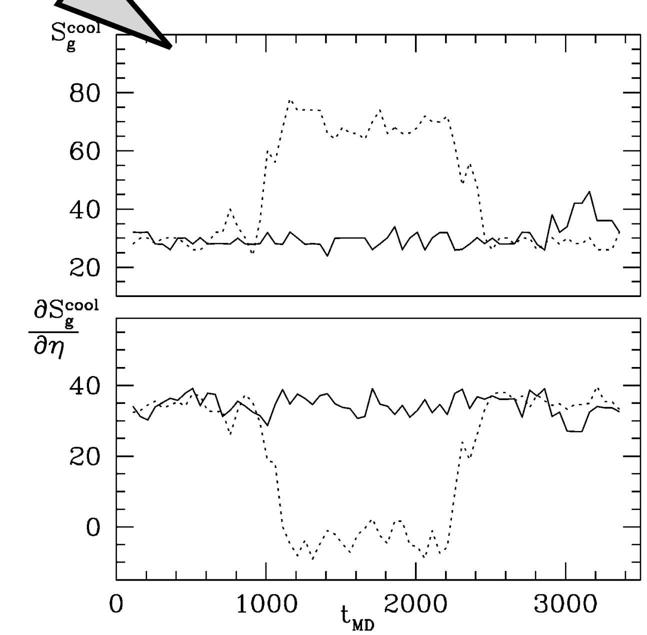
Why, and what is the underlying physics?

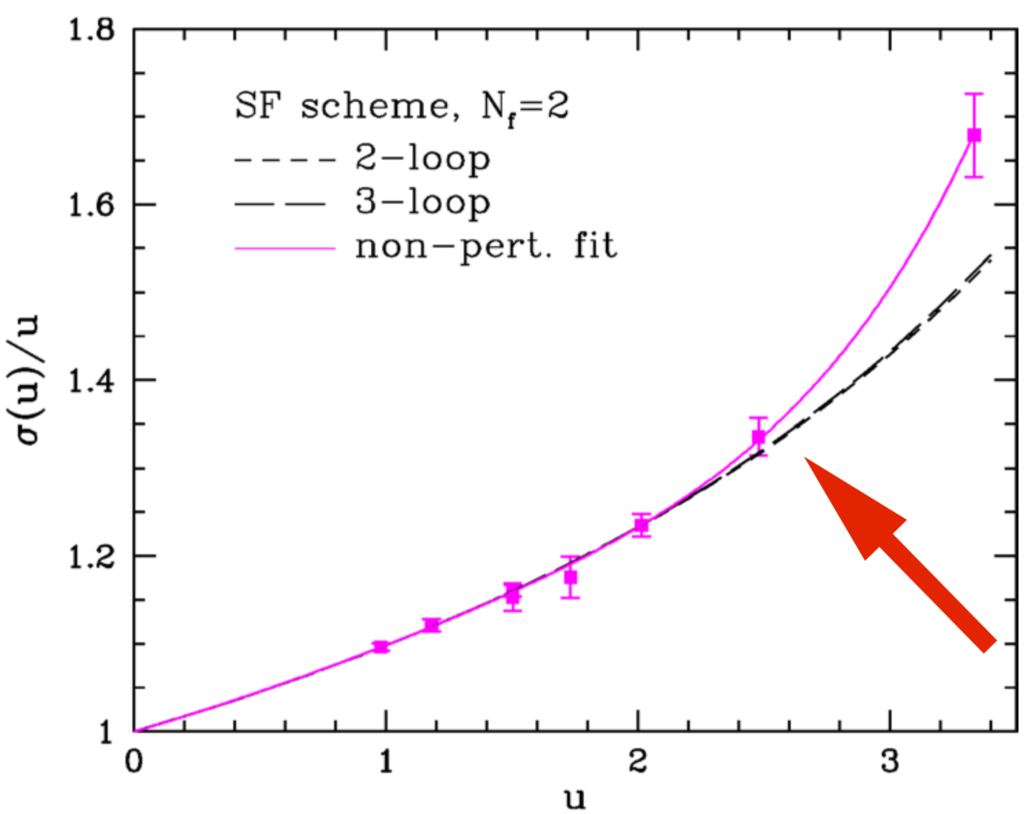
We need to understand femto physics better for the interpretation of the running coupling $g^2(L)$ in the presence of tunneling



Nf=16 weak coupling case study inside the conformal window shows the dynamics







Inside the conformal window: Nf=16 fundamental rep $SU(3)_c$ case study

Nf=16 important test of lattice technologies

From 2-loop beta function Banks-Zaks IRFP at $g^{*2} \approx 0.5$

Heller	early work SF
A. Hasenfratz	MCRG
Lattice Higgs Collab.	FSS and $g^2(L)$

α_{2l}	α_{3l}	α_{4l}
0.0416	0.0397	0.0398

Ryttov and Shrock

$\alpha = g^2 / 4\pi$

γ_{2l}	γ_{3l}	γ_{4l}
0.0272	0.0258	0.0259

Running coupling $g^2(L)$ evolving with L $\quad g^2(L) \to g^{*2}$, as $L \to \infty$ infrared limit

(evolution of finite volume spectrum?)

At small $g^2(L)$ the zero momentum components of the gauge field dominate the dynamics: Born-Oppenheimer approximation

Originally it was applied to pure-gauge system Luscher, van Baal

Small volume dynamics of QCD has spectrum which adiabatically evolves into hadron spectrum with rapid crossover around L ~ 0.7 fm

Method turns into important large volume dynamics around weak coupling fixed point inside conformal window

SU(3) 3^3=27 gauge vacua (electric fluxes) --> 2^3=8 massless fermion vacua (pbc)

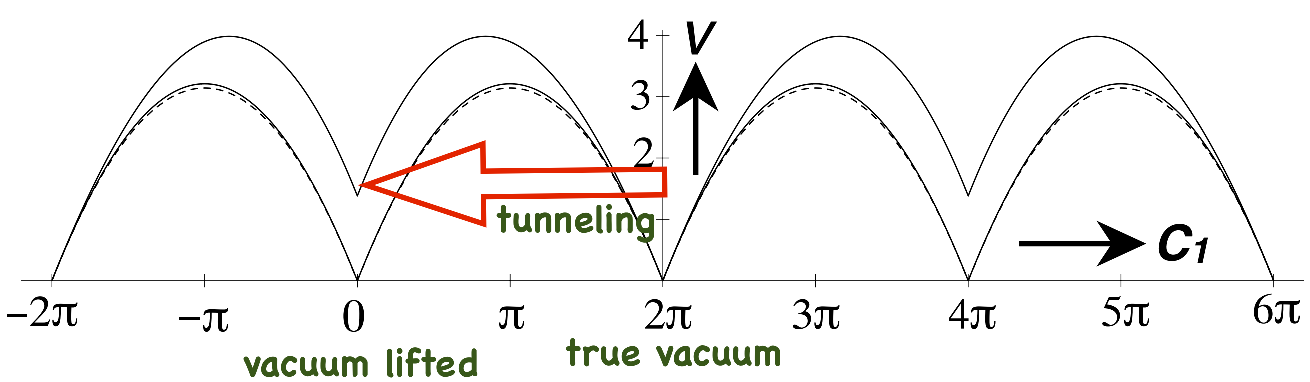
recent renewed interest:
Yaffe, Unsal
DeGrand, Hoffmann
others ...

$A_i(\mathbf{x}) = T^a C_i^a / L$ <-- zero momentum mode of gauge field

For $SU(3)$, $T_1 = \lambda_3/2$ and $T_2 = \lambda_8/2$

$$V_{\rm eff}^{\mathbf{k}}(\mathbf{C}^b) = \sum_{i>j} V(\mathbf{C}^b[\mu_b^{(i)} - \mu_b^{(j)}]) - N_f \sum_i V(\mathbf{C}^b \mu_b^{(i)} + \pi \mathbf{k}) \qquad \mu^{(1)} = (1,1,-2)/\sqrt{12} \text{ and } \mu^{(2)} = \tfrac{1}{2}(1,-1,0)$$

SU(2) V_{eff} shown for simplification:



Effective potential shows the effects of massless fermions van Baal

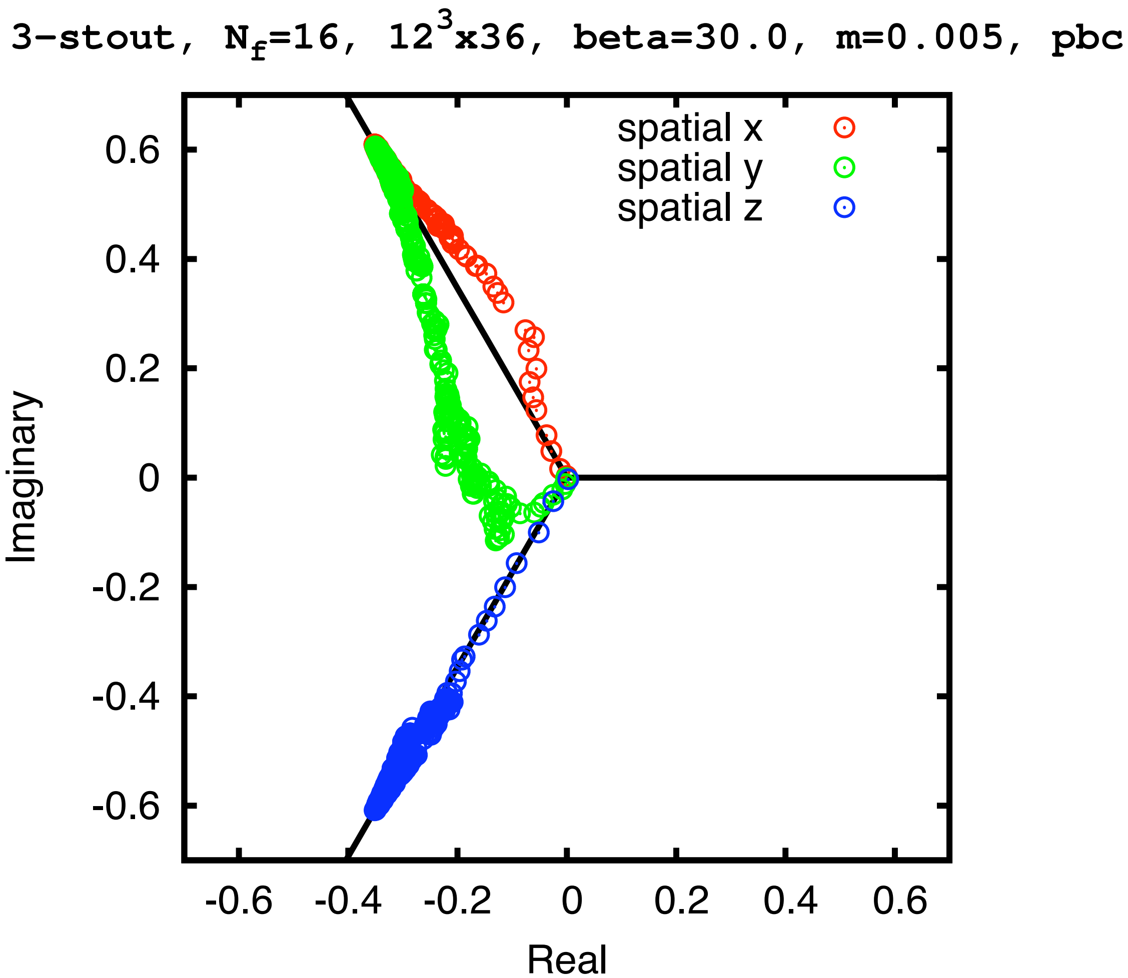
Fermions develop a gap in the spectrum

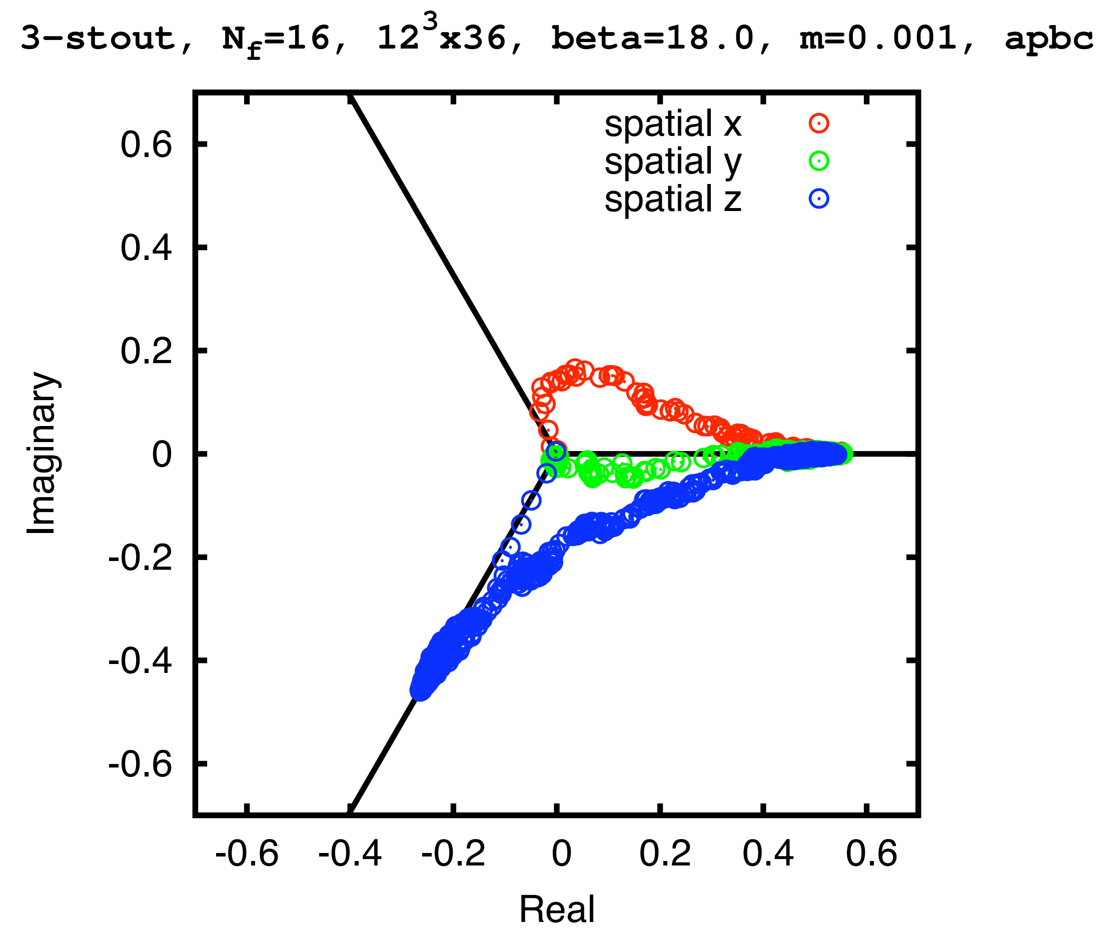
$\sim \pi / L$ k=(0,0,0) periodic
k=(1,1,1) antiperiodic

Low excitations of Hamiltonian (Transfer Matrix) scale with
will evolve into glueball states for large L $\sim g^{2/3}(L)/L$

Three scales of dynamics
scale 1: on smallest scale WF is localized on one vacuum
scale 2: tunneling sets in across vacua
scale 3: spill over the barrier - confinement scale

Nf=16 inside conformal window femto volume with tunneling





How is this effecting running coupling calculations?

Outline

- Composite Higgs Mechanism at the LHC
lattice BSM goals in Theory Space
world-wide lattice BSM effort
lattice resources (GPU technology)

- Below the Conformal Window
lattice specific: cut-off, volume, fermion mass
RG flow and lattice continuum physics
BSM specific χPT
m=0 chiral limit and finite volume issues

- Inside the conformal window
RG flow and lattice continuum physics
finite size scaling
running coupling and tunneling
Nf=16 case study

- Outlook
from workshop discussions: new input into lattice projects?

Summary and outlook

- We have technology to deal with lattice specific issues: cut-off, volume, fermion mass
  RG flow and lattice continuum physics
  BSM specific χPT
  m=0 chiral limit and finite volume issues

- Inside the conformal window
  RG flow and lattice continuum physics
  importance of finite size scaling
  running coupling and tunneling
  Nf=16 case study

- Outlook
  we have only seen so far the tip of the iceberg of what lattice BSM can do
  for example: FSS analysis of current correlators in m->0 limit   Lattice Higgs Collaboration
  phenomenology   Strong Lattice Dynamics Collaboration
  workshop discussions: new input into lattice projects?

My colleagues in the Lattice Higgs Collaboration:

Zoltan Fodor, Kieran Holland, Daniel Nogradi, Chris Schroeder, Ricky Wong