The locality problem for two tastes of staggered fermions

Francesco Knechtli
Humboldt Universität zu Berlin

Fermilab, June 22, 2004

Work in collaboration with B. Bunk, M. Della Morte and K. Jansen

Outline

- Introduction
- Analytic study of a candidate two-taste operator
- Numerical results
- Conclusions and Outlook

See hep-lat/0403022

Introduction

Naive Dirac operator on the lattice

$$D_{\text{naive}} \quad = \quad \frac{1}{2}\gamma_\mu(\partial^\star_\mu + \partial_\mu) + m$$

describes 16 Dirac fermion copies or tastes

$\exists\ M$ such that:

- $\det(aM) = \{\det(aD_{\text{naive}})\}^{1/4}$ $\surd$
- M describes 4 tastes as $a \longrightarrow 0$ $\surd$

Answer: staggered fermion operator

$$M \quad = \quad \sum_\mu \frac{1}{2}\eta(x,\mu)(\partial^\star_\mu + \partial_\mu) + m$$

acts on fields $\chi(x),\ \bar{\chi}(x)$ with one independent Grassmann variable per site

$\exists\ D$ such that:

- $\det(aD) = \{\det(aM)\}^{1/4}$?
- D describes 1 Dirac fermion as $a \longrightarrow 0$?

Introduction

Construction of the taste basis for staggered fermions

- *free theory in momentum space* [H.S. Sharatchandra, H.J. Thun and P. Weisz, 1981]:

  4 degenerate tastes, non-local fields in coordinate space
- *in coordinate space* [H. Kluberg-Stern, A. Morel, O. Napoly and B. Petersson, 1983]:

  4 hypercube taste-fields, O(a^2) taste-changing interactions (even in the free theory!)

How to simulate 2 dynamical quarks?

- reduced staggered formalism [H.S. Sharatchandra, H.J. Thun and P. Weisz, 1981]:

  χ lives only on odd sites and $\bar{\chi}$ only on even sites
  - complex determinant, no U(1) axial symmetry [C. van den Doel and J. Smit, 1983]
- the square root trick [in 2D QED: E. Marinari, G. Parisi and C. Rebbi, 1981]:

  take $\sqrt{\det(aM)}$ as the Boltzmann weight
  - in the classical continuum limit taking square root $=$ quenching 2 out of 4 tastes [C.W. Bernard and M.F.L. Golterman, 1994]
  - this is not a first principle formulation

Introduction

The locality problem at a glance

1. QCD partition function on the lattice, only local operators appear

$$\mathcal{Z} \;=\; \int_{U,\bar{\psi},\psi} \exp\left\{-S_g(U) + a^4 \sum_x \bar{\psi}(x) D \psi(x)\right\}$$

2. Integration over fermionic fields $\bar{\psi}, \psi$ ends up with non-local effective action

$$\mathcal{Z} \;=\; \int_U \det(aD) \exp\{-S_g(U)\} = \int_U \exp\{-S_g(U) + \mathrm{tr}\ln(aD)\}$$

For two tastes of staggered fermions the starting point is 2. with an effective action

$$S_{\mathrm{eff}} = -S_g(U) + \frac{1}{2}\mathrm{tr}\ln(aM) \quad \leftrightarrow \quad \sqrt{\det(aM)}$$

- how can renormalizability and universality be discussed?
- how can causality be established with a non-local action?

$\Rightarrow$ need a local formulation in terms of fundamental degrees of freedom

Introduction

The problem is hidden behind present simulations . . .

- algorithms deal with the square root of the determinant, which can be
  - $\longrightarrow$ treated using pseudofermion fields
  - $\longrightarrow$ simulated with
    - R algorithm (extrapolation to the zero molecular dynamics step–size limit) [S. Gottlieb, W. Liu, D. Toussaint, R.L. Renken and R.L. Sugar, 1987],
    - PHMC algorithm (exact algorithm) [JLQCD, S. Aoki et al., 2003]
    - RHMC algorithm (exact algorithm) [M.A. Clark and A.D. Kennedy, 2003]
- quark propagators are computed using M, sources project onto the desired valence taste components, adjustments are needed for flavor singlets like η' [C.W. Bernard and M.F.L. Golterman, 1994; DeGrand 2003]

. . . but is there!

Introduction

Is the continuum theory defined by taking the Boltzmann weight $\sqrt{\det(aM)}$ local? $\Leftrightarrow$

Can a fermion operator D with kernel

$$aD\psi(x) \quad = \quad a^4 \sum_y G(x,y)\psi(y)$$

be found, which satisfies

$$\det(aD) = \sqrt{\det(aM)} \quad \text{and} \quad \|G(x,y)\| \leq C\mathrm{e}^{-\gamma\|x-y\|_{\mathrm{E}}/a},$$

with C and $\gamma > 0$ independent of U?
($\|G(x,y)\|$ is the operator norm, $\|x-y\|_{\mathrm{E}}$ is the Euclidean norm)
Only local gauge paths are allowed in D

This is a requirement for universality to hold [P. Hernández, K. Jansen and M. Lüscher, 1999; F. Niedermayer, 1999]

Analytic study of a candidate two-taste operator

Numerical simulations of 4 tastes of staggered fermions use

$$(M^\dagger M)_e$$

$M^\dagger M$ acting on fields defined on the *even* sites only [O. Martin and S.W. Otto, 1985]

$$\det(aM) \quad = \quad \det(a^2 (M^\dagger M)_e)$$

In this work we investigate the **candidate**

$$D \quad = \quad \sqrt{(M^\dagger M)_e}$$

defined by a series of Chebyshev polynomials which approximates the unique hermitean positive definite square root of $(M^\dagger M)_e$

Analytic study of a candidate two-taste operator

Free theory

$M^\dagger M$ acts like a Laplace operator on the 16 sublattices with lattice spacing $2a$

Kernel $G(x,y)$ of $\sqrt{M^\dagger M}$ on one sublattice

$$G(x,y) = \int_{-\pi/(2a)}^{\pi/(2a)} \frac{d^4p}{\pi^4} \sqrt{(am)^2 + \sum_\mu \sin^2(p_\mu a)}\, \mathrm{e}^{ip(x-y)}\,, \qquad \frac{x_\mu - y_\mu}{a} \text{ even for all } \mu$$

Its continuum version is $(y=0)$

$$\int \frac{d^4p}{(2\pi)^4} \sqrt{p^2+m^2}\, \mathrm{e}^{ipx} \quad = \quad -\frac{1}{4\pi^2}\frac{m^2}{\|x\|_{\mathrm{E}}^3}\left(1 + \frac{3}{m\|x\|_{\mathrm{E}}} + \frac{3}{m^2\|x\|_{\mathrm{E}}^2}\right) \mathrm{e}^{-m\|x\|_{\mathrm{E}}}$$

The operator D is non-local in the free continuum limit at the scale

$$r_{\mathrm{loc}}^{(\mathrm{free})} \quad = \quad 1/m$$

Analytic study of a candidate two-taste operator

- the analytical results prove the non-locality of the continuum free theory defined by the operator $D = \sqrt{(M^\dagger M)_e}$
- it is very unlikely that the interacting theory will turn out to be local, at best one can hope that the bound $r_{\rm loc} = 1/m$ becomes for example

$$r_{\rm loc} \quad = \quad \frac{1}{\text{large hadron mass}} \quad ?$$

(similar studies in the overlap case showed that the analytical bound is only poorly satisfied on a real MC gauge ensemble, see [P. Hernández, K. Jansen and M. Lüscher, 1999])

Numerical results

Locality study [P. Hernández, K. Jansen and M. Lüscher, 1999] : consider source field

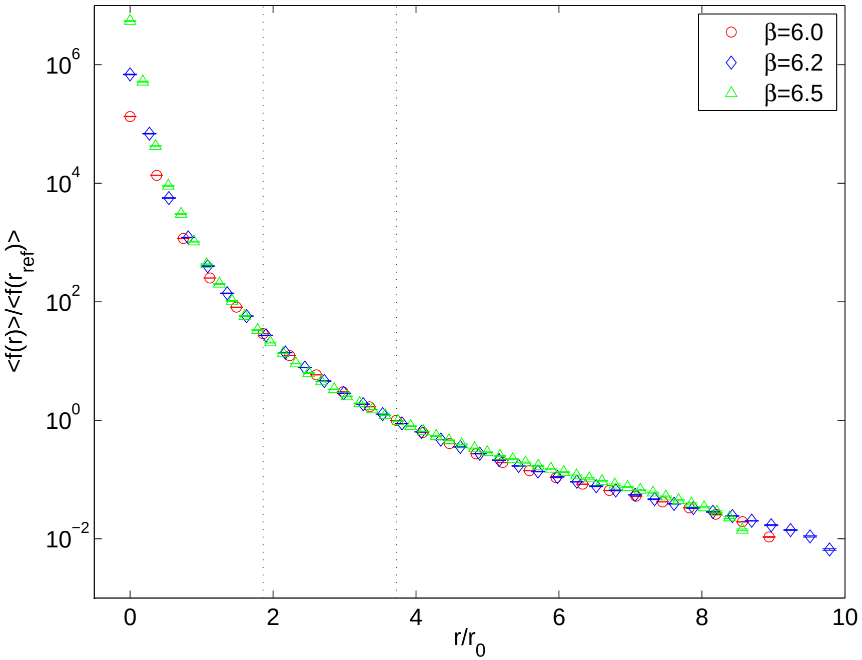
$$\xi_c(x) = \begin{cases} 1 & \text{if } x = y \text{ and } c = 1 \\ 0 & \text{otherwise} \end{cases}$$

compute decay properties of

$$\begin{aligned} \psi(x) &= aD\xi_c(x) \\ f(r) &= \max\left\{ \|\psi(x)\| \,\middle|\, \|x-y\|_1 = r \right\} \\ \|x-y\|_1 &= \sum_\mu \min\{|x_\mu - y_\mu|, L - |x_\mu - y_\mu|\} \quad \text{“taxi-driver distance”} \end{aligned}$$

- $\|\psi(x)\|$ is the norm in SU(3) color space
- $\|x-y\|_1/a$ is the number of links for the shortest path between x and y using the periodicity of the lattice, largest possible value is $2L/a$
- quenched study, three β values $(6.0\,,\ 6.2\,,\ 6.5)$ on a line of approximately constant Goldstone pion π_{G} mass $r_0 m_{\mathrm{G}} = 1.30(3)$
- two sets of physical volumes $Lm_{\mathrm{G}} \approx 4$ and $Lm_{\mathrm{G}} \approx 6$, periodic boundary conditions for gauge and fermion fields

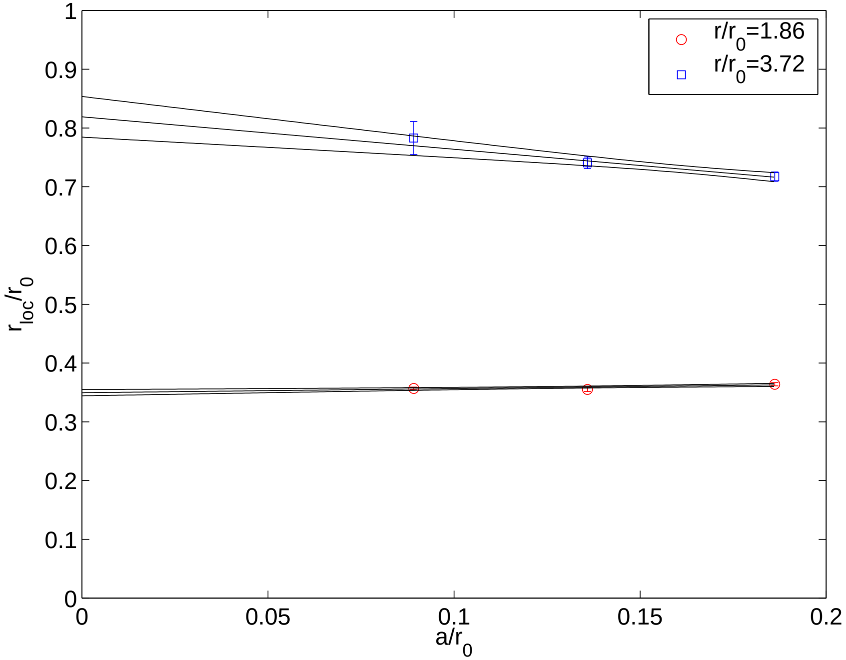
Numerical results



decay of $\langle f(r)\rangle / \langle f(r_{\mathrm{ref}})\rangle$

$$r_{\mathrm{ref}}/r_0 = 3.72$$

$r \ll \Lambda_{\mathrm{QCD}}^{-1}$ perturbative (polynomial in $1/r$)



localization range $\langle f(r)\rangle \propto \mathrm{e}^{-r/r_{\mathrm{loc}}(r)}$

$$r_{\mathrm{loc}}(r/r_0 = 1.86)m_{\mathrm{G}} \quad = \quad 0.455(12)$$

$$r_{\mathrm{loc}}(r/r_0 = 3.72)m_{\mathrm{G}} \quad = \quad 1.06(5)$$

a local operator has $r_{\mathrm{loc}}(r) = O(a)\,,\ \forall r$

Conclusions

- We have studied the square root of the staggered Dirac operator $(M^\dagger M)_e$ as a **candidate** to define a 2 tastes theory of staggered fermions

- Analytic results in the free field limit prove that this theory is non-local at the scale of the inverse quark mass

- Simulations in quenched QCD on a line of constant Goldstone pion mass $r_0 m_{\mathrm{G}} = 1.30(3)$ give the continuum limit result

$$1.06(5) \leq \quad r_{\mathrm{loc}} m_{\mathrm{G}} \quad \leq 2.8(6)$$

The upper bound is for the volume $L m_{\mathrm{G}} \approx 6$

Outlook

- ▷ We do not expect improved actions like Asqtad or HYP to show a better behavior for the square root operator as smoothing the gauge links only makes the situation closer to the free case
- ▷ The question, whether a local operator D with Boltzmann weight $\sqrt{\det((M^\dagger M)_e)}$ exists, is left open
- ▷ Present dynamical simulations do not use this setup: if local D is found then
  - configurations generated by present algorithms are safe
  - unitarity problem: D will dictate the appropriate Green's functions for a 2 taste theory, most likely not the ones built with the 4 taste operator M