Axial and tensor charge of the nucleon with dynamical fermions

A status report

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long term goal:
understand the quark mass (pion mass) dependence
and
the volume dependence
of the nucleon mass, g_A, the tensor charge, ...

tools

- lattice QCD simulations with $N_f = 2$ dynamical quarks (QCDSF-UKQCD)
- chiral effective field theories (chEFT)
  low-energy theories incorporating the constraints from (spontaneously broken) chiral symmetry
  parametrisation of the dependence on the quark-mass, volume, ...

achievements

- control over the chiral extrapolation and the thermodynamic limit of simulation results
- determination of (phenomenologically relevant) coupling constants in effective Lagrangians

lattice spacing dependence can be included in chEFT
in our simulations: $a \approx 0.1$ fm
poor control over lattice artefacts $\longrightarrow$ we must neglect them

however: improved fermions, non-perturbative renormalisation, ...

Chiral perturbation theory and finite size effects

chiral perturbation theory (chPT) originally used to describe the m_π dependence of low-energy quantities by means of an effective field theory based on effective pion, . . . fields

QCD in finite box of linear size L:

for $L \to \infty$, $m_\pi \to 0$ finite size effects dominated by pions "propagating around the world"

$\to$ chEFT can be used to calculate volume dependence

applied to "standard" chPT $\to$ p expansion:
$m_\pi = O(p)$, $L^{-1} = O(p)$ $\quad \Rightarrow m_\pi L = O(p^0)$

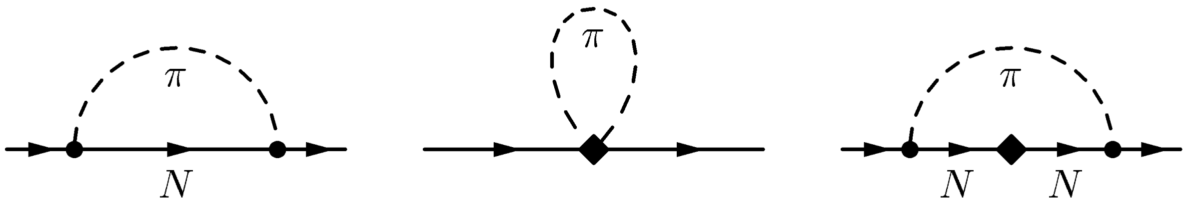
see: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83
Hasenfratz and Leutwyler, Nucl. Phys. B343 (1990) 241

different versions of chEFT with nucleons

- HBchPT (heavy baryon chPT): non-relativistic nucleons
  small parameter p
- SSE (small scale expansion): HBchPT + explicit Δ degrees of freedom
  small parameter ϵ
- relativistic baryon chPT (Becher and Leutwyler)
  small parameter p

paradigm: nucleon mass (Nucl. Phys. B689 (2004) 175 [hep-lat/0312030])

relativistic $SU(2)_f$ baryon chiral perturbation theory
(Becher and Leutwyler, Eur. Phys. J. C9 (1999) 643)



(a) (b) (c)

order p^3 order p^4

$$m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3 + \left[e_1^r(\lambda) - \frac{3}{64\pi^2 f_\pi^2}\left(\frac{g_A^2}{m_0} - \frac{c_2}{2}\right) - \frac{3}{32\pi^2 f_\pi^2}\left(\frac{g_A^2}{m_0} - 8c_1 + c_2 + 4c_3\right)\ln\frac{m_\pi}{\lambda}\right] m_\pi^4 + \frac{3g_A^2}{256\pi f_\pi^2 m_0^2} m_\pi^5 + O(m_\pi^6)$$

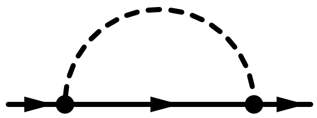
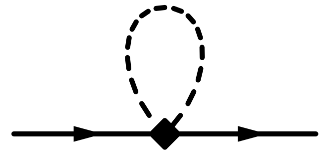
Becher and Leutwyler; Procura, Hemmert and Weise

parameters (coupling constants, to be taken in the chiral limit):

m_0	nucleon mass in the chiral limit
g_A	axial coupling constant of the nucleon
f_π	pion decay constant
$e_1^r(\lambda)$	counterterm at the renormalisation scale λ
$c_1,\ c_2,\ c_3$	

evaluation of the theory in a finite (spatial) volume: integral over the (spatial components of the) loop momenta replaced by a sum over the discrete set of momenta allowed by the boundary conditions

periodic boundary conditions for box length L: $p_i = \frac{2\pi}{L}\ell_i \, , \ell_i \in \mathbb{Z}$

Only graph (a)  and graph(b) 

contribute:

$$m_N(L) - m_N(\infty) = \Delta_a(L) + \Delta_b(L) + O(p^5)$$

$$\Delta_a(L) = \frac{3g_A^2 m_0 m_\pi^2}{16\pi^2 f_\pi^2} \int_0^\infty \mathrm{d}x \sum_{\vec{n}}{}' K_0\left(L|\vec{n}|\sqrt{m_0^2 x^2 + m_\pi^2(1-x)}\right)$$

$$\Delta_b(L) = \frac{3m_\pi^4}{4\pi^2 f_\pi^2} \sum_{\vec{n}}{}' \left[(2c_1 - c_3)\frac{K_1(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi} + c_2 \frac{K_2(L|\vec{n}|m_\pi)}{(L|\vec{n}|m_\pi)^2}\right]$$

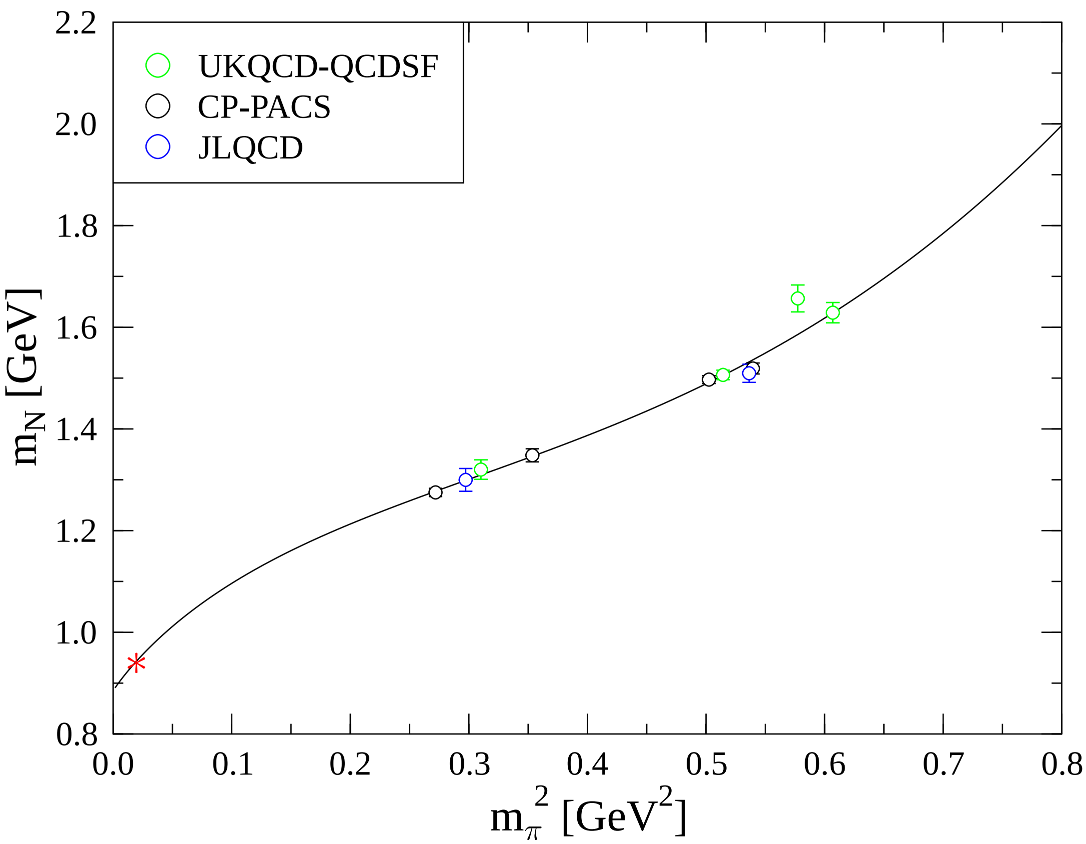
n_j: number of times the pion travels around the spatial box in the j direction

Comparison with $N_f = 2$ data for the nucleon mass

see Nucl. Phys. B689 (2004) 175 [hep-lat/0312030]

how to set the scale? $\longrightarrow r_0 = 0.5$ fm reliable?

	gauge field action	fermion action
UKQCD + QCDSF	standard Wilson	non-pert. improved clover
JLQCD	standard Wilson	non-pert. improved clover
CP-PACS	RG improved	mean field improved clover



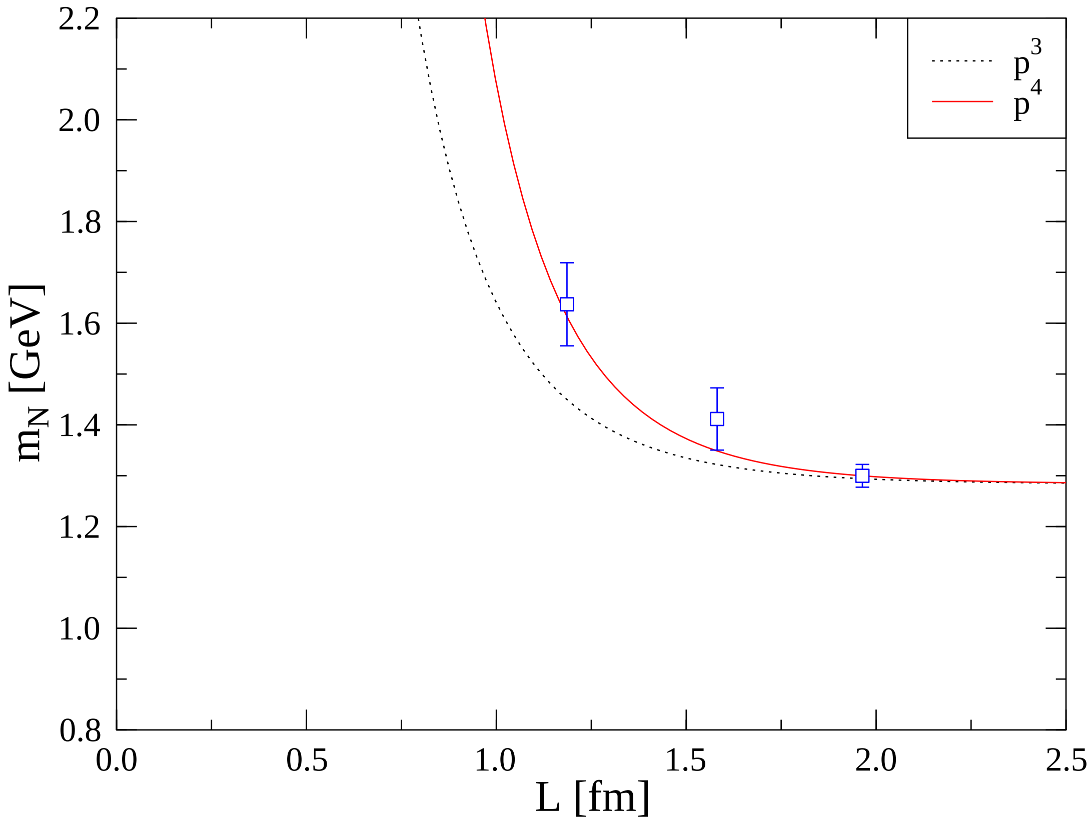
fit of the "infinite" volume data

$\longrightarrow$ phenomenologically reasonable values of the parameters

parameters from "infinite volume" fit $\longrightarrow$ evaluate the finite size corrections

$$m_N(L) = m_N(\infty) + \underbrace{\overbrace{\Delta_a(L)}^{p^3} + \Delta_b(L)}_{p^4}$$

$m_N(\infty)$: $m_N(L)$ on the largest lattice agrees with the Monte Carlo value
m_π: take the value from the largest lattice



$m_\pi = 545$ MeV

similarly good agreement for even higher masses
($m_\pi = 717$ MeV, 732 MeV)

Monte Carlo data for nucleon matrix elements ($N_f = 2$)

	Coll.	β	κ_{sea}	volume
1	QCDSF	5.20	0.1342	$16^3 \times 32$
2	UKQCD	5.20	0.1350	$16^3 \times 32$
3	UKQCD	5.20	0.1355	$16^3 \times 32$
4	UKQCD	5.20	0.13565	$16^3 \times 32$
5	UKQCD	5.20	0.1358	$16^3 \times 32$
6	QCDSF	5.25	0.1346	$16^3 \times 32$
7	UKQCD	5.25	0.1352	$16^3 \times 32$
8	QCDSF	5.25	0.13575	$24^3 \times 48$
9	UKQCD	5.26	0.1345	$16^3 \times 32$
10	UKQCD	5.29	0.1340	$16^3 \times 32$
11	QCDSF	5.29	0.1350	$16^3 \times 32$
12	QCDSF	5.29	0.1355	$12^3 \times 32$
13	QCDSF	5.29	0.1355	$16^3 \times 32$
14	QCDSF	5.29	0.1355	$24^3 \times 48$

UKQCD: Allton et al., Phys. Rev. D65 (2002) 054502

analysis in progress
$\longrightarrow$ in the following: very recent (preliminary) results

other groups in this field: RIKEN-BNL-Columbia-KEK, LHPC, ZeRo

comparison with chEFT: continuum results needed (in physical units)
set the scale using $\longrightarrow r_0 = 0.5$ fm lattice artefacts?

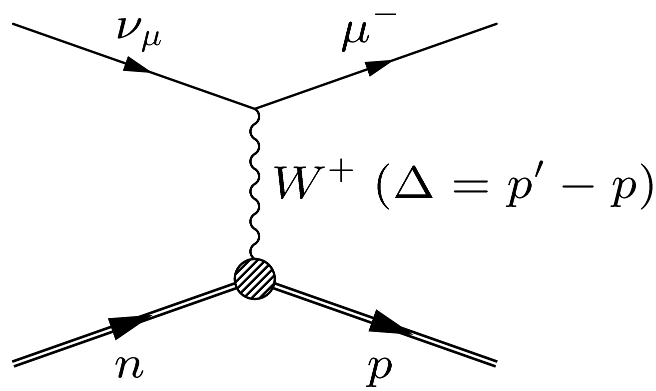
compared to mass computations:

additional difficulties for the axial and tensor charge of the nucleon!

- operators not yet improved
- operators have to be renormalised
  Rome-Southampton method with linear chiral extrapolation at fixed β
- quark-line disconnected contributions hard to evaluate
  $\longrightarrow$ consider only flavour-nonsinglet quantities

Axial form factor of the nucleon (g_A)

Neutrino-neutron scattering:



scattering matrix element

$$T_{fi} = \frac{G}{\sqrt{2}} \bar{u}_\mu(k'_\mu, s'_\mu)\gamma_\mu(1-\gamma_5)u_\nu(k_\nu, s_\nu)\langle p(p', s')|J^{+\mu}|n(p, s)\rangle$$

with the current $J^{+\mu} = \bar{u}\gamma^\mu(1-\gamma_5)d\cos\theta_c$ and the decomposition

$$\langle p(p', s')|J^{+\mu}|n(p, s)\rangle = -\bar{u}_p(p', s')\gamma^\mu\gamma_5 g_A(\Delta^2)u_n(p, s)\cos\theta_c + \cdots$$

β-decay ($n \longrightarrow pe^-\bar{\nu}_e$) gives $g_A(0)$

Using current algebra one gets for $A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$

$$\begin{aligned}&\langle p(p', s')|A_\mu^{u-d}|p(p, s)\rangle \\ &\quad = \bar{u}(p', s')\left[\gamma_\mu\gamma_5 g_A(\Delta^2) + \gamma_5\frac{\Delta_\mu}{2m_N}h_A(\Delta^2)\right]u(p, s)\end{aligned}$$

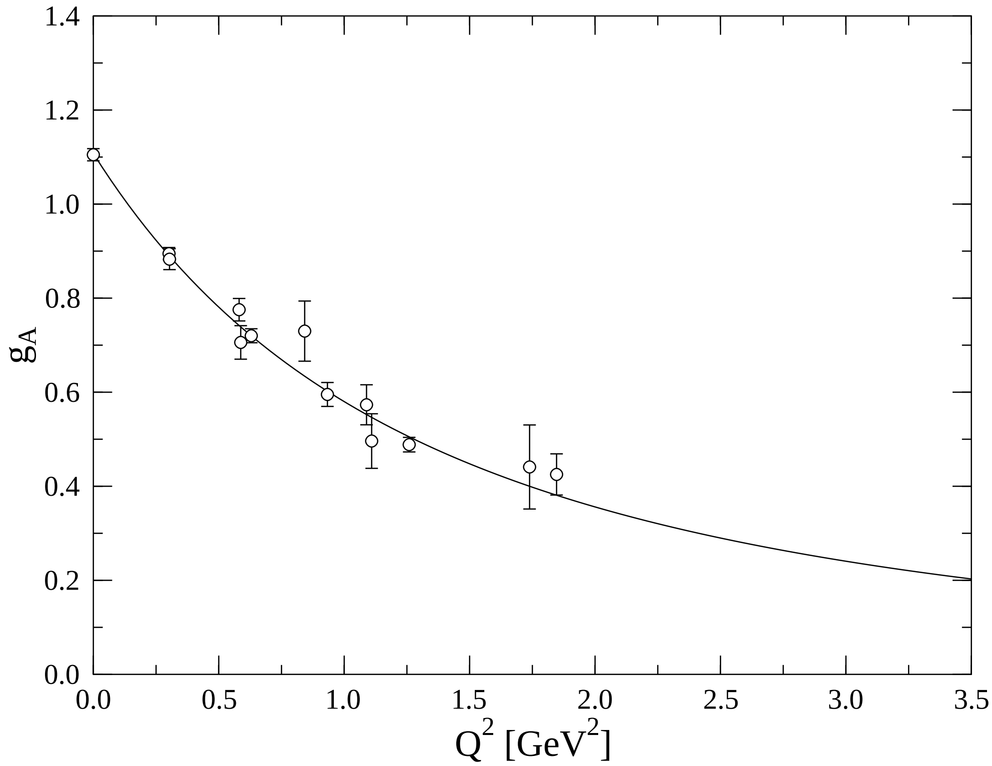
Experimental results give phenomenological (dipole) fits:

$$g_A(\Delta^2) = g_A(0)\left(1 - \Delta^2/m_A^2\right)^{-2}$$

$$g_A(0) = 1.267 \qquad m_A \sim 1.0\text{GeV}$$

form factor (with dipole fit)

$\beta = 5.25$, $\kappa = 0.13575$, $m_\pi = 0.557$ GeV



$m_D = 1.62(3)$ GeV

lattice artefacts visible?

mass dependence of $g_A(\Delta^2 = 0)$ in $O(\epsilon^3)$ SSE

Hemmert, Procura, Weise, Phys. Rev. D68 (2003) 075009

$$
\begin{aligned}
g_A^{SSE}(m_\pi^2) &= g_A^0 - \frac{(g_A^0)^3 m_\pi^2}{16\pi^2 f_\pi^2} \\
&+ 4\left\{ C^{SSE}(\lambda) + \frac{c_A^2}{4\pi^2 f_\pi^2}\left[\frac{155}{972}\, g_1 - \frac{17}{36}\, g_A^0\right] + \gamma^{SSE} \ln\frac{m_\pi}{\lambda} \right\} m_\pi^2 \\
&+ \frac{4 c_A^2 g_A^0}{27\pi f_\pi^2 \Delta} m_\pi^3 + \frac{8}{27\pi^2 f_\pi^2}\, c_A^2 g_A^0 m_\pi^2 \sqrt{1 - \frac{m_\pi^2}{\Delta^2}}\, \ln R \\
&+ \frac{c_A^2 \Delta^2}{81\pi^2 f_\pi^2}\left(25 g_1 - 57 g_A^0\right)\left\{ \ln\left[\frac{2\Delta}{m_\pi}\right] - \sqrt{1 - \frac{m_\pi^2}{\Delta^2}}\ln R \right\} + O(\epsilon^4)
\end{aligned}
$$

with

$$
\begin{aligned}
\gamma^{SSE} &= \frac{1}{16\pi^2 f_\pi^2}\left[\frac{50}{81}\, c_A^2 g_1 - \frac{1}{2}\, g_A^0 - \frac{2}{9}\, c_A^2 g_A^0 - (g_A^0)^3\right] \\
R &= \frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}
\end{aligned}
$$

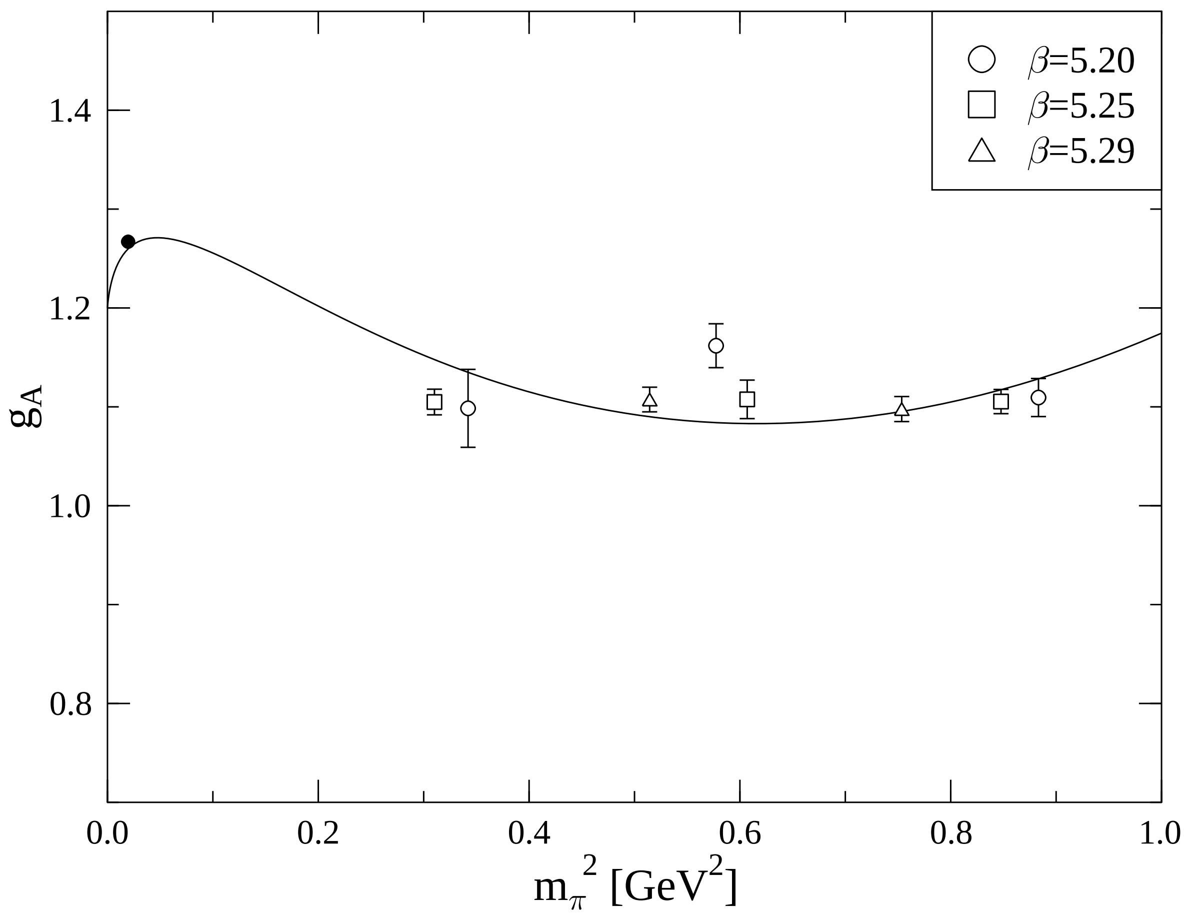
parameters:

g_A^0	axial coupling constant of the nucleon in the chiral limit
Δ	nucleon Δ mass splitting in the chiral limit
c_A, g_1	$N\Delta$, $\Delta\Delta$ axial coupling constants
$C^{SSE}(\lambda)$	counterterm at the renormalisation scale λ

only for illustrative purposes:

fix:	g_A^0	f_π	c_A	Δ
	1.2	92.4 MeV	1.125	0.2711 GeV

fit:	g_1	$C^{SSE}(\lambda = 1\text{ GeV})$
	5.42(6)	$-3.30(5)$

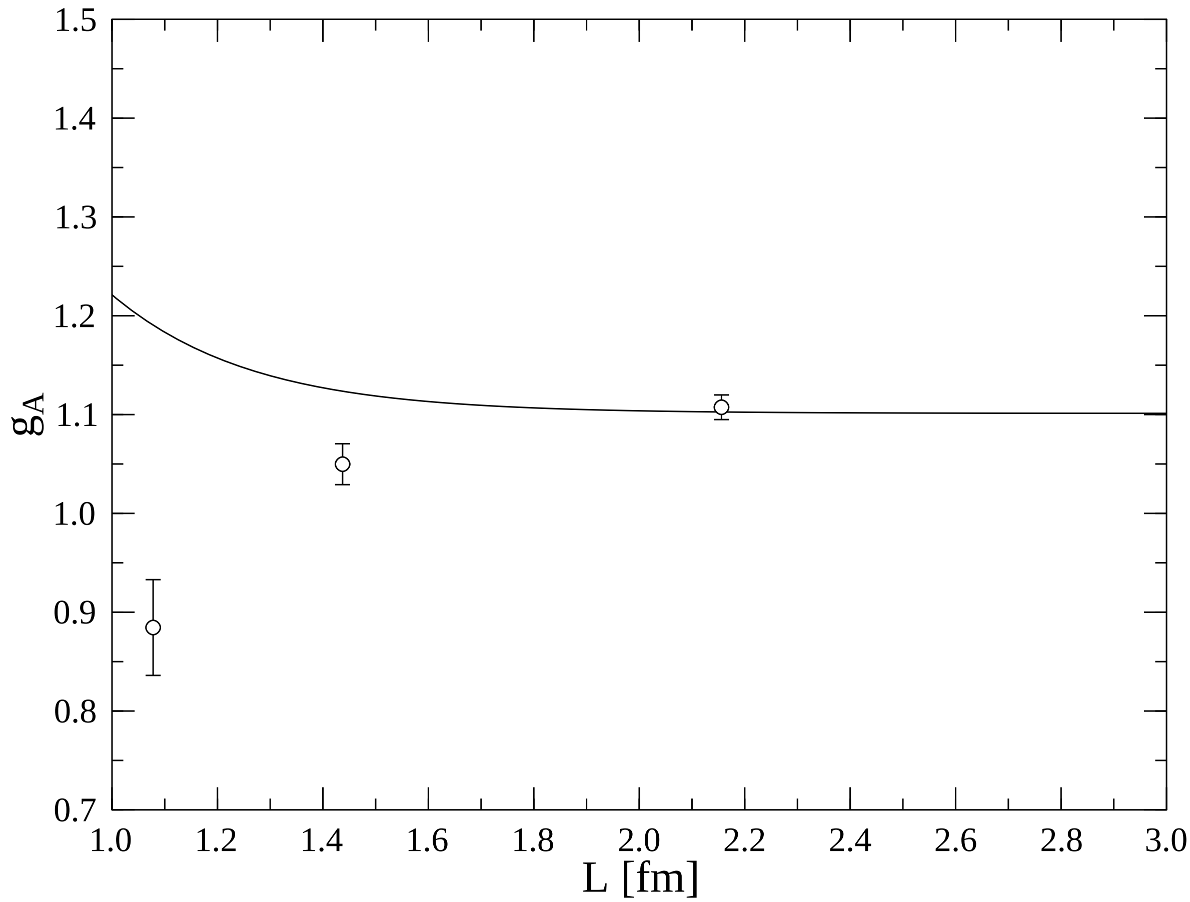


- weak mass dependence
- chiral logarithm dominates only for very small pion masses

finite size effect for g_A to $O(p^3)$ in HBchPT:

$$g_A(L) - g_A(\infty) = -\frac{g_A^0 m_\pi^2}{4\pi^2 f_\pi^2}\left(1 + \tfrac{2}{3}(g_A^0)^2\right) \sum_{\vec{n}}{}' \frac{K_1(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi} + \frac{(g_A^0)^3 m_\pi^2}{6\pi^2 f_\pi^2} \sum_{\vec{n}}{}' K_0(L|\vec{n}|m_\pi)$$

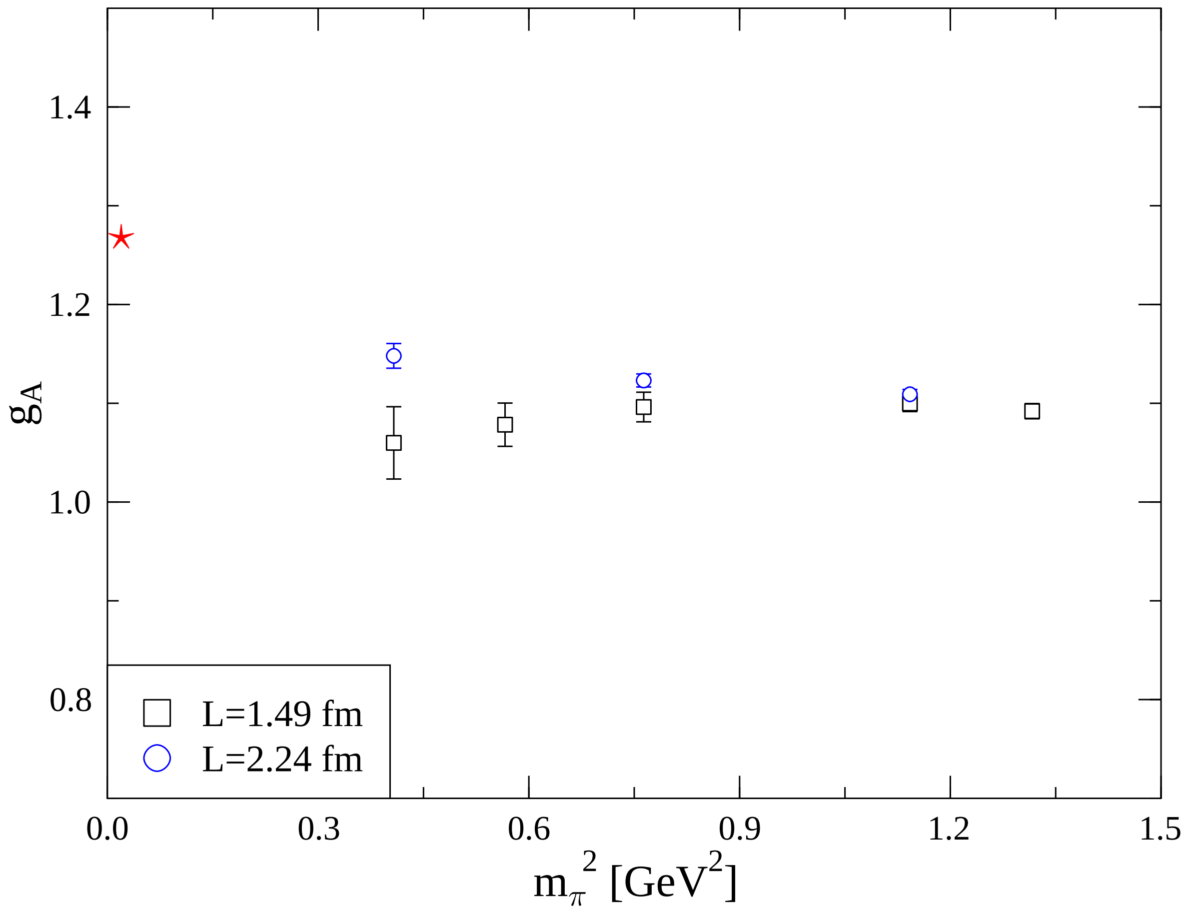
Beane and Savage, hep-ph/0404131 (with Δ effects included)
Hemmert, Weise, Wollenweber



$m_\pi = 0.717$ GeV

see also: Sasaki, Orginos, Ohta, Blum, Phys. Rev. D68 (2003) 054509

Quenched data



similar finite size effects have been found by other groups

see, e.g., Sasaki, Orginos, Ohta, Blum, Phys. Rev. D68 (2003) 054509

Tensor charge of the nucleon (δq)

tensor charge δq $\leftrightarrow$ lowest moment of the transversity distribution h_1

$$\langle p, s|\bar{q}\mathrm{i}\sigma_{\mu\nu}\gamma_5 q|p, s\rangle = \frac{2}{m_N}(s_\mu p_\nu - s_\nu p_\mu)\delta q \qquad s^2 = -m_N^2$$

stationary proton (only $p_0 \neq 0$):

$$\mathrm{i}\sigma_{\mu 0}\gamma_5 = \gamma_0\gamma_\mu\gamma_5 \qquad \mu \neq 0$$

non-relativistic limit: "quarks are eigenstates of γ_0 with eigenvalue 1"

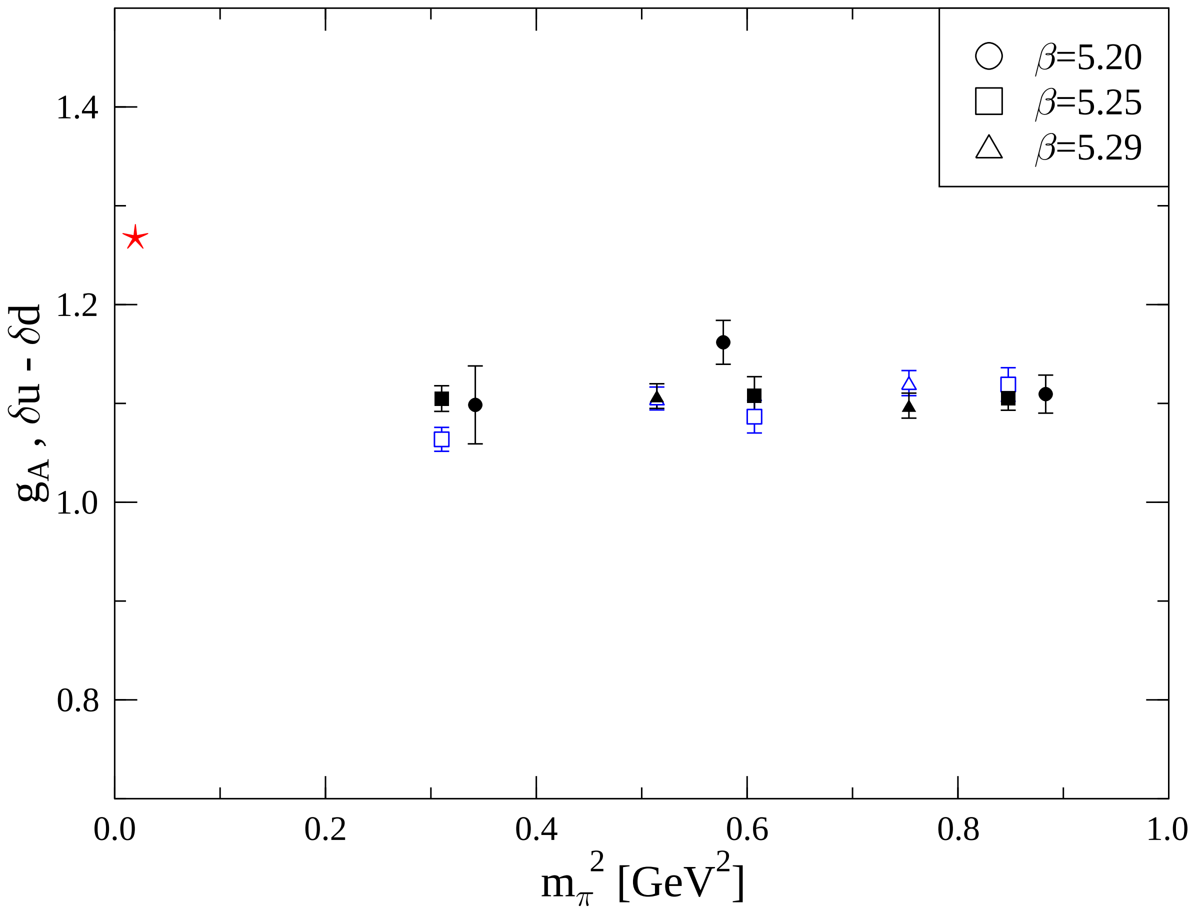
$$\frac{2}{m_N}s_\mu p_0 \delta q = \langle p, s|\bar{q}\gamma_0\gamma_\mu\gamma_5 q|p, s\rangle = \langle p, s|\bar{q}\gamma_\mu\gamma_5 q|p, s\rangle = 2 s_\mu \Delta q$$

Hence $\delta q = \Delta q$ in the non-relativistic limit and the comparison of δq and Δq gives us an impression of how relativistic the quarks in the nucleon are.

remember: $g_A \equiv g_A(0) = \Delta u - \Delta d$

$\longrightarrow$ compare g_A and $\delta u - \delta d$

$\delta u - \delta d$ and g_A



$\delta u - \delta d$ (blue symbols) in the $\overline{\text{MS}}$ scheme at $\mu^2 = 4\,\text{GeV}^2$

Erroneously, RGI values for $\delta u - \delta d$ were shown in the talk.

little difference between $\delta u - \delta d$ and g_A

more generally:

$$\langle p', s' | \bar{q} \mathrm{i} \sigma^{\mu\nu} q | p, s \rangle$$

$$= \bar{u}(p', s') \Big[\mathrm{i}\sigma^{\mu\nu} A_{T10}(\Delta^2) + \frac{\bar{p}^\mu \Delta^\nu - \bar{p}^\nu \Delta^\mu}{m_N^2} A'_{T10}(\Delta^2) + \frac{\gamma^\mu \Delta^\nu - \gamma^\nu \Delta^\mu}{2m_N} B_{T10}(\Delta^2) \Big] u(p, s)$$

with $\Delta = p' - p$, $\bar{p} = \frac{1}{2}(p' + p)$

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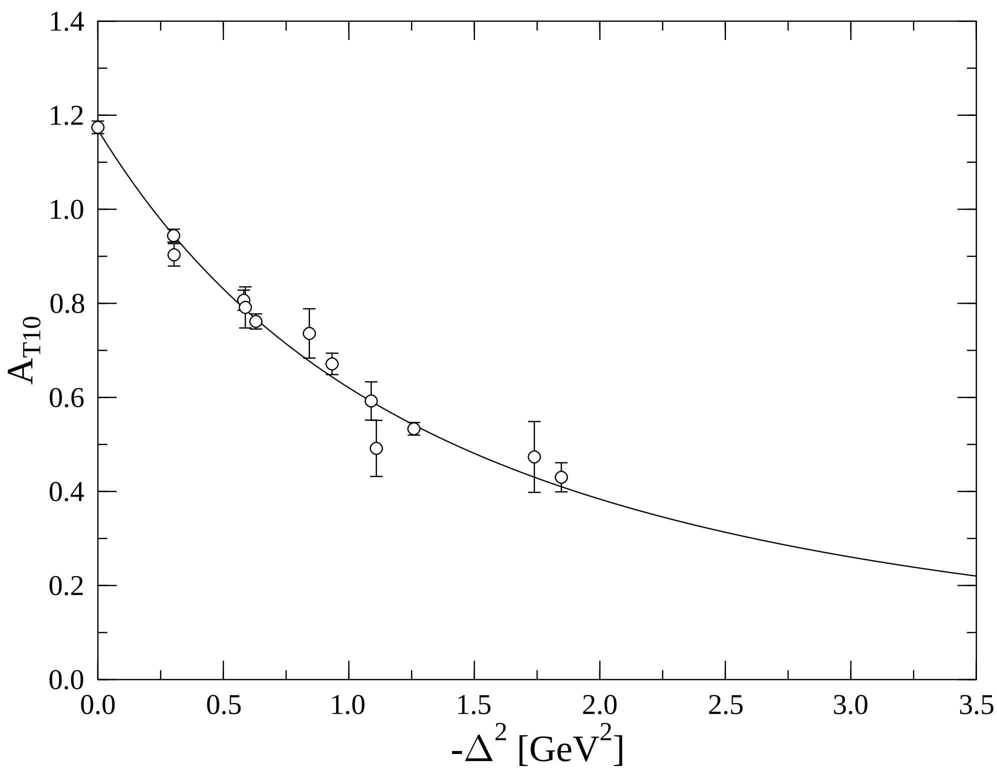
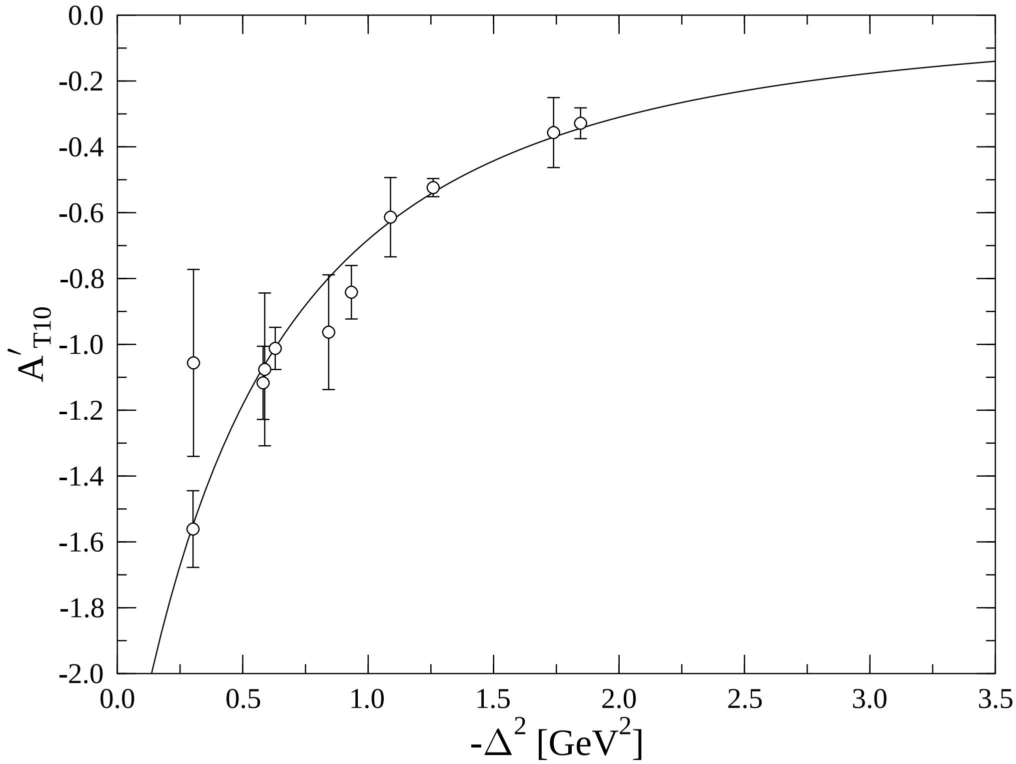
form factors (with dipole fits)

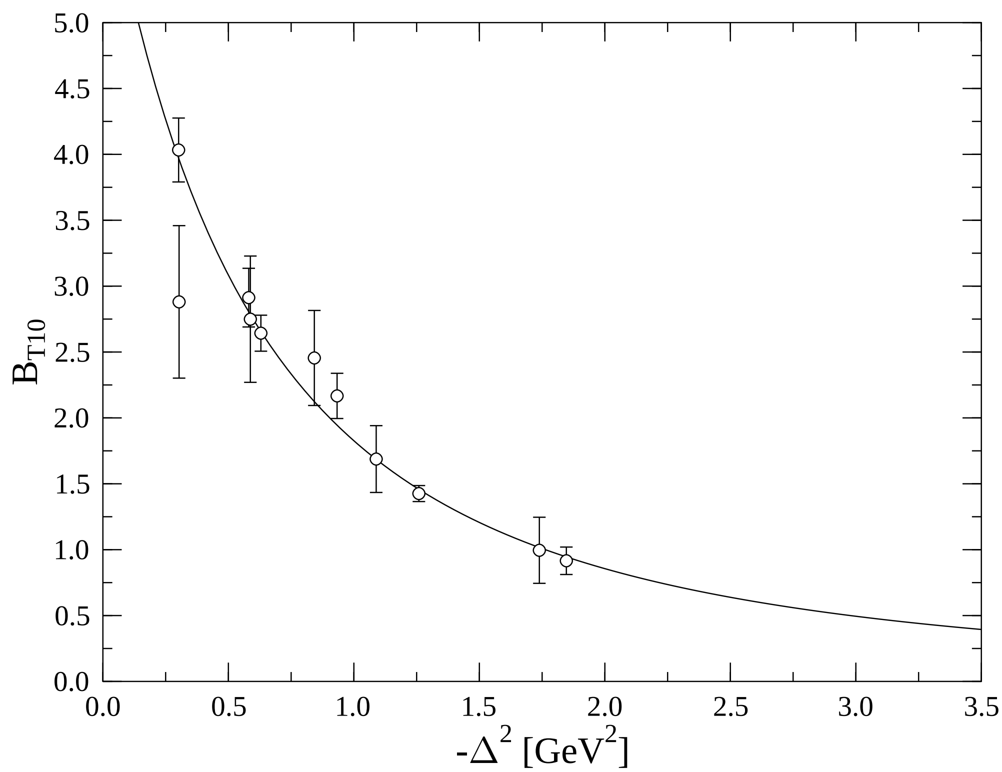
$\beta = 5.25$, $\kappa = 0.13575$, $m_\pi = 0.557$ GeV



$m_D = 1.64(3)$ GeV



$m_D = 1.04(6)$ GeV



$m_D = 1.08(5)$ GeV

Summary and outlook

- results for m_N, g_A, $\delta u - \delta d$, $\ldots$ from $N_f = 2$ simulations with clover fermions
- Do we understand them?
- chEFT in a finite volume $\longrightarrow$ expansion in powers of m_π and L^{-1} with coefficients depending on $m_\pi L$
- formalism applied to the nucleon mass surprisingly successful
  - for the pion mass dependence
  - for the volume dependence

  using relativistic SU(2)$_\mathrm{f}$ baryon chiral perturbation theory up to $O(p^4)$
- for the axial charge g_A of the nucleon
  - pion mass dependence compatible (?) with chPT
    $O(\epsilon^3)$ small scale expansion
  - volume dependence inconsistent with chPT
    $O(p^3)$ HBchPT
- for the tensor charge of the nucleon little seems to be known in chPT
  difference between tensor charge and axial charge small

for the future:

- Monte Carlo simulations:
  smaller quark masses (and lattice spacings)
- chEFT:
  higher order calculations for g_A
  calculations for the tensor charge