# A numerical study of a confined $Q\bar{Q}$ system in compact U(1) lattice gauge theory in 4D

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## Outline

- Motivations and general features of the model
- The method
- Focus onto the observables
- Results and plots
- Conclusions and perspectives

## Motivations and general features of the model

### General motivation

- Confinement: a long-standing problem in elementary particle Physics
- Strong interactions: SU(3) color gauge symmetry...
- ...but in the currently observed physical spectrum: colorless states only
- Precise theoretical explanation is still missing...
- ...although several possible mechanisms/effective theories have been suggested
- Numerical studies on the lattice may refute or confirm their predictions
- A ("meson-like") $Q\bar{Q}$ system: the simplest confined system
- It is possible that different gauge theories share the same (qualitative) mechanism for confinement
- Compact U(1) lattice gauge theory in 4D has a confined phase, analogous to non-Abelian gauge theories

## Motivations and general features of the model

### Compact U(1) lattice gauge theory in 4D

- A simple example of a confining gauge theory
- A pure gauge theory; $U_\mu(x) \in$ U(1) variables defined on the oriented bonds of a hypercubic lattice
- Wilson Action: $S = \beta \sum_{\mathrm{p}} (1 - \mathrm{Re} U_{\mathrm{p}})$
- $S = S(\{U_{\mathrm{p}}\}) \longrightarrow$ invariance w.r.t. local (site) U(1) gauge transformations
- Two different regimes:
  1. $0 < \beta < \beta_c = 1.0111331(21)^*$ confined phase
  2. $\beta > \beta_c$ deconfined ("Coulomb-like") phase



*G. Arnold, B. Bunk, Th. Lippert, K. Schilling, hep-lat/0210010

## Exploiting the duality properties of the theory

This theory enjoys a duality property:

- a group Fourier transform allows to map the partition function and observable VEV's to a dual formulation*
- whose fundamental degrees of freedom take values in $\mathbb{Z}$.
- In 4D, the dual model is again a gauge model
- with an interaction of "ferromagnetic" nature:

$$Z = (2\pi)^{4N} \prod \sum_{\star l} e^{-\beta} I_{|d^{\star} l|}(\beta)$$

$^{\star}l = {}^{\star}l_{\mu}(x) \in \mathbb{Z} \longrightarrow$ a 1-form defined on dual lattice links

$d^{\star}l \longrightarrow$ discretized exterior derivative of $^{\star}l$ (a 2-form associated with dual lattice plaquettes)

An exact, analytical mapping that allows to get results for the U(1) theory from simulations of the dual, "integer-valued" model

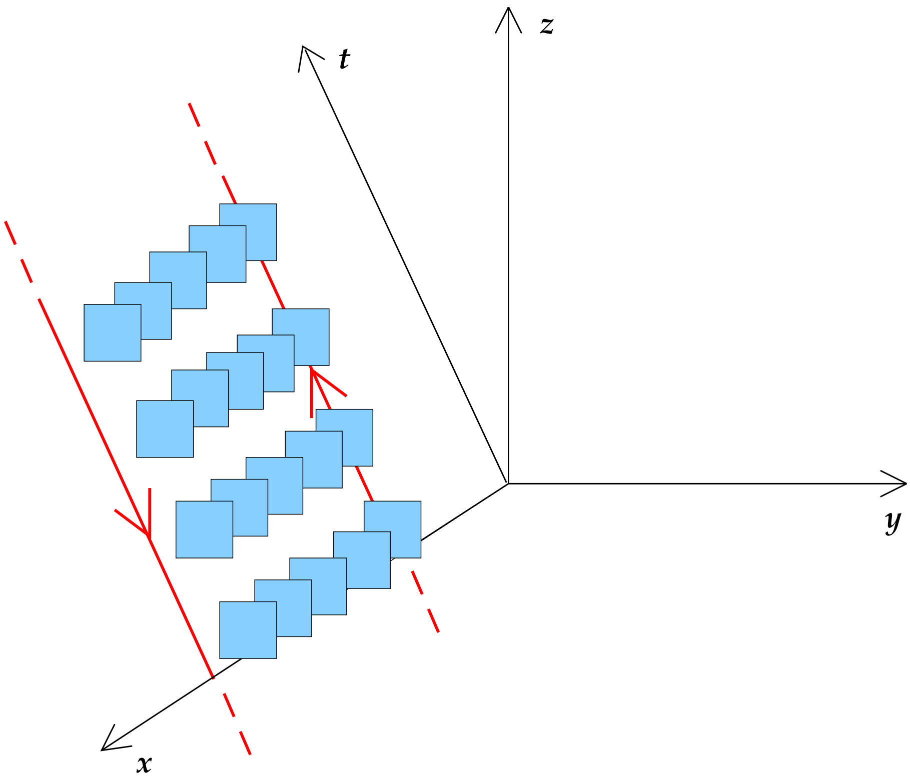


*See also M. Zach, M. Faber, P. Skala, hep-lat/9705019

## The $Q\bar{Q}$ system mapped to the dual model

The $Q\bar{Q}$ pair (Polyakov loops in the original model) is mapped to a stack of topological defects (${}^{\star}n$) in the dual lattice:

$$Z_{Q\bar{Q}} = (2\pi)^{4N} \prod \sum_{{}^{\star}l} e^{-\beta} I_{|d^{\star}l + {}^{\star}n|}(\beta)$$



## Advantages and limits: a comparison with other methods

Features:

- S/N ratio exponential decay problem is overcome
- computational advantages from integer-valued variables
- the method can be used for different gauge theories[*] — with further practical advantages in 3D
- it was already used in studies of $\mathbb{Z}_2$ LGT in 3D[†]
- a possible alternative to the multi-level algorithm[‡], which has proved to induce exponential error reduction in several gauge theories[§]
- including compact U(1) LGT in 4D[¶]
- unfortunately, the duality technique is not available for physically more interesting groups like SU(N)



[*] R. Savit, 1979

[†] M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch, P. Provero, hep-lat/9609041
M. Caselle, M. Hasenbusch, M. P., hep-lat/0211012
K. J. Juge, J. Kuti, C. Morningstar, hep-lat/0401032

[‡] M. Lüscher, P. Weisz, hep-lat/0108014

[§] M. Lüscher, P. Weisz, hep-lat/0207003
S. Kratochvila, Ph. de Forcrand, hep-lat/0209094
P. Majumdar, hep-lat/0211038
M. Caselle, M. Pepe, A. Rago, hep-lat/0310005

[¶] Y. Koma. M. Koma, P. Majumdar, hep-lat/0309003
Y. Koma. M. Koma, P. Majumdar, hep-lat/0311016

## Focus onto the observables

### Flux-tube profile

- Dual superconductor picture:

$$\mathcal{L}_{DLG} = -\frac{1}{4}F^2 + |\mathcal{D}\chi|^2 - \lambda(|\chi|^2 - v^2)^2$$

- Classically, at large enough distance from the Dirac string sheet:

$$E_x(r) \propto m^2 K_0\,(mr)$$

- String quantum fluctuations can be included*: they induce logarithmic growth of flux tube width†

- From the complete $E_x$ profile: check of rotational invariance — lattice artifacts?

- Role of finite lattice size effects?

- Scaling of dimensionless quantities at different values of $\beta$



*M. Zach, M. Faber, W. Kainz, P. Skala, hep-lat/9508017

†M. Caselle, F. Gliozzi, U. Magnea, S. Vinti, hep-lat/9510019 for $\mathbb{Z}_2$ in 3D

## Focus onto the observables

### Polyakov loop correlator

- The bosonic string scenario predicts*:

$$\langle P^\dagger(R)P(0)\rangle = \frac{e^{-\sigma RL+k}}{\left[\eta\left(i\frac{L}{2R}\right)\right]^{D-2}}$$

$\eta$ being the Dedekind function:

$$\eta(\tau) = q^{\frac{1}{24}}\prod_{n=1}^{+\infty}(1-q^n) \quad ; \quad q = e^{2\pi i\tau}$$

- Correspondingly, the confining potential reads†:

$$V(R) = -\frac{1}{L}\ln\langle P^\dagger(R)P(0)\rangle \simeq \sigma R - \frac{\pi(D-2)}{24R} + \dots$$

- Inclusion of a possible “boundary term contribution” in the effective action yields:

$$V(R) \simeq \sigma R - \frac{\pi(D-2)}{24R}\left(1+\frac{b}{R}\right) + \dots$$

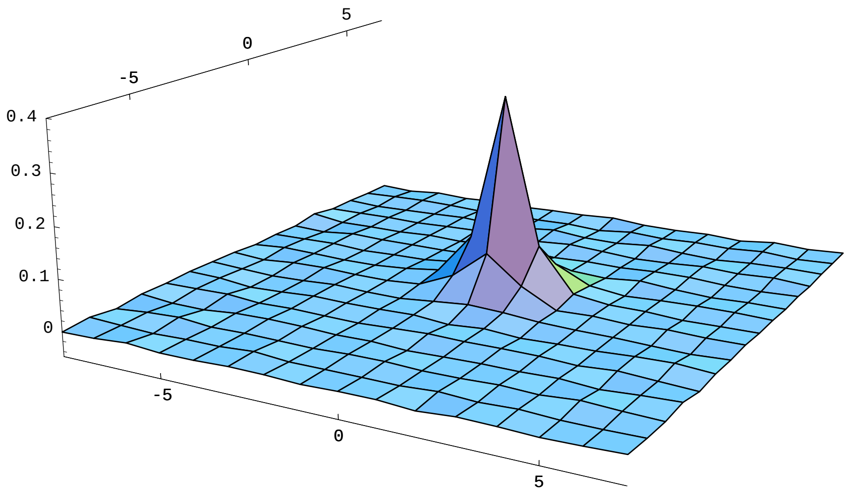
- Open questions . . .



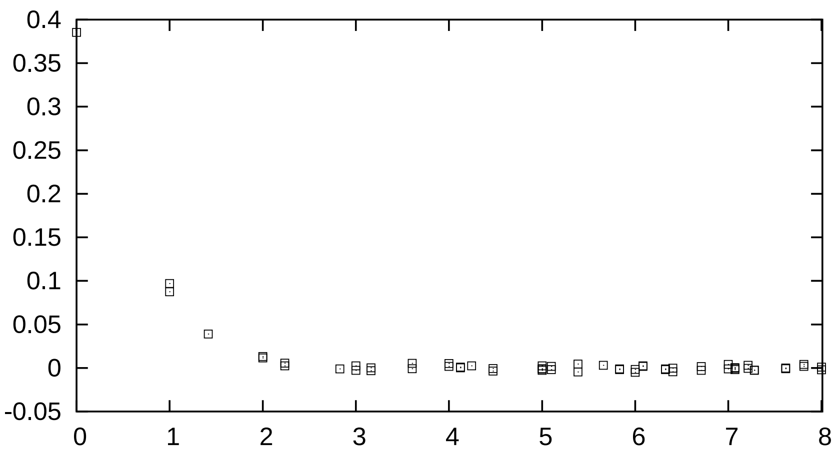
*M. Minami, 1978
M. Flensburg, C. Peterson, 1987

†M. Lüscher, K. Symanzik, P. Weisz, 1980

## Numerical results for the flux-tube profile: an example




$E_x$ profile in the $(y,z)$ mid-plane between the charges
(16[4] lattice, $\beta = 0.96$, $d_{Q\bar{Q}} = 3a$)

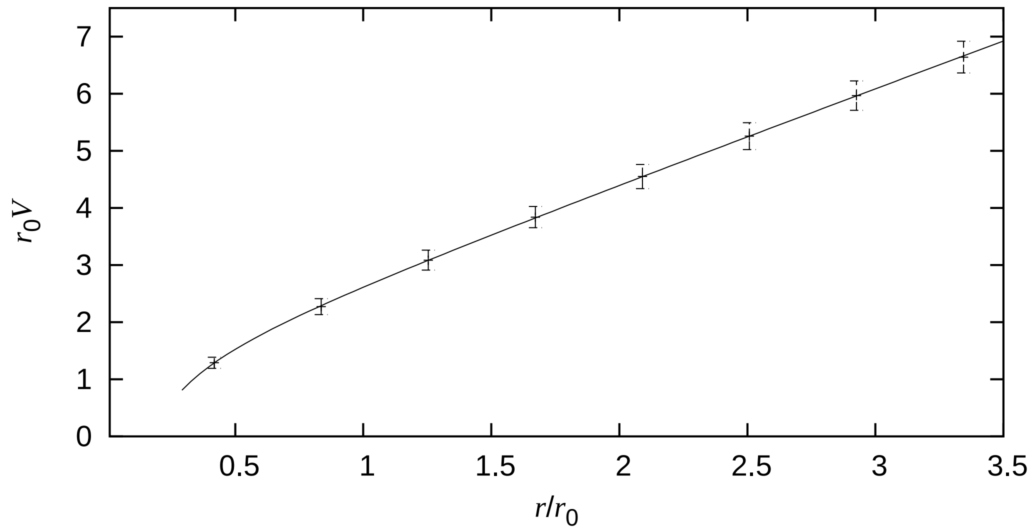



$E_x$ as a function of $\rho = \sqrt{y^2 + z^2}$

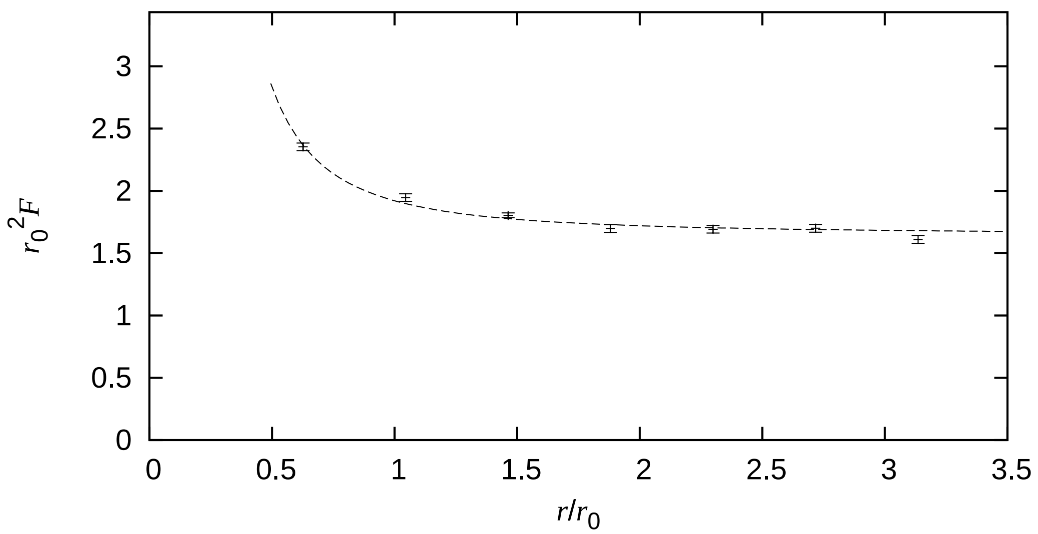
## Discussion

- Check of rotational invariance
- Comparing with results from lattices of different volume: finite size effects are under control
- Behaviour as $\beta \longrightarrow \beta_c$: the sharply peaked distribution gets flatter and flatter
- Behaviour as $d_{Q\bar{Q}}$ increases: the peak height decreases, width gets broader
- Possible fit of the expected behaviour at large enough $\rho$

## A look at the interquark potential and force




Sommer's scale* is introduced ($r_0 \simeq 2.3a$ at $\beta = 0.98$).




The fit yields: $\sigma a^2 = 0.289(3)$, with $\chi^2_{\text{red}} = 1.4$.



*R. Sommer, hep-lat/9310022

## Discussion

- The algorithm proves to be particularly efficient to study the interquark force, especially at large distances
- Well-defined scaling behaviour as $\beta$ is changed
- In agreement with other results published in literature, the effective string scenario is confirmed at large distances . . .
- . . . whereas at short distances the picture breakdown seems not to be completely cured by including a boundary term
- A possible effective pattern at short distances appears to be *non universal*, *i.e.* dependent on the gauge theory

## Conclusions and perspectives

### Summary

- We have simulated compact U(1) lattice gauge theory in 4D
- Duality allows to use a highly efficient algorithm
- The electric field profile induced by two static charges was observed
- Interquark potential and force were studied

### Agenda

- Increasing statistics
- Larger lattice runs
- Further investigations about the confining potential