Chiral extrapolations with small sea quark mass data in two-flavor lattice QCD

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1. Introduction

Features of lattice QCD

[Advantage]

- **Non-perturbative calculations**

[Disadvantage]

- **Statistical and systematic errors**
  - **Continuum extrapolation**($a \to 0$)
    $\to$ **improvement of actions and operators**
  - **Renormalization factor**
    $\to$ **non-perturbative renormalization method**
  - **Finite size effect(studied well)**
  - **Chiral extrapolation**

[Chiral extrapolation]

In lattice QCD, sea quark masses
(in two-flavor cases, $m_{ud} \equiv (m_u + m_d)/2$)
are heavier than those in our real world
because of computational costs
$\rightarrow$ **lattice data must be extrapolated**
to the physical point.

lattice $m_{PS}/m_V \rightarrow$ **real** $m_\pi/m_\rho = 0.17$

m_{PS}/m_V	**0.17**	**0.35**	**0.60**	**0.80**
m_{quark}^{AWI}[MeV]	~ 5	**13**	**63**	**91**

- **Long extrapolations may involve sizable systematic errors**
  $\rightarrow$ **more realistic simulations are desirable**

2. Simulations

$N_f = 2$ full QCD simulations

[Action and run parameters]

- **Small sea quark masses** ($m_{PS}/m_V = 0.60 - 0.35$, $m_{quark}^{AWI} = 63 - 13$ **MeV**)
- **Coarse lattices** ($a = 0.2$**[fm];**$L = 2.4, 3.2$**[fm]**)
- **RG gauge + tadpole-improved Clover quark**
- **HMC with the simple leapfrog scheme**
- **Measurements : every 5 trajectory (1 conf.)**

[Statistics]

Combined with our previous work, CP-PACS,2002

$[12^3 \times 24]$

κ_{sea}	m_{PS}/m_V	# traj.
0.14090	0.80	6250
0.14300	0.75	5000
0.14450	0.70	7000
0.14585	0.60	4000
0.14640	0.55	5250
0.14660	0.50	4000
0.14705	0.40	4000
0.14720	0.35	1400

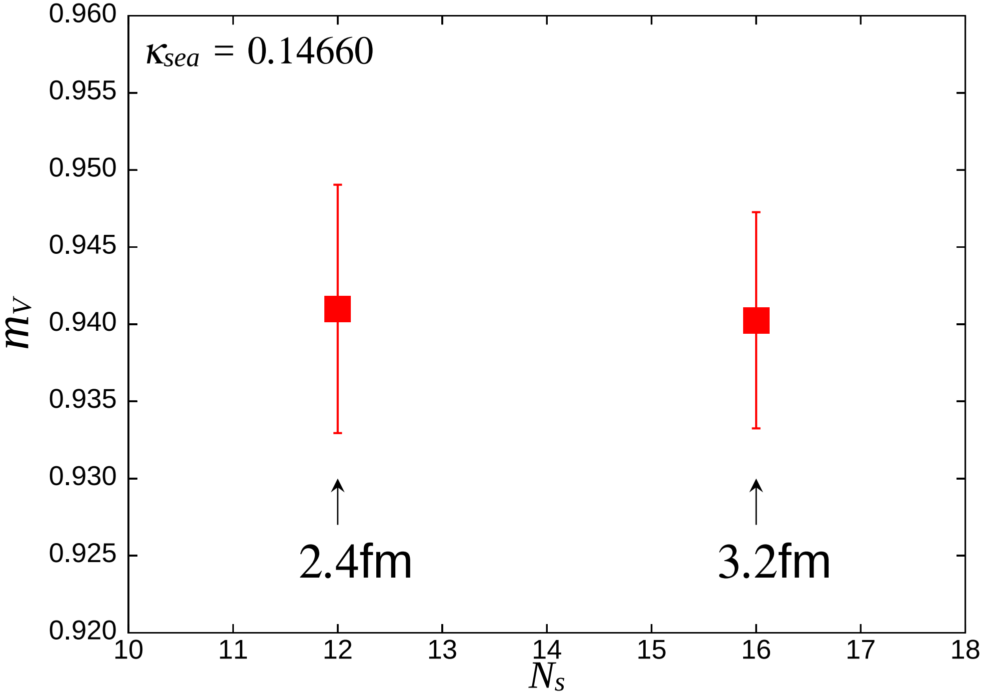
3. Results

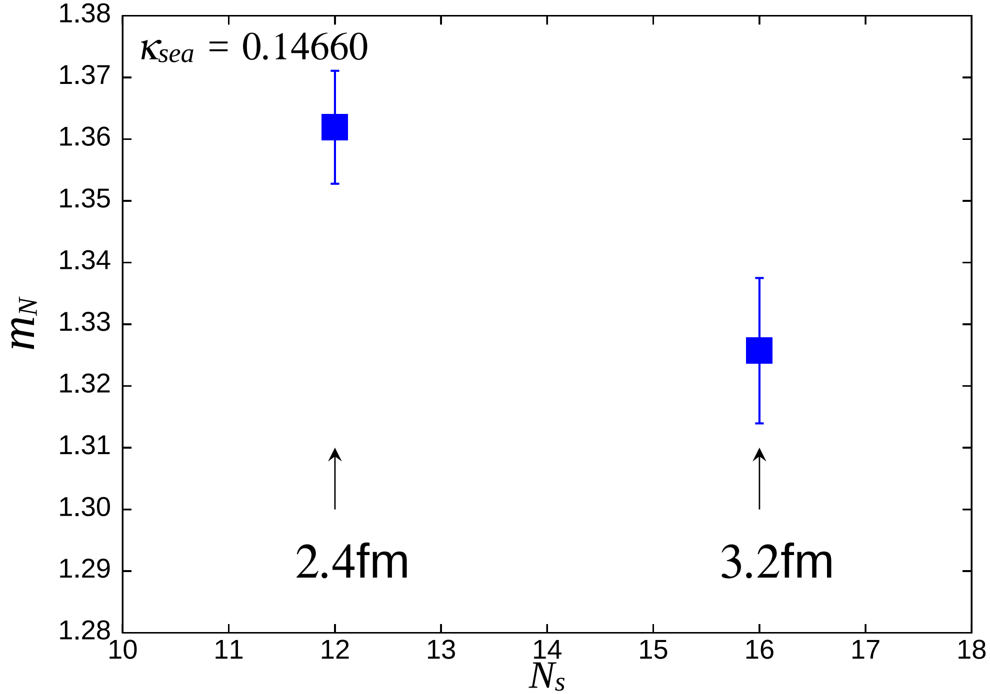
[Finite size effect]

$L = 2.4$fm vs $L = 3.2$fm

- **No finite size effects in meson quantities** $(m_{PS}, m_V, m_{quark}^{AWI})$

- $1 - 3\%(0.8 - 2.3\sigma)$ **decreases in baryon quantities** (m_N, m_Δ)
  $\rightarrow$ **we concentrate on the meson sector**





[Chiral extrapolation]
Parameterize lattice data O with m_{quark} for chiral extrapolations

$$O = f(m_{quark})$$

Choice of the extrapolation function f

- **polynomial(conventional method)**
- **Chiral perturbation(ChPT) formulae**
- **Wilson ChPT(WChPT) formulae**

Polynomial (conventional) extrapolation

[Vector meson mass]

We parameterize vector meson masses m_V as a function of pseudoscalar meson masses m_{PS}

$$m_V = A + Bm_{PS}^2 + Dm_{PS}^4 + Fm_{PS}^6 \quad \textbf{for } m_{PS}/m_V = 0.80 - 0.35$$

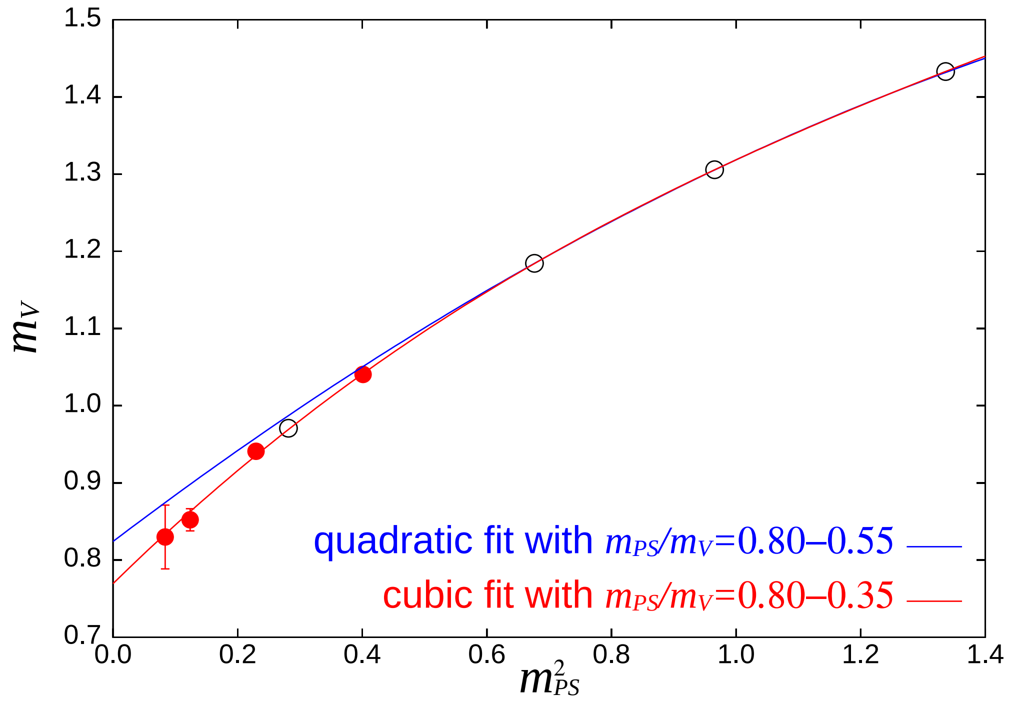
$$m_V = A + Bm_{PS}^2 + Dm_{PS}^4 \quad \textbf{for } m_{PS}/m_V = 0.80 - 0.55$$

(previous fit) **CP-PACS,2002**

- **Check the validity of the previous quadratic fit from $m_{PS}/m_V = 0.80 - 0.55$ by comparing with new small sea quark mass data $m_{PS}/m_V = 0.50 - 0.35$**

- **We use m_{PS}^2 instead of m_{quark} for comparison with the previous work (we parameterize m_{PS}^2 as a function of m_{quark} later)**

- **Systematic deviations from the previous fit are observed in small sea quark mass region**
  $\rightarrow$ **Sea quark mass dependence was underestimated by the previous quadratic extrapolation from $m_{PS}/m_V = 0.80 - 0.55$**
  The deviation in the chiral limit is $7\%(3.6\sigma)$
  $\rightarrow$ **sizable systematic error!**



[Pseudoscalar meson mass]

We parameterize pseudoscalar meson masses m_{PS}
as a function of VWI quark masses m_{quark}^{VWI}

$$
\begin{aligned}
m_{PS}^2 &= Bm_{quark}^{VWI} + C(m_{quark}^{VWI})^2 + D(m_{quark}^{VWI})^3 + E(m_{quark}^{VWI})^4 \\
&\quad \textbf{for } m_{PS}/m_V = 0.80 - 0.35 \\
m_{PS}^2 &= Bm_{quark}^{VWI} + C(m_{quark}^{VWI})^2 \\
&\quad \textbf{for } m_{PS}/m_V = 0.80 - 0.55 \textbf{(previous fit)} \quad \textbf{CP-PACS,2002}
\end{aligned}
$$

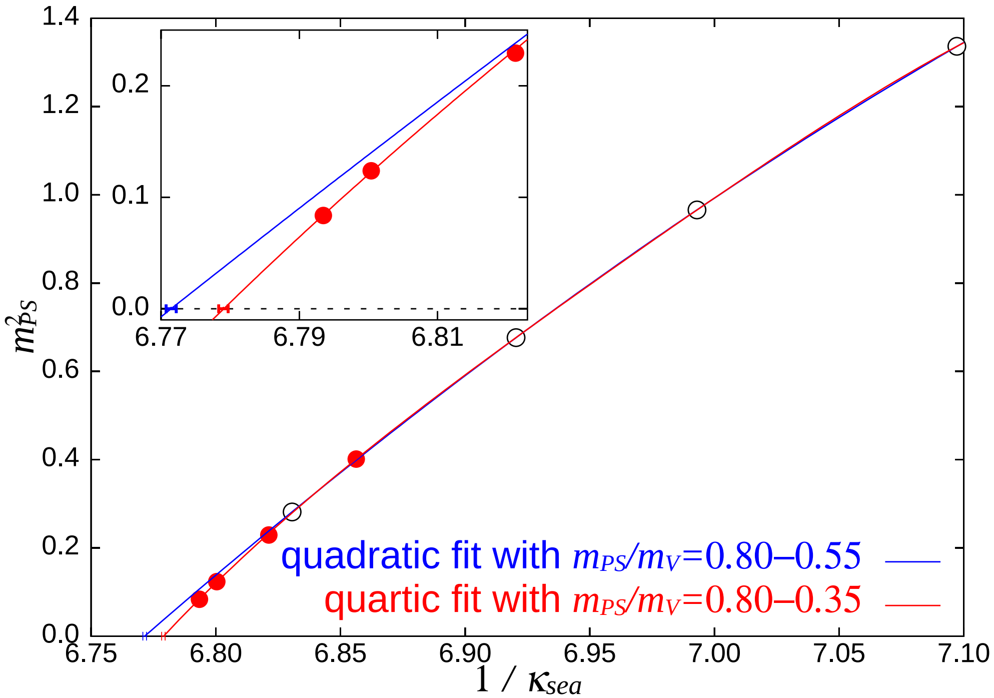
where

$$
m_{quark}^{VWI} = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)
$$

- **Up to** $(m_{quark}^{VWI})^4$ **terms are needed to describe our whole data of** $m_{PS}/m_V = 0.80 - 0.35$ **with a reasonable** $\chi^2/dof = 1.4$

- **Large deviations from the previous fit are also observed in small sea quark mass region (10σ difference in κ_c) $\longrightarrow$ sizable systematic error!**

- **The quadratic fit form can not describe our data of** $m_{PS}/m_V = 0.80 - 0.35 (\chi^2/dof = 10)$
  cf. $m_{PS}/m_V = 0.80 - 0.55 (\chi^2/dof = 0.1)$



Chiral extrapolations based on ChPT

- **In the case of polynomial extrapolations, more higher order terms may be needed if smaller sea quark mass data are available**
  → ChPT may give a guide for extrapolations, which is expected to describe the sea quark mass dependence around the chiral limit.

ChPT : a low energy effective theory of QCD

Weinberg,1979;Gasser and Leutwyler,1984,1985

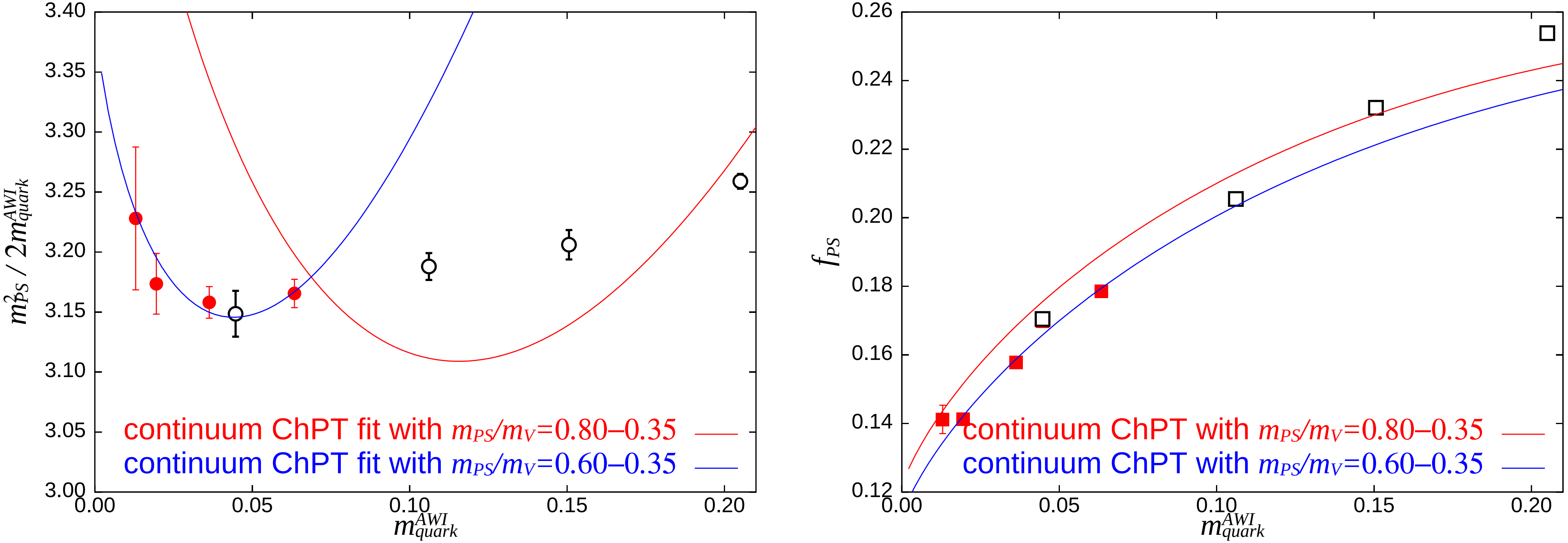
m_{PS}/m_V	**0**	**0.17**	**0.35**	**0.60**	**0.80**
m_{quark}^{AWI}[MeV]	**0**	~ 5	**13**	**63**	**91**

[Simultaneous fit to m_{PS}^2 and f_{PS} with one-loop ChPT]

$$m_{PS}^2 = 2m_{quark}B_0\left(1 + \frac{m_{quark}B_0}{(4\pi f)^2}\log\frac{2m_{quark}B_0}{\Lambda_3{}^2}\right)$$

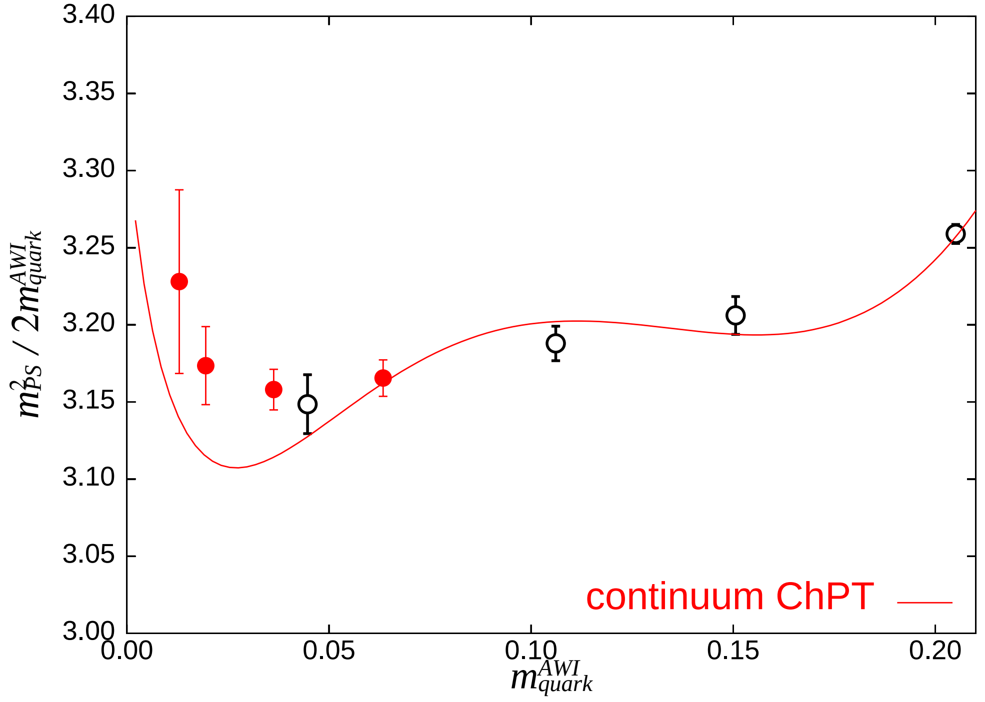
$$f_{PS} = f\left(1 - \frac{2m_{quark}B_0}{(4\pi f)^2}\log\frac{2m_{quark}B_0}{\Lambda_4{}^2}\right)$$

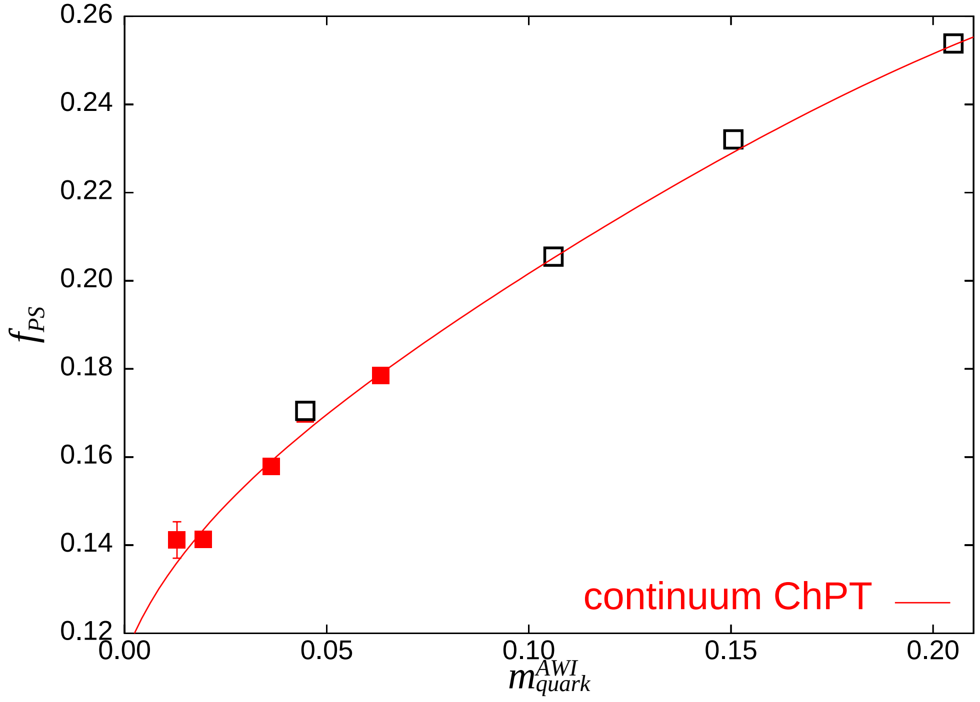
- **ChPT fails to describe our data of** $m_{PS}/m_V = 0.80 - 0.35$
- **To get a reasonable** $\chi^2/dof \sim 4$**, we must drop the heavy data** $\rightarrow$ **ChPT log may appear only below** $m_{PS}/m_V \sim 0.4$



[Simultaneous two-loop ChPT fit to m_{PS}^2 and f_{PS}]

- **ChPT fails to describe our data of $m_{PS}/m_V = 0.80 - 0.35$ (f_{PS} is OK but mass is not)**
- **Dropping the heavy data gives a similar wavy fit curve**





[Reasons of failure of the continuum ChPT formulae]

- **Sea quark masses are still too large for the one-loop and two-loop formulae**
  $\rightarrow$ ChPT log may appear only below $m_{PS}/m_V \sim 0.4$

- **Chiral symmetry is explicitly broken for the Wilson-type fermions**
  $\rightarrow$ Modify ChPT for Wilson-type fermions(WChPT)
  Sharpe and Singleton,1998; Rupak and Shoresh,2002; O.Baer *et.al.*,2003; S.Aoki,2003

$$
\begin{aligned}
\mathcal{L}_{WChPT} &= \frac{f^2}{4}\Big(1 + c_0(S^0 - 1)\Big)\,\mathbf{Tr}\left(\partial_\mu U \partial^\mu U\right) + c_1 S^0 + c_2 (S^0)^2 \\
S_0 &\equiv \mathbf{Tr}(U + U^\dagger), \qquad U \equiv \exp\left(i\sum_{a=1}^{3}\frac{\pi_a\sigma^a}{f}\right) \in SU(N_f = 2)
\end{aligned}
$$

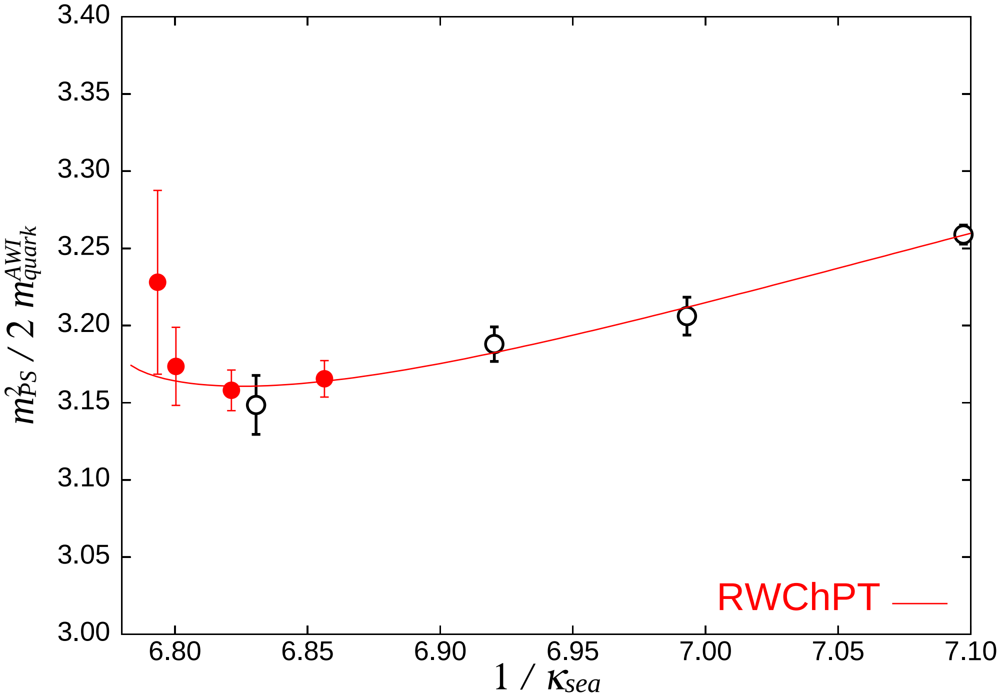
$$
\begin{aligned}
c_0 &= W_0 a, \\
c_1 &= W_1 a + B_1 m_{quark}, \\
c_2 &= W_2 a^2 + V_2 (m_{quark} a) + O(m_{quark}^2).
\end{aligned}
$$

where $W = V = 0$ recovers the continuum ChPT Lagrangian

[Resummed WChPT formulae at one-loop] S.Aoki,2003

$$m_{PS}^2 = Am_{quark}^{VWI}\left(-\log\left(\frac{m_{quark}^{VWI}}{\Lambda_0}\right)\right)^{\omega_0}\left(1+\omega_1^{PS}m_{quark}^{VWI}\log\left(\frac{Am_{quark}^{VWI}}{\Lambda_3^2}\right)\right)$$

$$m_{quark}^{AWI} = m_{quark}^{VWI}\left(-\log\left(\frac{m_{quark}^{VWI}}{\Lambda_0}\right)\right)^{\omega_0}\left(1+\omega_1^{AWI}m_{quark}^{VWI}\log\left(\frac{Am_{quark}^{VWI}}{\Lambda_{3,AWI}^2}\right)\right)$$



[WChPT fit with our all data of $m_{PS}/m_V = 0.80 - 0.35$]

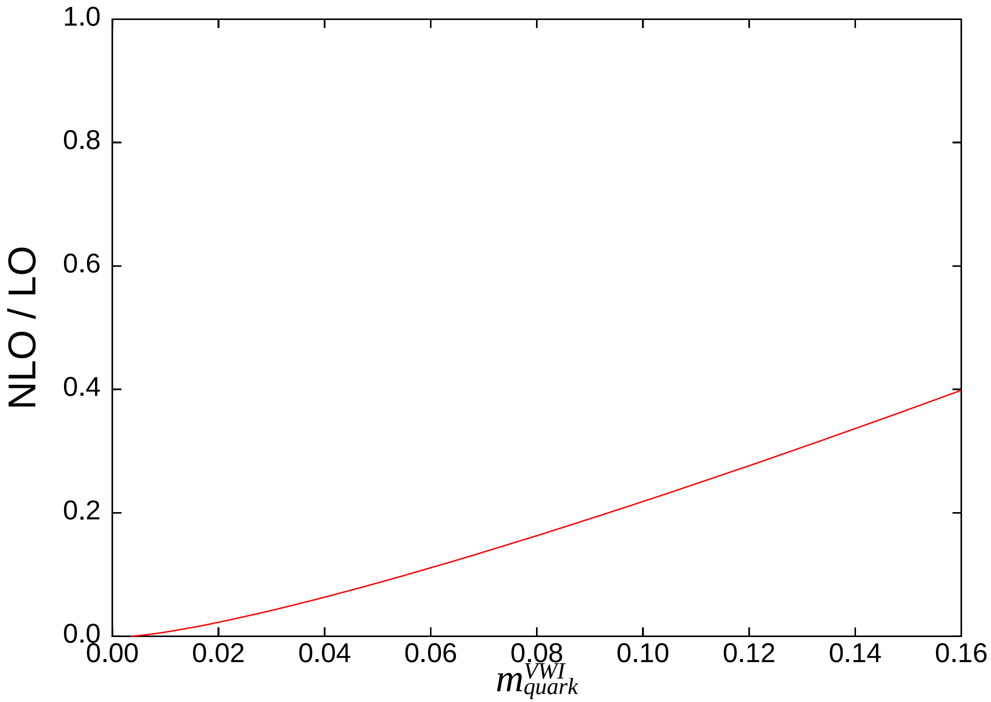
- **The WChPT formulae describe our data well($\chi^2/dof = 1.6$), in contrast to the continuum ChPT case.**
- $\Lambda_3 = 0.15(15)$**[GeV] is comparable with the phenomenological estimate(**$0.2 < \Lambda_3 < 2$**[GeV]** Gasser and Leutwyler,1984 **)**
- $O(a)$ **contribution** $(\omega_1^{PS} - \omega_1^{AWI})$ **suppresses the curvature in** $m_{PS}^2/2m_{quark}^{AWI}$ **to approximately** 10% **of the continuum ChPT**

[Convergence check of the resummed WChPT]

Convergence is reasonable

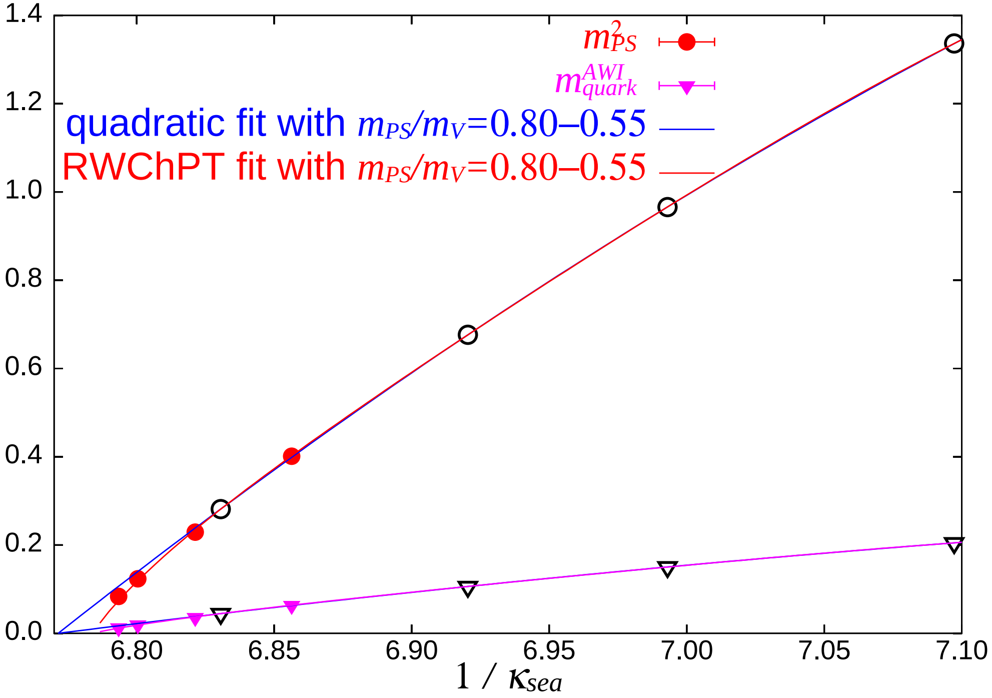
At the heaviest point,

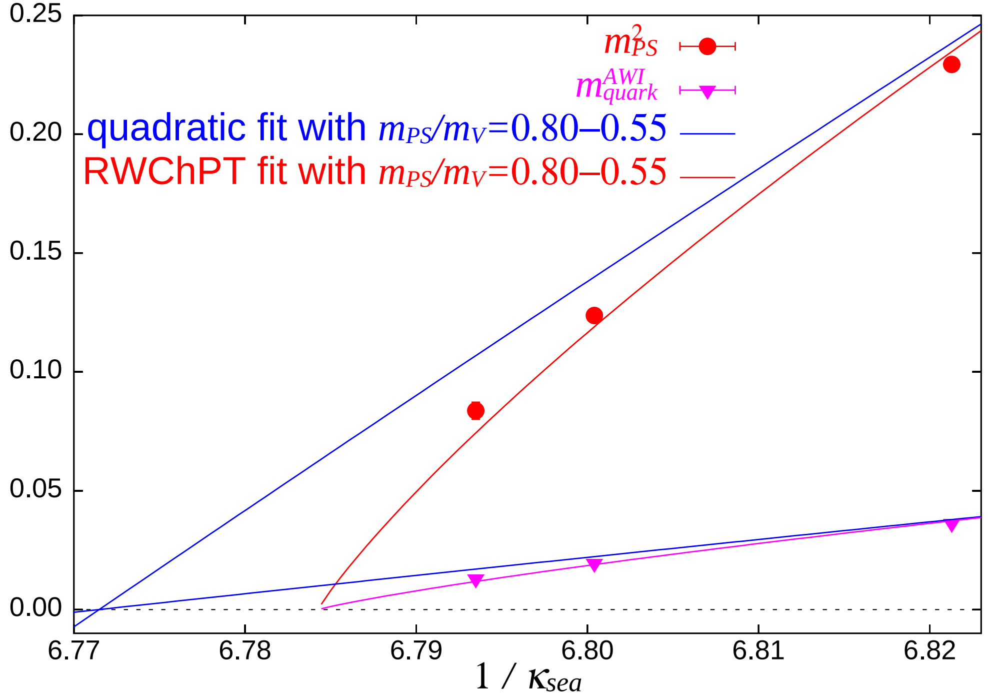
- **Tree : 71%**
- **one-loop : 29%**



[Resummed WChPT extrapolation from intermidiate quark masses]

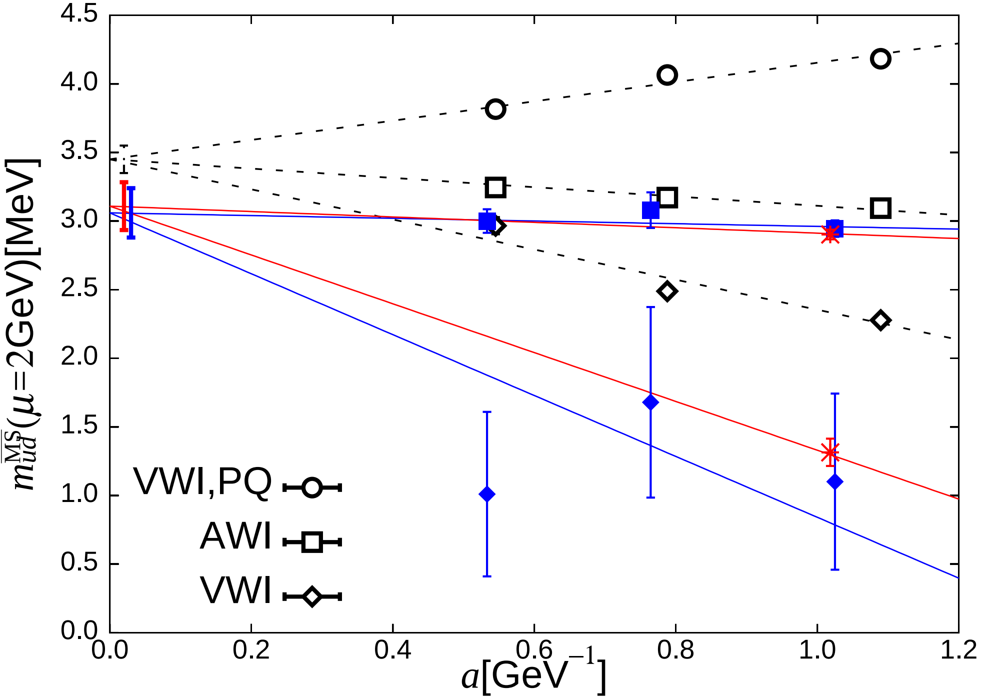
- **The WChPT formulae with the previous heavy data** $m_{PS}/m_V = 0.80 - 0.55$ **predicted the light data of** $m_{PS}/m_V = 0.50 - 0.35$ **better than polynomials** $\rightarrow$ **WChPT may work for intermidiate quark masses**





[Reanalysis of quark masses with the resummed WChPT]

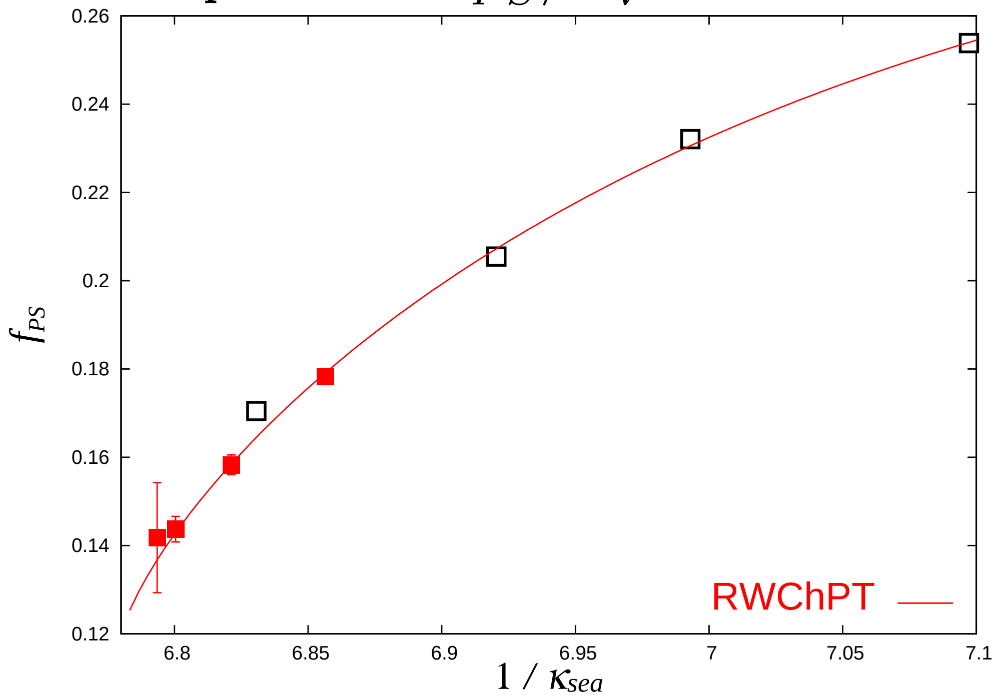
$$m_{ud}(\overline{\mathbf{MS}}, \mu = 2\mathbf{GeV}) = 3.11(17)[\mathbf{MeV}](3.45(10)[\mathbf{MeV}] \ \ \mathbf{cf.\ polynomial})$$

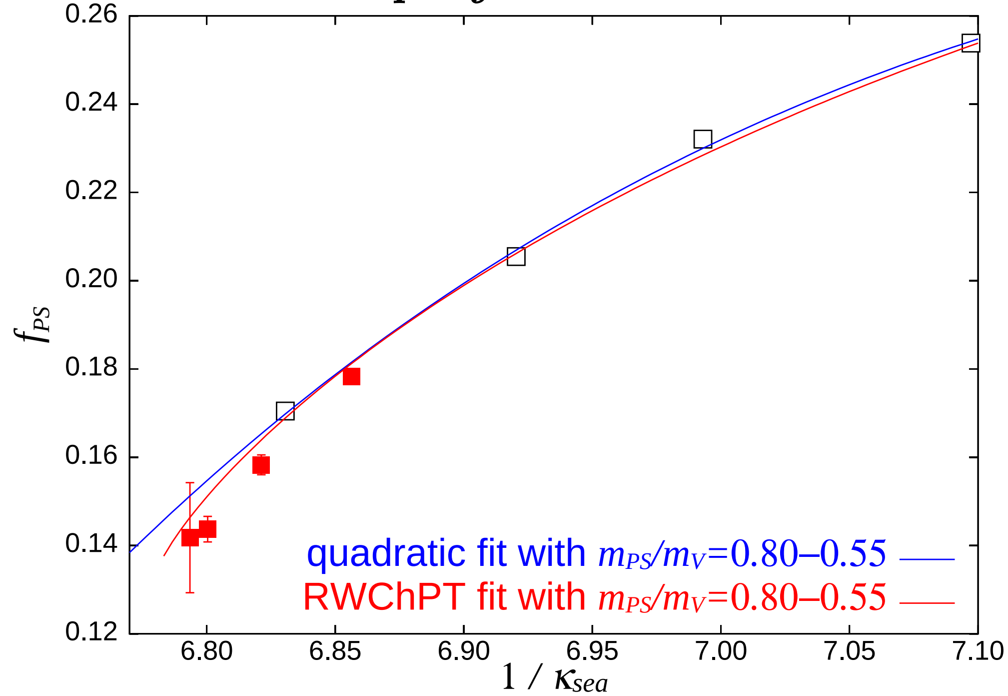


[WChPT formula for the pseudoscalar decay constant]

$$f_{PS} \quad = \quad f\left(1 - \omega_1^{f_{PS}} m_{quark}^{VWI} \log\left(\frac{A m_{quark}^{VWI}}{\Lambda_4^2}\right)\right)$$

- **The WChPT describes our data of** $m_{PS}/m_V = 0.80 - 0.35$ **(**$\chi^2/dof = 3.6$**)**
- $\Lambda_4 = 2.44(13)$ **is comparable with(a little larger than) the phenomenological estimate (**$\Lambda_4 = 1.26 \pm 0.14$**[GeV] Colangelo** *et al.*,**2001)**
  $\rightarrow$ **The scaling check is needed**
- **WChPT formula with the previous data** $m_{PS}/m_V = 0.80 - 0.55$ **predicts** $m_{PS}/m_V = 0.50 - 0.35$ **data better than polynomial**





5. Conclusion

We studied light hadron spectrum using several chiral extrapolations with our data of $m_{PS}/m_V = 0.80 - \mathbf{0.35}$. Our results imply that it is better to employ WChPT formulae instead of conventional polynomials for chiral extrapolations.

[Polynomial extrapolation]

- **Quadratic fits with heavy data at $m_{PS}/m_V = 0.80 - 0.55$ did not describe the behavior at small sea quark masses**

[ChPT extrapolation]

- **Our data do not show a quark mass dependence of ChPT $\rightarrow$ ChPT may be applicable only below $m_{PS}/m_V \sim 0.4$**
  cf. UKQCD,qq+q 2004

[WChPT extrapolation]

- **WChPT ensures extrapolations toward small sea quark masses**
- **WChPT formulae describe our data of** $m_{PS}/m_V = 0.80 - 0.35$ **(in contrast to the continuum ChPT formulae)**
- **Convergence of one-loop WChPT formulae is reasonable**
- **WChPT fit to** $m_{PS}/m_V = 0.80 - 0.55$ **predicted** $m_{PS}/m_V = 0.50 - 0.35$ **data better than polynomials**

[Prediction by WChPT formulae]

$$
\begin{array}{rrcll}
 & m_{ud}(\overline{\mathbf{MS}}, \mu = 2\mathbf{GeV}) & = & 3.11(17)[\mathbf{MeV}] & \textbf{by RWChPT} \\
\textbf{cf.} & m_{ud}(\overline{\mathbf{MS}}, \mu = 2\mathbf{GeV}) & = & 3.45(10)[\mathbf{MeV}] & \textbf{by a polynomial}
\end{array}
$$

[Future works]

- **Check the validity of WChPT formulae at smaller lattice spacings**