Excited hadrons from improved interpolating fields

C. B. Lang

Inst. f. Physik / Theoretische Physik

Universität Graz

with

Tommy Burch - Christof Gattringer - Leonid Glozman –
Reinhard Kleindl - Andreas Schäfer

BernGrazRegensburg-QCD collaboration

(Computations at Leibniz-RZ München Hitachi SR8000-F1)

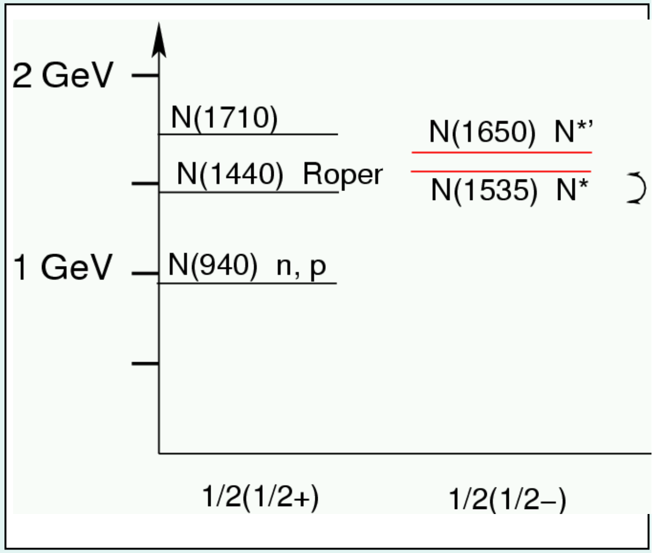
hep-lat/0405006

See also: hep-lat/0309036 =
NP B PS 129/130 (2004) 251
hep-ph/0307073 =
PR D69 (2004) 094513

UNI GRAZ

Low lying hadron spectrum

- The nucleons have unusual level ordering
  - Flavor-spin Goldstone boson exchange (chiral quark model: Glozman/Riska)?
  - Structure of Roper state?
    (3-quark, 4+1 quark, …?)



- The pion is different from all other mesons
  - Role of the Goldstone mode?

Confinement vs. Chiral Symmetry Breaking

“Chirally improved” Dirac operator

Approximate GW-fermions:

systematic (diagrammatic) expansion and reduction of GWC to set of algebraic equations:

$$D = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in P_\alpha} c_p^\alpha \left\langle l_1 l_2 \dots l_{|p|} \right\rangle$$

Gattringer
PRD 63 (2001) 114501;
Gattringer et al.
Nucl. Phys. B697 (2001) 451)

Studies of ground state hadron spectrum on lattices up to 16^3x32:

BGR collab. Nucl.Phys. B677 (2004) 3

Correlation functions

$$\langle 0 | \bar{X}(t) X(0) | 0 \rangle = \sum_n \langle 0 | \bar{X} | n \rangle e^{-E_n t} \langle n | X | 0 \rangle$$

How to disentangle more than one state in a quantum channel?

→ **"Variational method"** (C. Michael 1985, Lüscher & Wolff 1990

- Use set of interpolating operators χ_i (as "complete" as possible)
- Compute all cross-correlations; the spectral representation reads:

$$C_{ij}(t) = \langle \bar{\chi}_i(t) \chi_j(0) \rangle = \sum_\alpha \bar{A}_i^\alpha A_j^\alpha \ \lambda_\alpha(t)$$

- Eigenvectors are dominated by physical states

$$C(t) u = \lambda(t) C(t_0) u$$

$$\lambda_\alpha(t) \propto e^{-tE_\alpha} (1 + O(e^{-t\Delta E_\alpha}))$$

…best overlap with physical states:

$$u^\alpha = \sum_i A_i^\alpha \chi_i$$

These can be analyzed like the usual correlation functions.

Nucleon interpolating operators

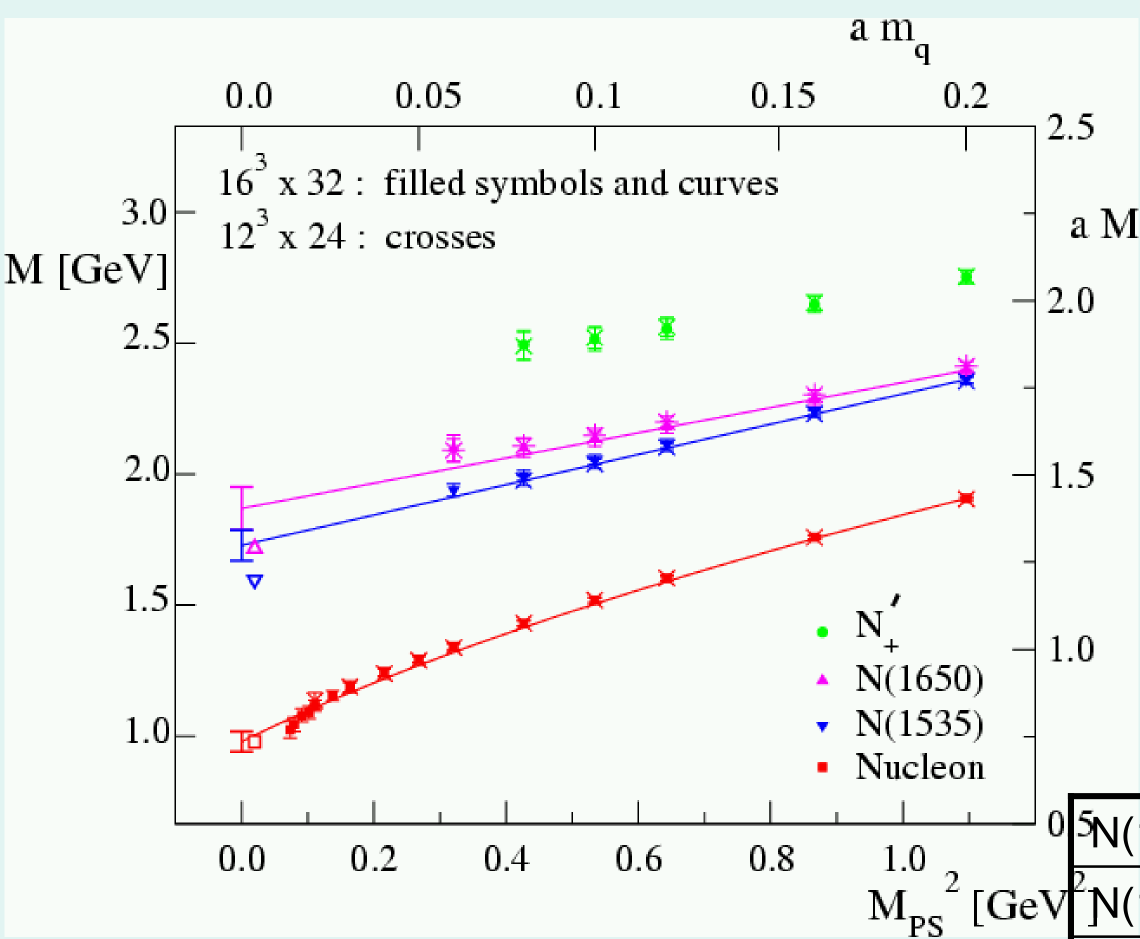
$$\chi_1(x) = \varepsilon_{abc} \left[u_a^T(x)\, C\gamma_5\, d_b(x) \right] u_c(x)$$

$$\chi_2(x) = \varepsilon_{abc} \left[u_a^T(x)\, C\, d_b(x) \right] \gamma_5\, u_c(x)$$

$$\chi_3(x) = i\,\varepsilon_{abc} \left[u_a^T(x)\, C\gamma_4\gamma_5\, d_b(x) \right] u_c(x)$$

Lowest two states in both parity sectors

Brömmel et al., PR D69 (2004) 094513



Where is the Roper?

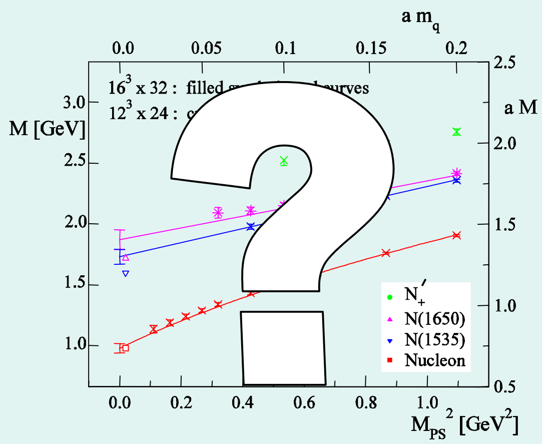
- N - N(1535) splitting increases towards chiral limit
- Splitting between neg. parity states
- **N_+' is not the Roper – maybe N(1710)?**

Mass ratios in "chiral" extrapolation: Exp.

N(1535)/N(938)	1.77(7)	1.63(2)
N(1650)/N(938)	1.91(9)	1.75(3)
N(1650)/N(1535)	1.08(5)	1.08(1)

UNI GRAZ

Where is the Roper?



Roper + Level switching observed in:

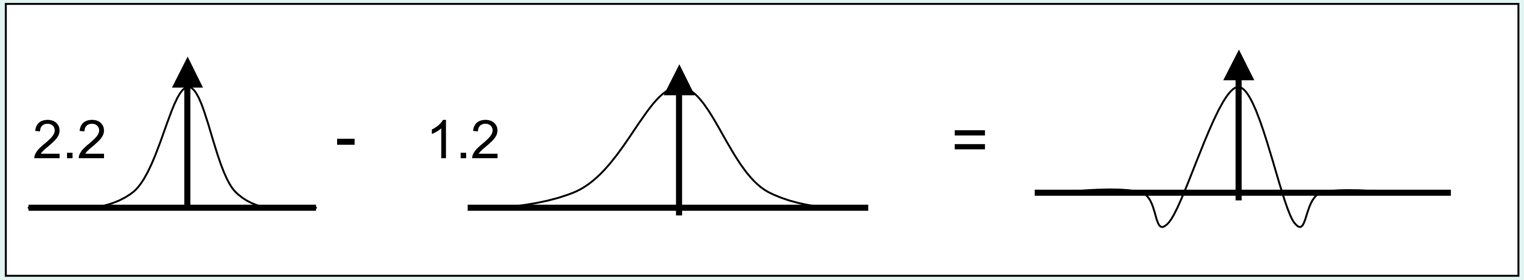
Chen et al. (hep-ph/0306199):
Overlap action, Constrained Bayesian fits,
Quenched “ghost” analysis η’N:
Roper / N(1535) level switching window:
0.3..0.6 GeV

Sasaki et al. (e.g. nucl-th/0305014,
Prog.Theor.Phys. Suppl. 151 (2003) 143):
Wilson action, maximum entropy method,
Roper / N(1535) level switching window:
0.6..0.9 GeV

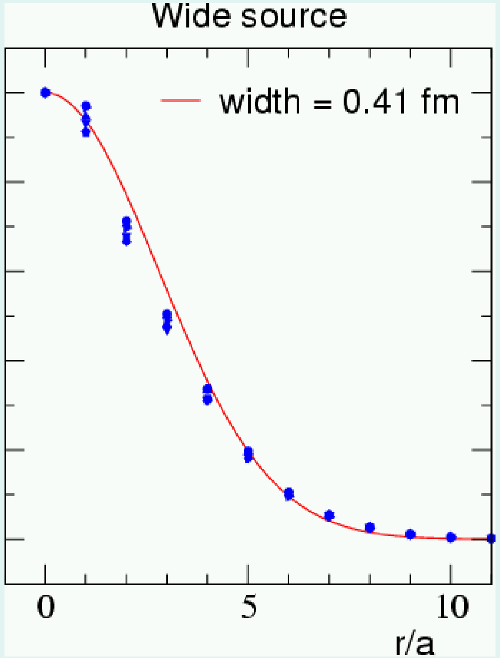
Improve the operators

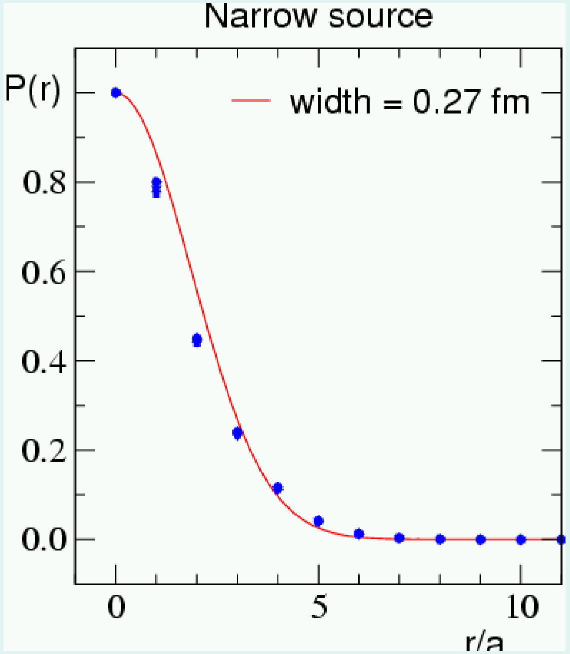
- Excited states have nodes!
- The interpolating operators are built from quark sources
- Quark sources are
  - (wall sources)
  - (point sources)
  - Gaussian shapes (Jacobi smearing)

→ Combine quark sources with different widths!



Constructing more operators





Quark propagators:
source - sink:

- narrow→point
- wide→point

(Point can be smeared to any shape)

	Quark-source combinations	Operator types	Cross correlations
Mesons	4	≤2	up to 8x8
Baryons	8	≤3	up to 24x24

Results on 12^3x24 lattices: Baryons

Interpolating fields are combinations of different width quark sources (e.g. in χ_1):

(nnn), (nnw), (nwn), (wwn)

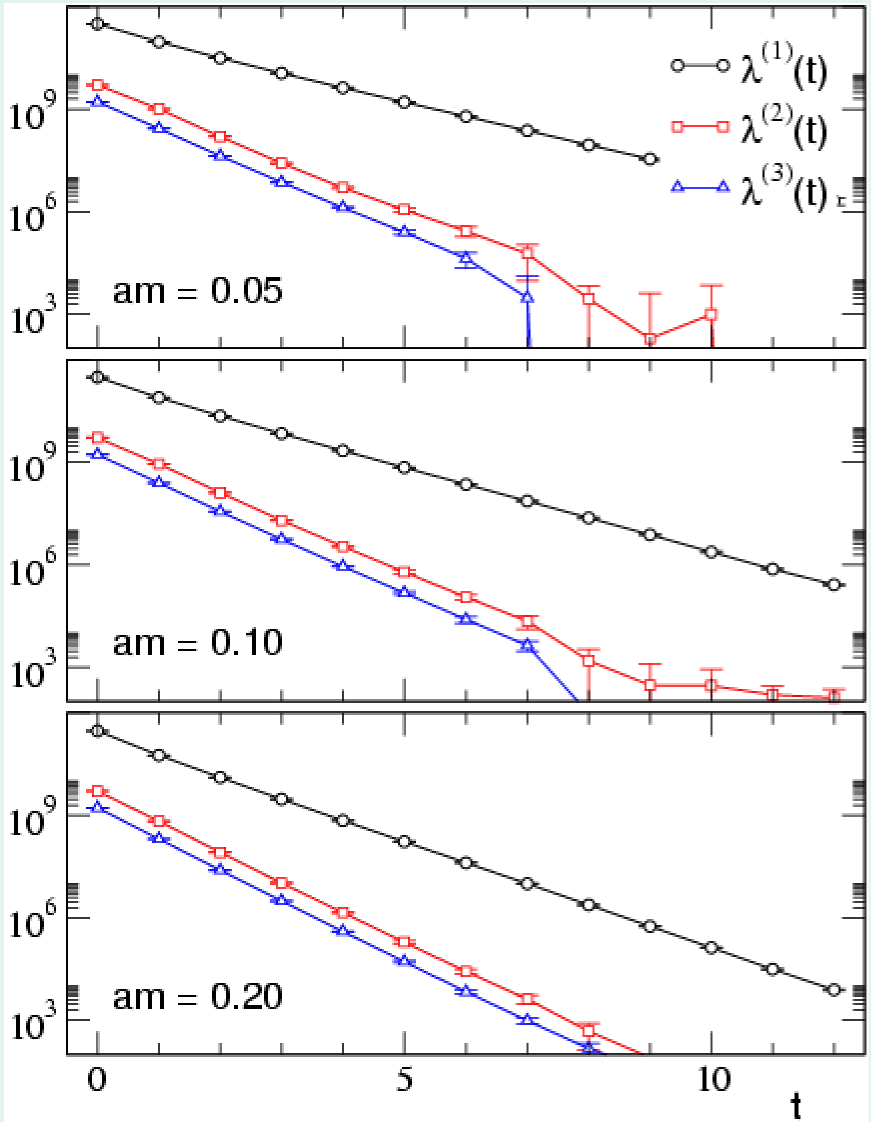
↓

4x4 correlation matrix :

…diagonalize…

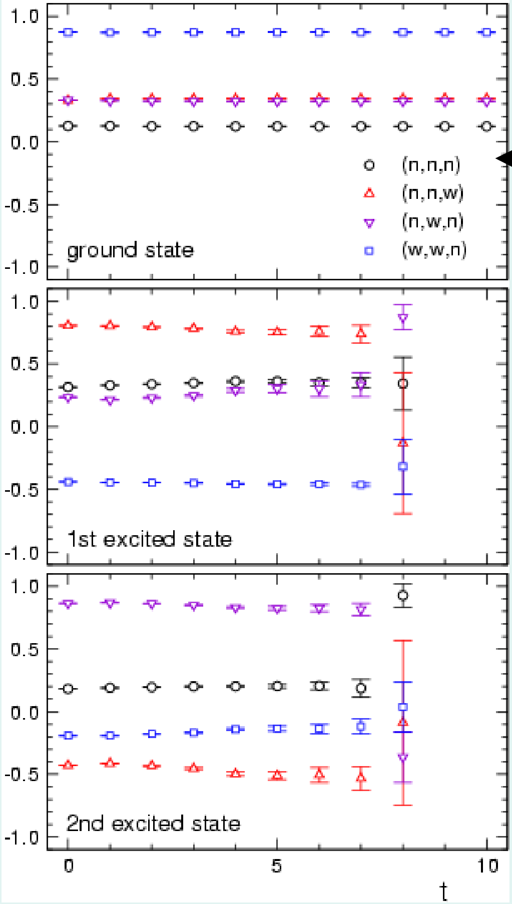
…fit to $\lambda_i(t) \sim \exp(-t\, m_i)$

Lowest 3 eigenvalues:
N, N(1440), N(1710)

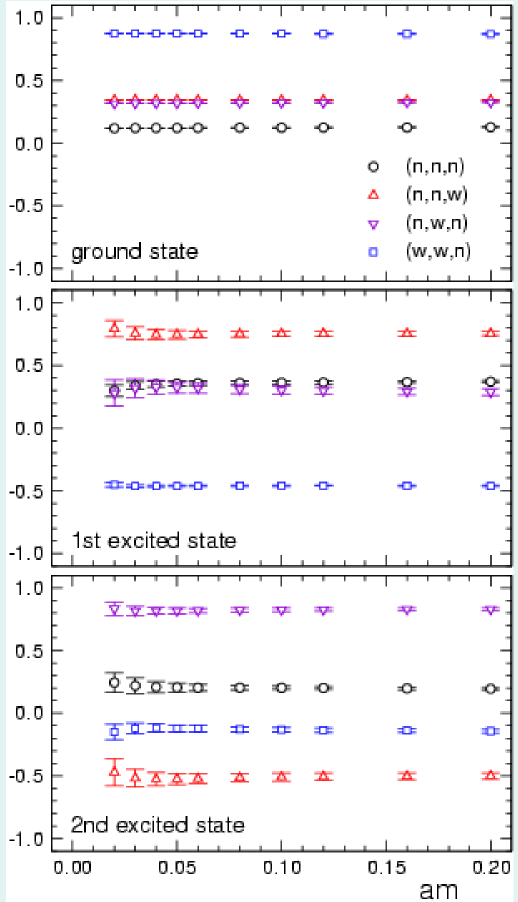


Operator content: coefficients

Eigenvectors
vs. t
(a m =0.10)



Eigenvectors
vs. m



Same signs: ground state has no node !

Different signs: excited states have a node !

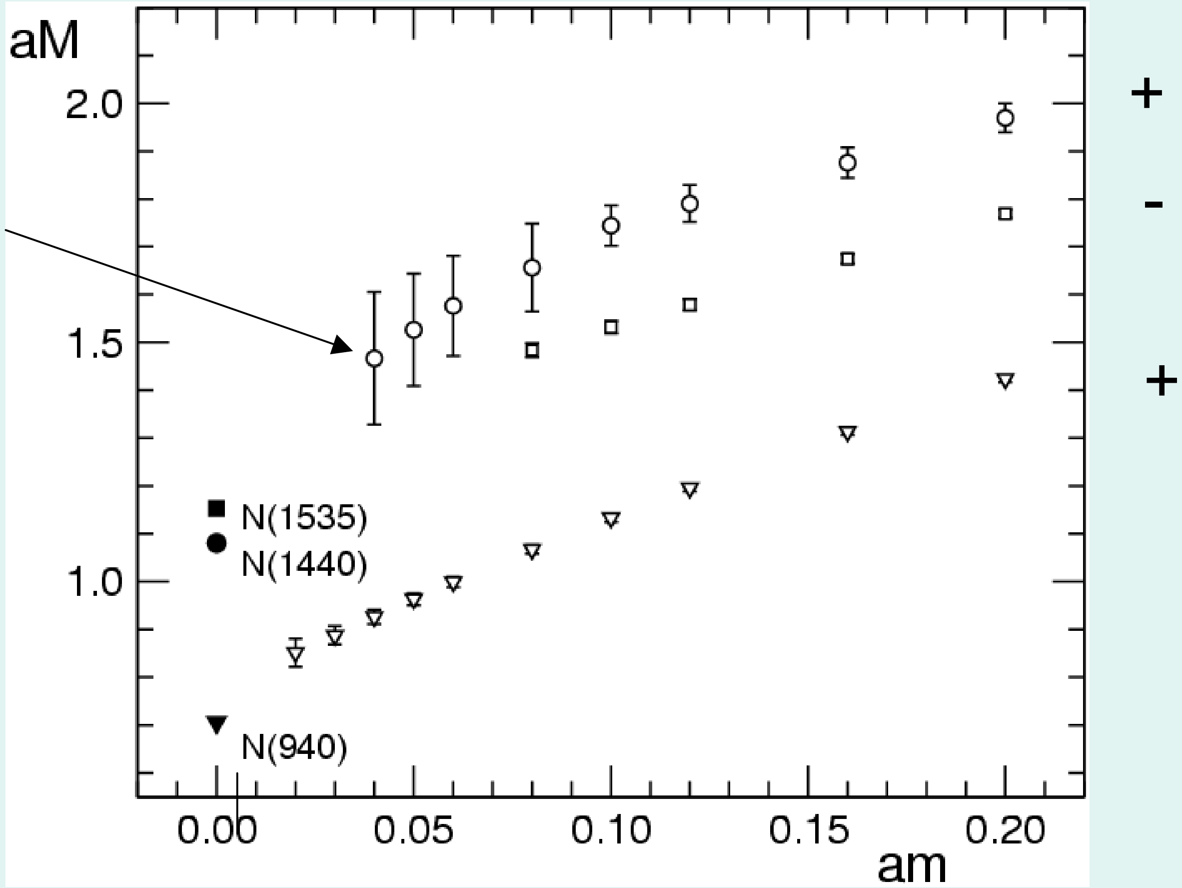
Results cont'd

Finally:
a signal of the Roper
(as a three quark state)

Chiral approach
affects different states
differently
(level crossing?)

Heavy quark limit:
confinement+ hyperfine
spin-spin interaction

Light quarks:
χSB effects become important



Can we be sure about the correct identification: Roper and/or N(1710)??

Roper

- is seen in χ_1 (like the nucleon) but only when different quark sources are combined allowing nodal w.f., (the ground state nucleon is seen)
- is orthogonal to the nucleon as a separate state, with smaller amplitude
- is it contaminated by N(1710)?

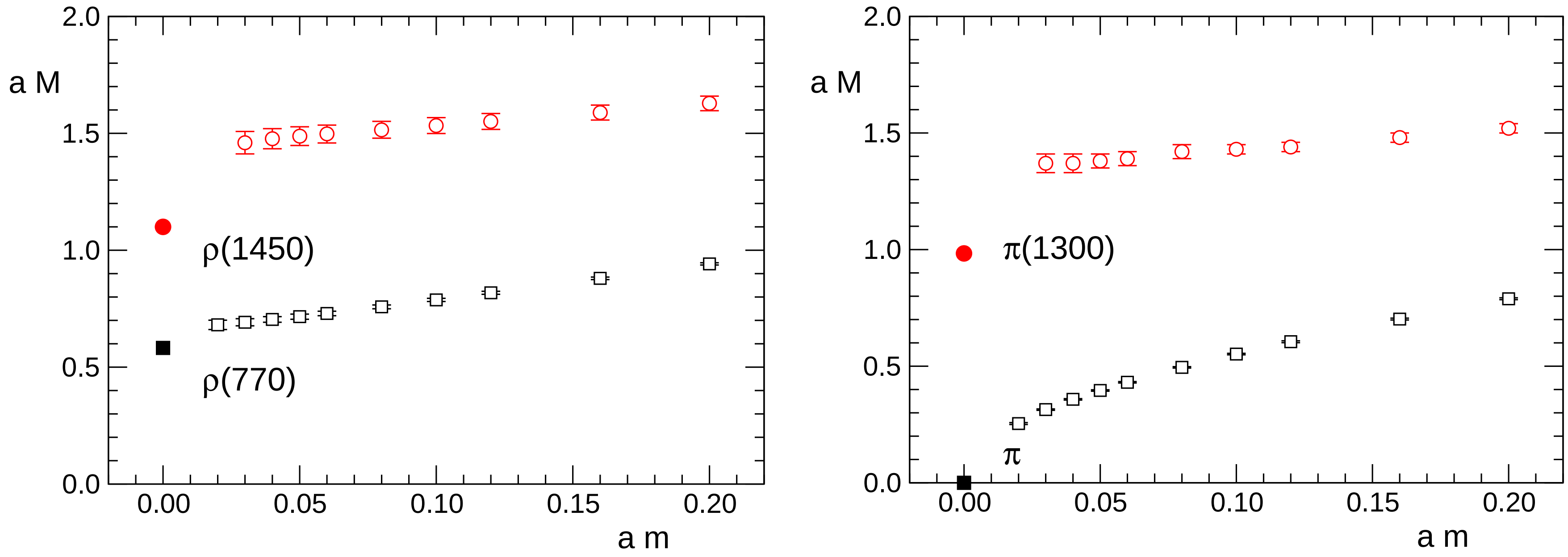
  Continuing from the heavy quark region where:

  - Roper 56-plet has positive parity for any two quarks (seen in χ_1, but not seen in χ_2, which has negative parity two-quark content)
  - N(1710) is in another multiplet containing positive and negative parity two-quark subsystems, and it is seen in χ_1 and in χ_2

  (n.b.: The nucleon is seen in χ_1, but not in χ_2)

Results: Mesons

Interpolating fields with diff. width quarks: **(nn), (nw), (ww)**



(excited states are 30% below the experiment – excited mesons are larger and have stronger finite volume influence?)

UNI GRAZ

Conclusions

- Understanding the Roper ab-initio is important and non-trivial!
- Variational analysis allows to disentangle mixed (physical) states
- Interpolating fields allowing for nodal "wave function" very useful to find the excited states

Questions and Outlook

- Volume dependence
- Level reordering towards the chiral limit?
- How to find even higher excitations
- Contact with χPT: Quenched pathologies, ghost signals?