N=D=2 Twisted Supersymmetry on a Lattice

based on hep-lat/0406029

Alessandoro D'Adda, Issaku Kanamori, Noboru Kawamoto and Kazuhiro Nagata

INFN Torino,Italy

Department of Physics, Hokkaido University, Japan

0. Introduction

SUSY on the Lattice

Kaplan, B D *et al*
So, H; Kato, M *et al*
Fujikawa, K
Catteral, S; Karamov, S
Sugino, F
⋮

Importance

Regularization of Bosons & Fermions
Regularization of Spacetime
Non-perturbative dynamics of SUSY models ...

Difficulties

Leibniz rule ...

What we have done

N=2 Twisted SUSY on 2D Lattice

with *Exact Leibniz rule* on the Lattice

based on N=D=2 Twisted *Non-Commutative* Superspace

1. N=D=2 Twisted SUSY Algebra

Witten, E
Kawamoto, N; Tsukioka, T

$$\{s_\alpha^{\ i}, s_\beta^{\ j}\} = 2\delta^{ij}(\gamma^\mu)_{\alpha\beta}\partial_\mu$$

$\mu,\nu = 1,2$: 2D Euclidean
$\alpha,\beta = 1,2$: spinor indices
$i,j = 1,2$: internal indices
$\gamma^1 = \sigma_3,\ \gamma^2 = \sigma_1,\ \gamma^5 = \gamma^1\gamma^2$

Dirac-Kahler Twist

$$s_\alpha^{\ i} = (\mathbf{1}s + \gamma^\mu s_\mu + \gamma^5 \tilde{s})_\alpha^{\ i}$$

s : Fermionic Scalar
s_μ : Fermionic Vector
$\tilde{s}$: Fermionic Pseudo Scalar

$$\{s, s_\mu\} = -i\partial_\mu \qquad \{\tilde{s}, s_\mu\} = i\epsilon_{\mu\nu}\partial_\nu$$

$\xrightarrow{\text{Lattice}}$

$$\{s, s_\mu\} = -i\Delta_{\pm\mu} \qquad \{\tilde{s}, s_\mu\} = i\epsilon_{\mu\nu}\Delta_{\pm\nu}$$

2. Leibniz rule & Non-Commutativity

Dimakis, A
Kanamori, I; Kawamoto, N
⋮

$$\{s_A, s_B\} = -if^\mu_{AB}\Delta_{\pm\mu} \qquad s_A = (s, s_\mu, \tilde{s})$$

R.H.S.: $\Delta_{\pm\mu}$ carries **Non-Commutative** Nature

$$\Delta_{\pm\mu}\phi(x) \equiv \pm[\phi(x \pm 2\hat{n}_\mu) - \phi(x)]$$
$$\Delta_{\pm\mu}(\phi(x)\psi(x)) = (\Delta_{\pm\mu}\phi(x))\psi(x) + \phi(x \pm 2\hat{n}_\mu)(\Delta_{\pm\mu}\psi(x))$$

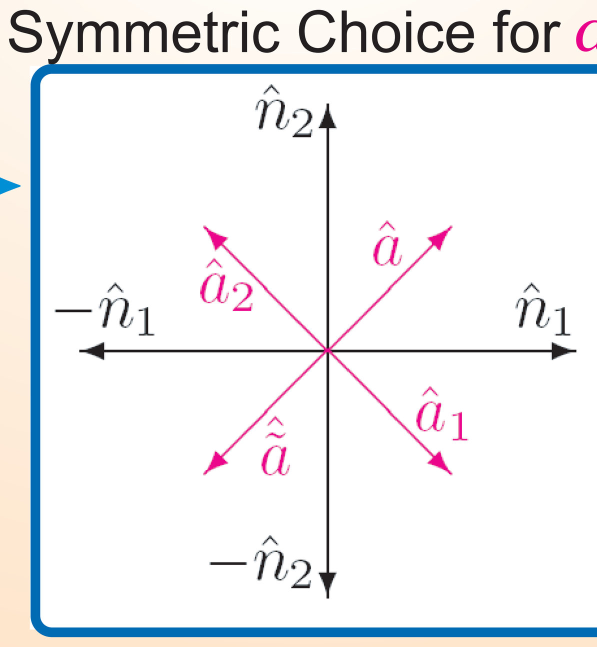
L.H.S.: Introduce **Non-Commutativity** on s_A

$$s_A\phi(x) \equiv \vec{s}_A\phi(x) - (-1)^{|\phi|}\phi(x+2\hat{a}_A)\vec{s}_A$$
$$s_A(\phi(x)\psi(x)) = (s_A\phi(x))\psi(x) + (-1)^{|\phi|}\phi(x+2\hat{a}_A)(s_A\psi(x))$$
$$\{s_A, s_B\}(\phi(x)\psi(x)) = (\{s_A, s_B\}\phi(x))\psi(x) + \phi(x+2\hat{a}_A + 2\hat{a}_B)(\{s_A, s_B\}\psi(x))$$

Consistency Conditions

$$\hat{a}_A + \hat{a}_B = +\hat{n}_\mu \quad \text{for} \quad \Delta_{+\mu}$$
$$\hat{a}_A + \hat{a}_B = -\hat{n}_\mu \quad \text{for} \quad \Delta_{-\mu}$$

Symmetric Choice for a_A



$$\because \quad \Delta_{+\mu}\phi(x)\psi(x) = \phi(x+2\hat{n}_\mu)\psi(x+2\hat{n}_\mu) - \phi(x)\psi(x)$$
$$= \phi(x+2\hat{n}_\mu)\psi(x+2\hat{n}_\mu) - \phi(x+2\hat{n}_\mu)\psi(x) + \phi(x+2\hat{n}_\mu)\psi(x) - \phi(x)\psi(x)$$
$$= \phi(x+2\hat{n}_\mu)[\psi(x+2\hat{n}_\mu) - \psi(x)] + [\phi(x+2\hat{n}_\mu) - \phi(x)]\psi(x)$$
$$= (\Delta_{+\mu}\phi(x))\psi(x) + \phi(x+2\hat{n}_\mu)(\Delta_{+\mu}\psi(x))$$

N=D=2 Twisted Lattice SUSY Algebra

$$\{s, s_\mu\} = -i\Delta_{+\mu} \qquad \{\tilde{s}, s_\mu\} = i\epsilon_{\mu\nu}\Delta_{-\nu}$$

3. N=D=2 Twisted Non-Commutative Superspace

$$\{Q, Q_\mu\} = i\Delta_{+\mu} \qquad \{\tilde{Q}, Q_\mu\} = -i\epsilon_{\mu\nu}\Delta_{-\nu}$$

$$\theta_A\phi(x) = (-1)^{|\phi|}\phi(x-2\hat{a}_A)\theta_A$$

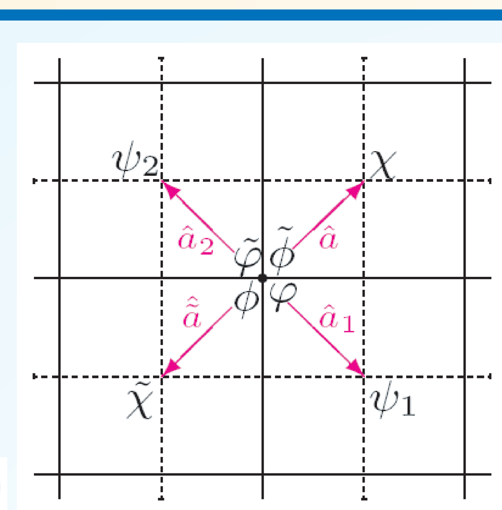
$$Q = \frac{\partial}{\partial\theta} + \frac{i}{2}\theta^\mu\Delta_{+\mu}, \qquad D = \frac{\partial}{\partial\theta} - \frac{i}{2}\theta^\mu\Delta_{+\mu},$$
$$Q_\mu = \frac{\partial}{\partial\theta^\mu} + \frac{i}{2}\theta\Delta_{+\mu} - \frac{i}{2}\tilde{\theta}\epsilon_{\mu\nu}\Delta_{-\nu}, \quad D_\mu = \frac{\partial}{\partial\theta^\mu} - \frac{i}{2}\theta\Delta_{+\mu} + \frac{i}{2}\tilde{\theta}\epsilon_{\mu\nu}\Delta_{-\nu},$$
$$\tilde{Q} = \frac{\partial}{\partial\tilde{\theta}} - \frac{i}{2}\epsilon_{\mu\nu}\theta^\mu\Delta_{-\nu}, \qquad \tilde{D} = \frac{\partial}{\partial\tilde{\theta}} + \frac{i}{2}\epsilon_{\mu\nu}\theta^\mu\Delta_{-\nu},$$

4. Twisted Wess-Zumino model

Chiral Conditions

$$D\Psi(x) = \tilde{D}\Psi(x) = 0, \qquad D_\mu\overline{\Psi}(x) = 0$$

$\Psi, \overline{\Psi}$: Bosonic



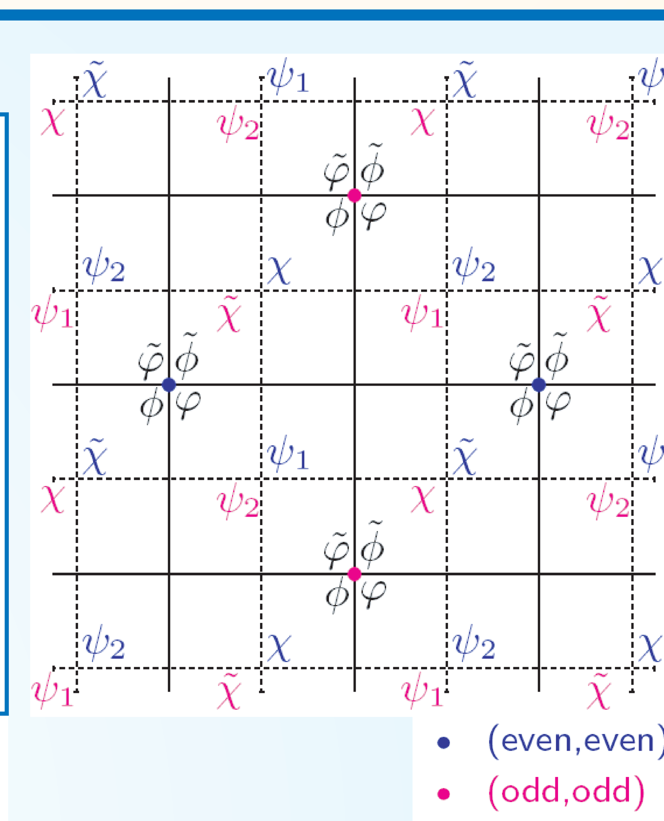
$$U \equiv e^{-\frac{i}{2}(\theta\theta_\mu\Delta_{+\mu} - \tilde{\theta}\epsilon^{\mu\nu}\theta_\mu\Delta_{-\nu})}$$

$$\Psi(x) = U^{-1}[\phi(x) + \theta_\mu\psi_\mu(x+\hat{a}_\mu) + \theta_1\theta_2\tilde{\phi}(x+\hat{a}_1+\hat{a}_2)]U$$
$$\overline{\Psi}(x) = U\Big[\varphi(x) + \theta\chi(x+\hat{a}) + \tilde{\theta}\tilde{\chi}(x+\hat{\tilde{a}}) + \theta\tilde{\theta}\tilde{\varphi}(x+\hat{a}+\hat{\tilde{a}})\Big]U^{-1}$$

SUSY inv. Action

$$S^{lat}_{WZ} = \int d^4\theta \sum_x (\overline{\Psi}(x)\Psi(x))$$
$$= \sum_x s\tilde{s}s_1s_2(\varphi(x)\phi(x))$$
$$= \sum_x \Big[i\chi(x-\hat{a})\Delta_{-\mu}\psi_\mu(x+\hat{a}_\mu) + i\tilde{\chi}(x-\hat{\tilde{a}})\epsilon_{\mu\nu}\Delta_{+\mu}\psi_\nu(x+\hat{a}_\nu) + \varphi(x)\Delta_{+\mu}\Delta_{-\mu}\phi(x) + \tilde{\varphi}(x+\hat{a}_1+\hat{a}_2)\tilde{\phi}(x+\hat{a}_1+\hat{a}_2)\Big]$$

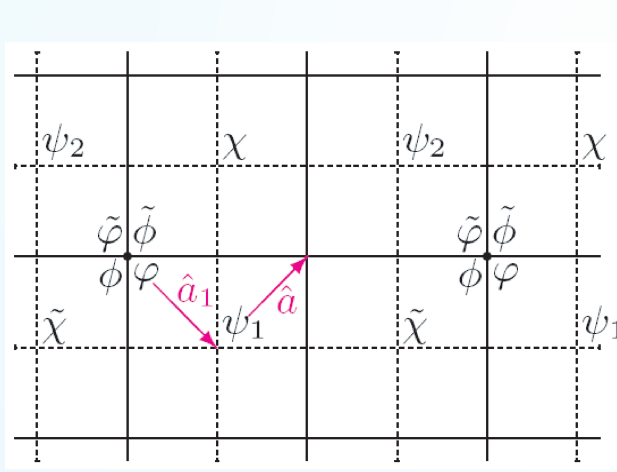
$$\sum_x = \sum_{(even,even)} + \sum_{(odd,odd)}$$



SUSY Trans. laws

$$s_A\Psi(x) = Q_A\Psi(x)$$
$$s_A\overline{\Psi}(x) = Q_A\overline{\Psi}(x)$$
$$\Downarrow$$
$$s_1\phi(x) = \psi_1(x+\hat{a}_1)$$
$$s\psi_1(x+\hat{a}_1) = -i\Delta_{+1}\phi(x) = -i[\phi(x+2\hat{n}_1) - \phi(x)]$$

etc..



$fields$	s	s_μ	$\tilde{s}$
$\phi(x)$	0	$\psi_\mu(x+\hat{a}_\mu)$	0
$\psi_\nu(x+\hat{a}_\nu)$	$-i\Delta_{+\nu}\phi(x)$	$-\epsilon_{\mu\nu}\tilde{\phi}(x+\hat{a}_1+\hat{a}_2)$	$i\epsilon_{\nu\rho}\Delta_{-\rho}\phi(x)$
$\tilde{\phi}(x+\hat{a}_1+\hat{a}_2)$	$i\epsilon_{\rho\sigma}\Delta_{+\rho}\psi_\sigma(x+\hat{a}_\nu)$	0	$-i\Delta_{-\rho}\psi_\rho(x+a_\mu)$
$\varphi(x)$	$\chi(x+\hat{a})$	0	$\tilde{\chi}(x+\hat{\tilde{a}})$
$\chi(x+\hat{a})$	0	$-i\Delta_{+\mu}\psi(x)$	$\tilde{\varphi}(x+\hat{a}+\hat{\tilde{a}})$
$\tilde{\chi}(x+\hat{\tilde{a}})$	$-\tilde{\varphi}(x+\hat{a}+\hat{\tilde{a}})$	$i\epsilon_{\mu\rho}\Delta_{-\rho}\varphi(x)$	0
$\tilde{\varphi}(x+\hat{a}+\hat{\tilde{a}})$	0	$i\epsilon_{\mu\rho}\Delta_{-\rho}\chi(x+\hat{a}) + i\Delta_{+\mu}\tilde{\chi}(x+\hat{\tilde{a}})$	0

Untwisting & Dirac-Kahler Mechanism

$$F_1(x+\hat{a}_1+\hat{a}_2) = \tfrac{1}{2}\left(\tilde{\phi}(x+\hat{a}_1+\hat{a}_2) - \tilde{\varphi}(x+\hat{a}_1+\hat{a}_2)\right), \quad \phi_1(x) = \tfrac{1}{2}(\phi(x) - \varphi(x))$$
$$F_2(x+\hat{a}_1+\hat{a}_2) = \tfrac{i}{2}\left(\tilde{\phi}(x+\hat{a}_1+\hat{a}_2) + \tilde{\varphi}(x+\hat{a}_1+\hat{a}_2)\right), \quad \phi_1(x) = \tfrac{i}{2}(\phi(x) + \varphi(x))$$
$$\xi_{\alpha i}(x) = \tfrac{1}{2}\left(\chi(x-\hat{a}) + \gamma_\mu\psi_\mu(x+\hat{a}_\mu) + \gamma_5\tilde{\chi}(x-\hat{\tilde{a}})\right)_{\alpha i}$$

Wess-Zumino action

$$S^{lat}_{WZ} = \sum_x \Big[-\phi_i(x)\Delta_{+\mu}\Delta_{-\mu}\phi_i(x) - F_i(x+\hat{a}_1+\hat{a}_2)F_i(x+\hat{a}_1+\hat{a}_2) + i\bar{\xi}_{i\alpha}(x)(\gamma_\mu)_{\alpha\beta}\frac{\Delta_{+\mu}+\Delta_{-\mu}}{2}\xi_{\beta i}(x) - i\bar{\xi}_{i\alpha}(x)(\gamma_5)_{\alpha\beta}\frac{\Delta_{+\mu}-\Delta_{-\mu}}{2}\xi_{\beta i}(x)(\gamma_5\gamma_\mu)_{ji}\Big]$$

1st diff. w.r.t. double size lattice — *2nd diff. ~ O(lat.const)*

5. Quantized BF Model

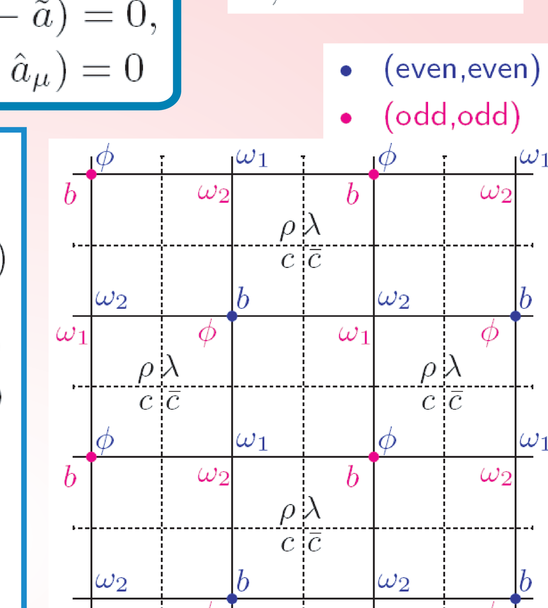
Kato, J; Kawamoto, N; Uchida, Y

Non-Abelian Chiral Conditions

$$D\Phi(x-\hat{a}) - i\Phi(x+\hat{a})\Phi(x-\hat{a}) = 0,$$
$$\tilde{D}\Phi(x-\hat{\tilde{a}}) = 0,$$
$$D_\mu\overline{\Psi}(x-\hat{a}_\mu) = 0$$

$\Phi, \overline{\Psi}$: Fermionic

$$S^{lat}_{BF} = \int d\theta^4 \sum_x \mathrm{tr}\ i\ \overline{\Psi}(x+\hat{a})\Phi(x+\hat{a})$$
$$= \sum_x \mathrm{tr}\ s\tilde{s}s_1s_2\big(-i\bar{c}(x+\hat{a})c(x+\hat{a})\big)$$
$$= \sum_x \mathrm{tr}\Big[\phi(x+\hat{a}-\hat{\tilde{a}})\big[\epsilon_{\mu\nu}\Delta_{+\mu}\omega_\nu(x-\hat{a}_\mu-\hat{\tilde{a}}) + (\epsilon_{\mu\nu}\omega_\mu\omega_\nu)(x+\hat{a}-\hat{\tilde{a}}) - \{c,\lambda\}(x+\hat{a}-\hat{\tilde{a}})\big] + b(x)\Delta_{-\mu}\omega_\mu(x+\hat{n}_\mu) - i\bar{c}(x+\hat{a})\Delta_{-\mu}D_{+\mu}c(x+\hat{a}) + i\rho(x-\hat{\tilde{a}})\lambda(x-\hat{\tilde{a}})\Big]$$
$$\Big(D_{+\mu}c(x) \equiv \Delta_{+\mu}c(x) + [\omega_\mu, c](x+\hat{n}_\mu)\Big)$$



$$[\omega_\mu, c](x) \equiv \omega_\mu(x+\hat{a})c(x-\hat{n}_\mu) - c(x+\hat{n}_\mu)\omega_\mu(x-\hat{a}),$$
$$(\epsilon_{\mu\nu}\omega_\mu\omega_\nu)(x) \equiv \epsilon_{\mu\nu}\omega_\mu(x+\hat{n}_\nu)\omega_\nu(x-\hat{n}_\mu),$$
$$\{c,\lambda\}(x) \equiv c(x+\hat{n}_1+\hat{a}_2)\lambda(x-\hat{a}) + \lambda(x+\hat{a})c(x-\hat{n}_1-\hat{a}_2)$$

$fields$	s	s_μ	$\tilde{s}$
$c(x+\hat{a})$	$-c(x+3\hat{a})c(x+\hat{a})$	$-i\omega_\mu(x+\hat{n}_\mu)$	0
$\omega_\nu(x+\hat{n}_\nu)$	$\Delta_{+\nu}c(x+\hat{a}) + [\omega_\nu, c](x+\hat{n}_\nu+\hat{a})$	$-i\epsilon_{\mu\nu}\lambda(x+\hat{n}_\nu+\hat{a}_\mu)$	$-\epsilon_{\nu\rho}\Delta_{-\rho}c(x+\hat{a})$
$\lambda(x+\hat{n}_1+a_2)$	$\epsilon_{\rho\sigma}\Delta_{+\rho}\omega_\sigma(x+\hat{n}_\sigma) + (\epsilon_{\rho\sigma}\omega_\rho\omega_\sigma)(x+\hat{n}_1+\hat{n}_2) - \{c,\lambda\}(x+\hat{n}_1+\hat{n}_2)$	0	$-\Delta_{-\rho}\omega_\rho(x+\hat{n}_\rho)$
$\bar{c}(x+\hat{a})$	$-ib(x+2\hat{a})$	0	$-i\phi(x+\hat{a}+\hat{\tilde{a}})$
$b(x+2\hat{a})$	0	$\Delta_{+\mu}\bar{c}(x+\hat{a})$	$-i\rho(x+2\hat{a}+\hat{\tilde{a}})$
$\phi(x+\hat{a}+\hat{\tilde{a}})$	$i\rho(x+2\hat{a}+\hat{\tilde{a}})$	$-\epsilon_{\mu\rho}\Delta_{-\rho}\bar{c}(x+\hat{a})$	0
$\rho(x+2\hat{a}+\hat{\tilde{a}})$	0	$-\Delta_{+\mu}\phi(x+\hat{a}+\hat{\tilde{a}}) - \epsilon_{\mu\rho}\Delta_{-\rho}b(x+2\hat{a})$	0

6. Summary & Outlook

N=D=2 Twisted SUSY on the Lattice

- **Exact SUSY Algebra for all Supercharges**
  ***keeping* *Leibniz rule* on the Lattice**
- **Twisted *Non-Commutative* Superspace**
  → Manifestly SUSY inv. Models on the Lattice

For Furure work

Super Yang-Mills on the Lattice

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