Lattice artefacts in SU(3) lattice gauge theory with a mixed fundamental and adjoint plaquette action

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Motivations

Choose a “convenient” gauge action

- to reduce discretization errors
- to improve chiral properties
  $\rightarrow$ suppression of dislocations

in view if the next unquenched simulations

Several approaches already investigated:

- Symanzik (Lüscher-Weisz)
- Renormalization Group (Iwasaki, DBW2, Perfect Actions)

In this work: AD HOC approach

SU(N) lattice plaquette gauge action: [Gonzales-Arroyo, Korthals Altes (1982)]

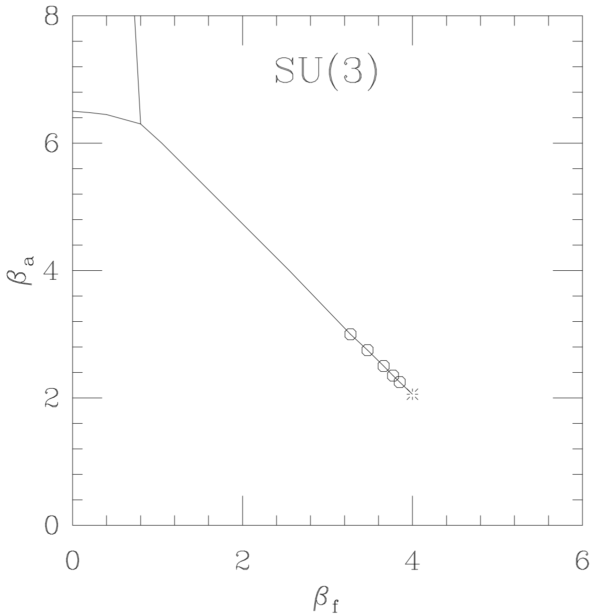
$$S = \sum_{\alpha} \tilde{\beta}_{\alpha} \sum_{P} \left[1 - \frac{1}{d(\alpha)} \mathrm{ReTr}_{\alpha} U_P \right]$$

$\rightarrow$ sum over irreducible representations α of SU(N)

$\mathrm{Tr}_a U = \mathrm{Tr}_f U^\dagger \mathrm{Tr}_f U - 1$

Fundamental + adjoint action:

$$S = \beta_f \sum_P \left[1 - \frac{1}{N}\mathrm{Re}\mathrm{Tr}_f U_P\right] + \beta_a \sum_P \left[1 - \frac{1}{N^2}\mathrm{Tr}_f U_P^\dagger \mathrm{Tr}_f U_P\right]$$



Phase diagram:
line of first order phase transition with endpoint located in
$(\beta_f, \beta_a) = (4.00(7), 2.06(8))$
[Blum et al (1996)]

At the endpoint: $am_{0^{++}} \to 0$
[Heller (1996)]
Responsable for large lattice artefacts in $m_{0^{++}}$ for $5.5 < \beta_f < 6.0$
for the Wilson (fundamental) action?

Proposal:

Fundamental + adjoint action with $\beta_a < 0$
can improve the scaling behavior?

$\beta_a = -2.0, -4.0$ fixed

- Finite temperature phase transition: evaluation of $\beta_{f,c}$ for $1/(aT_c) = 2, 3, 4, 6$
- Static quark potential: computation of the string tension (at $T = 0$) with variance reduction algorithms
- Glueball masses
  0^{++}: is expected to be the most sensitive quantity

Adjoint plaquettes with negative couplings:
[Gupta et al (1991)]
[Morningstar & Peardon (1999)]

Finite temperature phase transition

Critical temperature:

$$\frac{1}{T_c} = N_t a(\{\beta_f, \beta_a\}_c)$$

Determination of $\beta_{f,c}$ for $\beta_a = 0, -2, -4$ fixed

Method: study the probability distribution of $\Omega = \frac{1}{N_s^3} \sum_{\vec{x}} P_{\vec{x}}$

$P_{\vec{x}} = \mathrm{Tr}_f \prod_t U_{\vec{x},t,0}$ Polyakov loop

N_t; β_a	0.0	–2.0	–4.0
2	5.0948(6)	6.4475(6)	7.8477(6)
3	5.5420(3)	7.1603(3)	8.8357(4)
4	5.6926(2)	7.4433(3)	9.2552(6)
6	-	7.8056(5)	9.7748(11)

N_t	$\beta_{f,c}$	ref.
2	5.0933(7)	[Alves et al (1992)]
3	–	–
4	5.6927(4)	[Alves et al (1992)]
4	5.69254(24)	[Iwasaki et al (1992)]
4	5.6925(2)	[Beinlich et al (1999)]

For $\beta_a = 0$: consistent with the literature ($\beta_{f,c}$ computed from the peak in the susceptibility)

$N_t = 6, \beta_a = 0$:

$\beta_{f,c} = 5.89405(51)$

[Iwasaki et al (1992)]

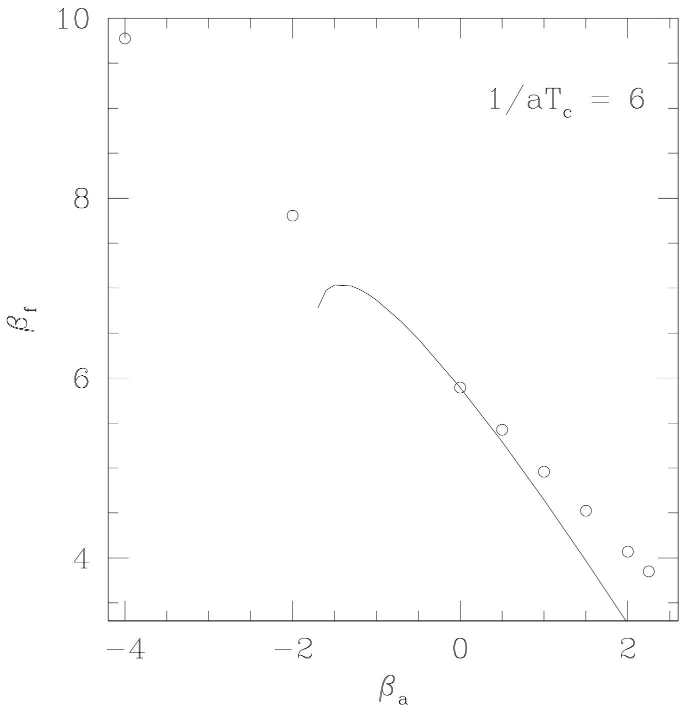
Lines of constant physics

Naive continuum limit:

$\frac{6}{g_0^2} = \beta_W = \beta_f + 2\beta_a \rightarrow$ equivalent Wilson coupling

At one loop : $\beta_W = \beta_f + 2\beta_a - 5\frac{\beta_a}{\beta_f+2\beta_a}$

Constant $\beta_W \rightarrow$ lines of constant physics



Deconfinement transition line for $1/(aT_c) = 6$ ($a \simeq 0.11$ fm):

Numerical results compared with one-loop prediction with $\beta_W = \beta_{f,c}|_{a=0.11\,\mathrm{fm},\ \beta_\mathrm{a}=0}$.

Results for $\beta_a > 0$ included [Blum et al, 1995]

$\rightarrow$ Perturbative predictions fail in describing the lines of constant physics for $\beta_a < 0$

Quadratic interpolation: $\beta_f = c_0 + c_1\beta_a + c_2\beta_a^2$ satisfactory for $a = 0.11\ \mathrm{fm}, 0.17\ \mathrm{fm}, 0.33\ \mathrm{fm}$

The static potential

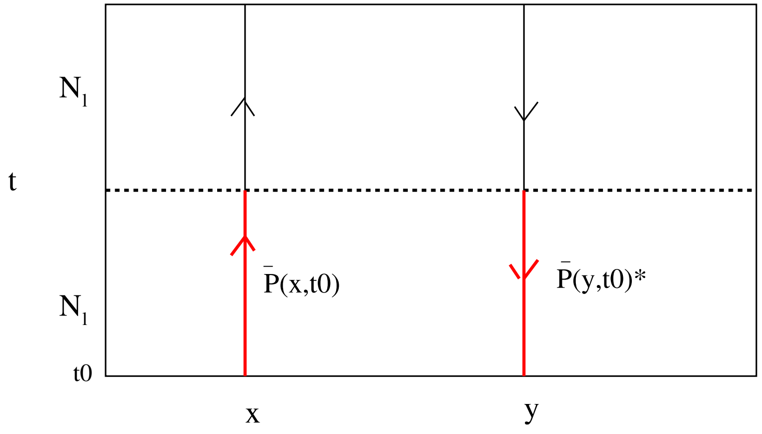
extracted from Polyakov loop correlation function

$$aV(r) = -\frac{1}{N_t}\left[\ln\langle P(x)^* P(y)\rangle + \epsilon\right]$$

Computed at $\beta_{f,c}, \beta_a = 0, -2, -4$, at $T = 0$ $(aN_t >> 1/T_c)$

Variant of Lüscher-Weisz method for error reduction [Lüscher, Weisz (2001)]

Factorization in temporal + spatial directions:
temporal direction divided in layers of thickness N_l +
lattice divided in blocks $b^3 \times N_l$



Polyakov loop correlation function:

$$\langle P(x)^* P(y)\rangle =$$

$$\langle\{\mathbb{T}(\vec{x},\vec{y},0)\mathbb{T}(\vec{x},\vec{y},a)\ldots\mathbb{T}(\vec{x},\vec{y},aN_t - a)\}\rangle_{\alpha,\alpha,\gamma,\gamma}$$

$$\mathbb{T}(\vec{x},\vec{y},t)_{\alpha\beta\delta\gamma} = U(\vec{x},t,0)^*_{\alpha\beta}U(\vec{y},t,0)_{\delta\gamma}$$

$$\langle\{\mathbb{T}(\vec{x},\vec{y},0)\mathbb{T}(\vec{x},\vec{y},a)...\mathbb{T}(\vec{x},\vec{y},aN_t-a)\}\rangle_{\alpha,\alpha,\gamma,\gamma} =$$

$$\langle\{[\mathbb{T}(\vec{x},\vec{y},0)...\mathbb{T}(\vec{x},\vec{y},aN_l)]_{sub,t}...[\mathbb{T}(\vec{x},\vec{y},aN_t-aN_l)...\mathbb{T}(\vec{x},\vec{y},aN_t)_{sub,t}]\}\rangle_{\alpha,\alpha,\gamma,\gamma}$$

$[\ ...\ ]_{sub,t}$: average under fixed boundaries between temporal layers

$$\downarrow$$

$$[\ ...\ ]_{sub,t} = [P(\vec{x},t)^*_{\alpha,\beta}P(\vec{y},t)_{\gamma,\delta}]_{sub,t}$$

$$\downarrow$$

$$[[P(\vec{x},t)^*_{\alpha,\beta}]_{sub,s}[P(\vec{y},t)_{\gamma,\delta}]_{sub,s}]_{sub,t}$$

$[\ ...\ ]_{sub,t}$: average under fixed boundaries of the spatial blocks

$P(\vec{x},t)$:segments of Polyakov loops

Advantage:
factorization in the spatial directions
Disadvantages:
less copies of the correlator, more parameters to be tuned

String tension

Ansatz:

$$V(r) = \sigma r + \mu - \frac{\pi}{12r}\left(1 + \frac{b}{r}\right) + O\left(\frac{1}{r^3}\right)$$

$b = 0.04\ \mathrm{fm}$ [Lüscher, Weisz (2002)]

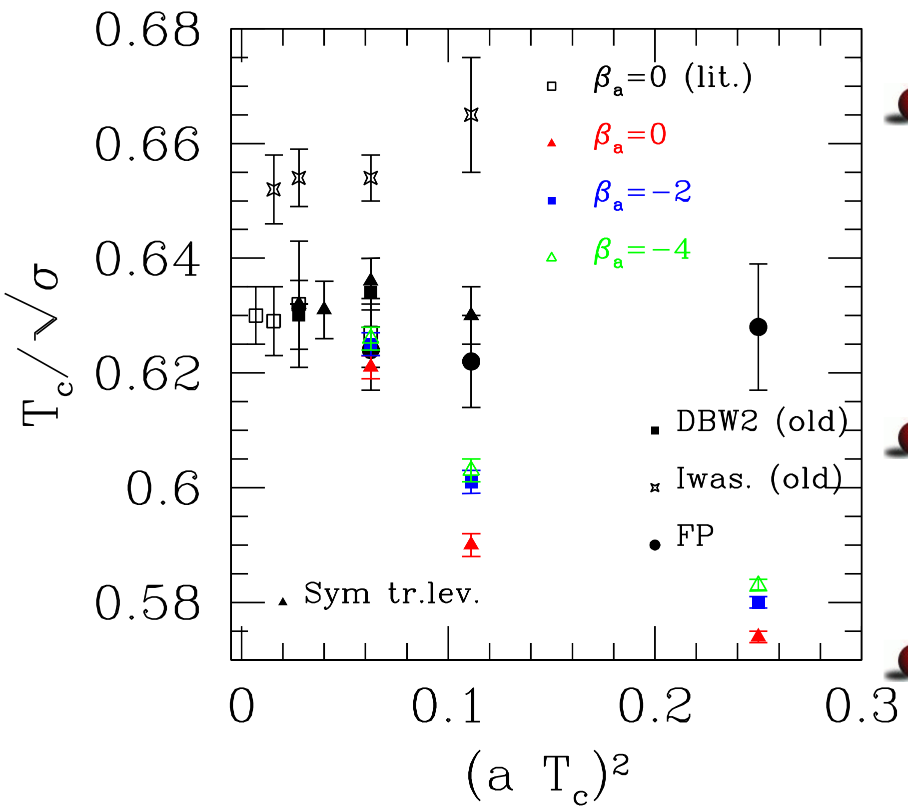
Example: $\beta_f = 5.6926$, $\beta_a = 0$:

r/a	$a^2\sigma_{naive}$	$a^2\sigma_{b=0}$	$a^2\sigma_{b=0,r_I}$	$a^2\sigma_{b=0.04\mathrm{fm},r_I}$	stat. error
3	0.2211	0.1793	0.1707	0.1600	0.0003
4	0.1902	0.1688	0.1664	0.1629	0.0004
5	0.1783	0.1654	0.1645	0.1630	0.0004
6	0.1727	0.1640	0.1637	0.1629	0.0006
7	0.1692	0.1630	0.1628	0.1623	0.0006
8	0.1667	0.1621	0.1620	0.1617	0.0011
9	0.1655	0.1618	0.1618	0.1616	0.0025

r_I: tree level improved definition of the force

Systematic errors under control

Scaling of $T_c/\sqrt{\sigma}$



- For $\beta_a = -2, -4$ the estimate is closer to the continuum limit than for $\beta_a = 0$ (Wilson action) but no significant improvement is observed
- for $1/(aT_c) = 4$ the results for $\beta_a = 0, -2, -4$ do not differ significantly
- Where is the continuum limit?

action	$T_c/\sqrt{\sigma}$
Wilson [Beinlich et al (1999)]	0.630(5)
Symanzik imp.[Beinlich et al (1999)]	0.634(8)
DBW2 [QCD-TARO (2000)]	0.627(12)
Iwasaki [CP-PACS (1999)]	0.651(12)
1-loop tadpole impr. [Bliss et al (1996)]	0.659(8)

Glueball masses

0^{++} glueball : large lattice artefacts for the Wilson action
40% at $a \simeq 0.15\mathrm{fm}$, 20% at $a \simeq 0.10\mathrm{fm}$.

$\rightarrow$ stringent test on the scaling behavior of alternative gauge actions

- Correlation matrices:

$$C^R_{kl}(t) = \langle S^R_k(t) S^R_l(0) \rangle - \langle S^R_k(t) \rangle \langle S^R_l(0) \rangle$$

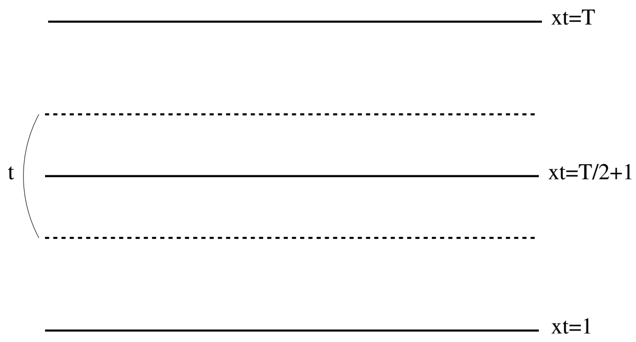
R = representation of the cubic group ($\rightarrow A_1^{++}$)
k, l = different operators with several smearing levels

$$S^R_n(t) = \tfrac{L^{-3/2}}{K} \sum_{\vec{x}} \sum_{i=1}^{d_n} c_n^{iR} W_n^i(\vec{x}, t), \quad n = 1, ..., 22,$$

$W_n^i(\vec{x}, t)$ spatial Wilson loops up to length 8
(7 operators selected, each with 4 smearing levels)

Error reduction algorithm

[Meyer (2003,2004)]

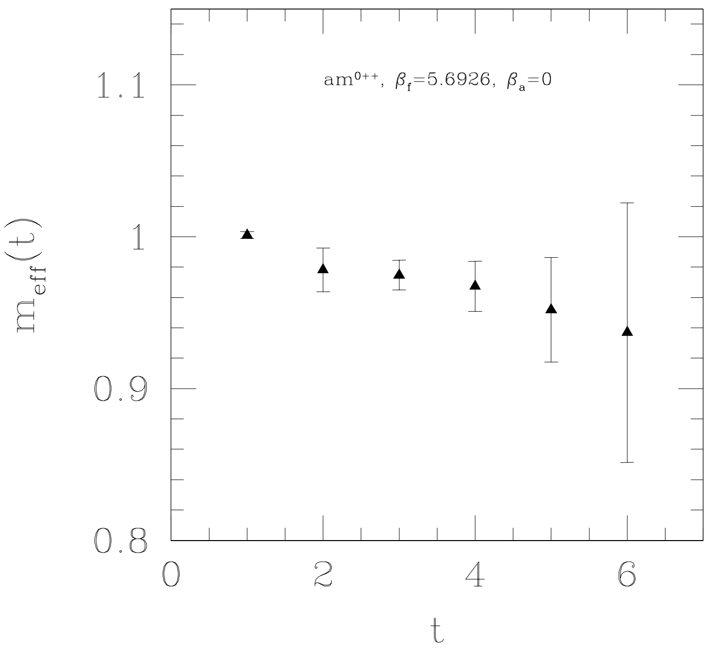


Global updatings+ sub-updatings with spatial links at time slices t_0, t_1 fixed

$t \geq 2$, t even:

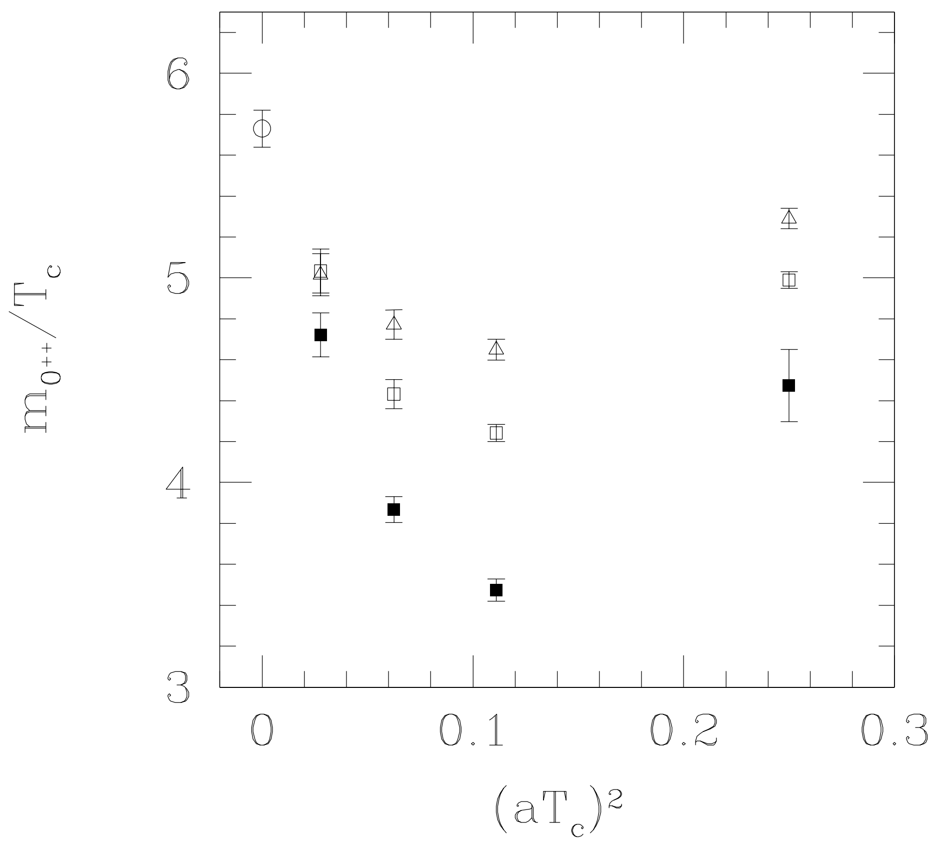
$C_{ij}(t) = \frac{1}{2} \sum_{t'=t_0,t_1} \langle O_i(t'+t/2) \rangle_{sub} \langle O_j(t'-t/2) \rangle_{sub}$

Masses extracted through the variational method
Example: $\beta_f = 5.6926$, $\beta_a = 0$



Results

β_f	β_a	$am_{0^{++}}$	t/a
5.0948	0	2.237(88)	3
6.4475	-2.0	2.495(20)	2
7.8477	-4.0	2.645(25)	2
5.5420	0	1.158(18)	3
7.1603	-2.0	1.414(14)	2
8.8357	-4.0	1.550(17)	2
5.6926	0	0.967(16)	4
7.4433	-2.0	1.108(18)	3
9.2564	-4.0	1.193(18)	3
5.89405	0	0.787(18)	4
7.8056	-2.0	0.839(18)	3
9.7748	-4.0	0.836(17)	3



Continuum limit: $m_{0^{++}} r_0 = 4.30(6)$

from [Teper (1998), Vaccarino & Weingarten (1999), Morningstar & Peardon (1999), Liu (2001)]

$T_c r_0 = 0.7498(50)$ [S.N. (2003)] $\rightarrow m_{0^{++}}/T_c = 5.73(9)$

[Morningstar, Peardon (2003)] : $m_{0^{++}}/T_c = 5.33$

Lattice artefacts: $a \simeq 0.11$ fm: $\beta_a = 0 :\sim 18\%$, $\beta_a = -2, -4 :\sim 12\%$
$a \simeq 0.17$ fm: $\beta_a = 0 :\sim 40\%$, $\beta_a = -2 :\sim 25\%$, $\beta_a = -4 :\sim 20\%$

Conclusions

- Small but not significant improvement of the scaling behavior of $T_c/\sqrt{\sigma}$ for mixed actions with negative adjoint coupling
- Significant reduction of lattice artefacts for the mass of the lightest glueball $m_{0^{++}}/T_c$
- Error reduction algorithms allow good precision and control of systematic errors and are necessary to extract reliable results from correlation functions at large distance
- Is for $\beta_a < 0$ the transfer matrix positive?
  Analitically: it is not trivial to prove the existence of a finite range of β_a where the transfer matrix is strictly positive
  Numerical study: for $\beta_a = -2, -4$ and all β_f considered: negative eigenvalues were found (but with absolute value « largest eingenvalue: not observed in the decay of two-point functions)
- Further studies are needed if one is interested in applying these actions in future dynamical QCD simulations:
  dislocations, spectrum of the Wilson-Dirac matrix

Simulation algorithm

- Cabibbo Marinari update $U \to U'$ (heatbath) with the action:

$$S_0 = \beta'_f \sum_P \left[1 - \tfrac{1}{N}\mathrm{ReTr}_f U_P\right]$$

  accepted with probability
  $A = \min\left[1, \exp(-S(U') + S_0(U') + S(U) - S_0(U))\right]$
  $\beta'_f < \beta_f$ tuned to optimize acceptance rate

- Overrelaxation update keeping S_f constant
  accepted with probability
  $A = \min\left[1, \exp(-S_a(U') + S_a(U))\right]$,

$$S_a(U) = \beta_a \sum_P \left[1 - \tfrac{1}{N^2}\mathrm{Tr}_f U_P^\dagger \mathrm{Tr}_f U_P\right]$$

Complete update: 1 Cabibbo-Marinari + M overrelaxation sweeps

Finite temperature phase transition

Critical temperature:

$$\frac{1}{T_c} = N_t a(\{\beta_f, \beta_a\}_c)$$

Determination of $\beta_{f,c}$ for $\beta_a = 0, -2, -4$ fixed

Method: study the probability distribution of $\Omega = \frac{1}{N_s^3} \sum_{\vec{x}} P_{\vec{x}}$

$P_{\vec{x}} = \mathrm{Tr}_f \prod_t U_{\vec{x},t,0}$ Polyakov loop

- In the thermodynamic limit:
  At the transition point, $p(|\Omega|)$ has a double peak structure
  the weight of each phase is the same,
  the ordered phase is threefold degenerate (Z_3 symmetry)
- On a finite lattice: one assigns
  config. with $|\Omega| < O_{min} \rightarrow$ disordered phase
  config. with $|\Omega| > O_{min} \rightarrow$ ordered phase
  O_{min}: minimum of $p(|\Omega|)$ between the two peaks

Weight of the phases: $P_{dis} = \int_0^{O_{min}} \mathrm{d}|\Omega|\ p(|\Omega|)$; $P_{order} = \int_{O_{min}}^{\infty} \mathrm{d}|\Omega|\ p(|\Omega|)$

Condition for the estimation of $\beta_{f,c}$: $\frac{P_{order}(\beta_{f,c})}{P_{dis}(\beta_{f,c})} = 3$

Spatial factorization