Density matrix RG approach to a two-dim bosonic model

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Introduction

- Yang-Mills gauge theory
- Confinement
- Vacuum state

Hamiltonian diagonalization

- No sign problem
- Vacuum as ground state
- Wave function
- Symmetry based on operator algebra

Compression of Hilbert space

Quantum Field Theory

- Infinite degrees of freedom
- Hamiltonian is too large

Numerical renormalization group

- K. Wilson (Kondo problem)
- S. White (DMRG: Density Matrix RG)

Density Matrix RG (DMRG)

Very successful in condensed matter physics.

- Applied to various chain models
  (1D Heisenberg, Hubbard, t-J, Kondo,...)
- 2D Hubbard with small lattices
- 1D models at finite temperature
- Massive Schwinger model (Byrnes et al, PRD66)

DMRG is basically a method for fermions.

Does it work well for bosons?

Fermion and boson

On a finite lattice

- Fermion: finite dimensional
- Boson: infinite dimensional

Essential difference.

DMRG needs to be tested in a bosonic model.

(1+1)-dim $\lambda\phi^4$ model

Critical coupling and exponent.

Hamiltonian lattice model

(1+1)-dim $\lambda\phi^4$ model

$$L = \int dx \left[\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - \mu_0^2 \phi^2) - \frac{\lambda}{4!}\phi^4 \right]$$

Hamiltonian on a lattice

$$H = \sum_{n=1}^{L} \left(\frac{1}{2a}\pi_n^2 + \frac{\mu_0^2 a}{2}\phi_n^2 + \frac{\lambda a}{4!}\phi_n^4 \right) + \frac{1}{2a}\sum_{n=1}^{L-1}(\phi_n - \phi_{n+1})^2.$$

Open boundary conditions.

Representation

Quantization

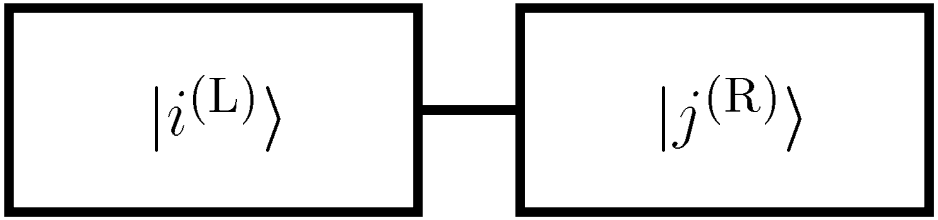
$$[\phi_m, \pi_n] = i\delta_{mn}$$

Fock-like operators

$$\begin{aligned} \phi_n &= \frac{1}{\sqrt{2}}(a_n^\dagger + a_n) \\ \pi_n &= \frac{i}{\sqrt{2}}(a_n^\dagger - a_n) \end{aligned}$$

$[a_m, a_n^\dagger] = \delta_{mn}$ and $a_n|0\rangle = 0$.

Compression of Hilbert space



$$
\begin{aligned}
|\Psi\rangle &= \sum_{i=1}^{N_\mathrm{L}} \sum_{j=1}^{N_\mathrm{R}} \Psi_{ij} |i^{(\mathrm{L})}\rangle |j^{(\mathrm{R})}\rangle \\
&= \sum_{k=1}^{N_\mathrm{L}} D_k |u_k^{(\mathrm{L})}\rangle |v_k^{(\mathrm{R})}\rangle.
\end{aligned}
$$

D_k are singular values of Ψ_{ij}

$$\Psi_{ij} = \sum_{k=1}^{N_\mathrm{L}} U_{ik} D_k V_{kj}$$

Density matrices

In actual numerical works

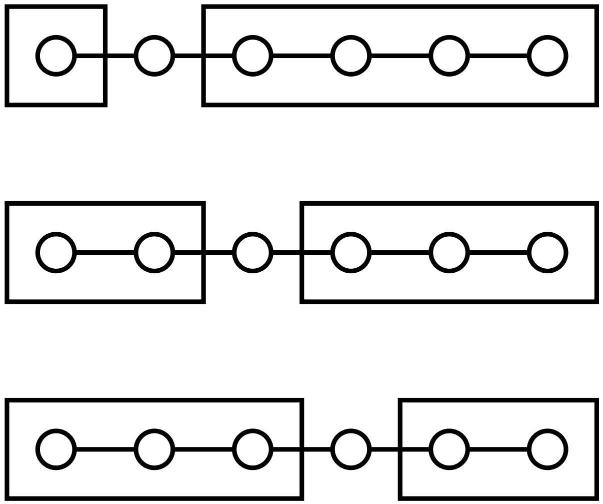
$$\rho^{(\mathrm{L})}_{ii'} = \sum_{j=1}^{N_{\mathrm{R}}} \Psi^*_{ij} \Psi_{i'j} = \sum_{k=1}^{N_{\mathrm{L}}} U^*_{ik} |D_k|^2 U_{i'k}$$

$$\rho^{(\mathrm{R})}_{jj'} = \sum_{i=1}^{N_{\mathrm{L}}} \Psi^*_{ij} \Psi_{ij'} = \sum_{k=1}^{N_{\mathrm{L}}} V^*_{kj} |D_k|^2 V_{kj'}$$

Diagonalization of the density matrices

$\to D$, U, and V

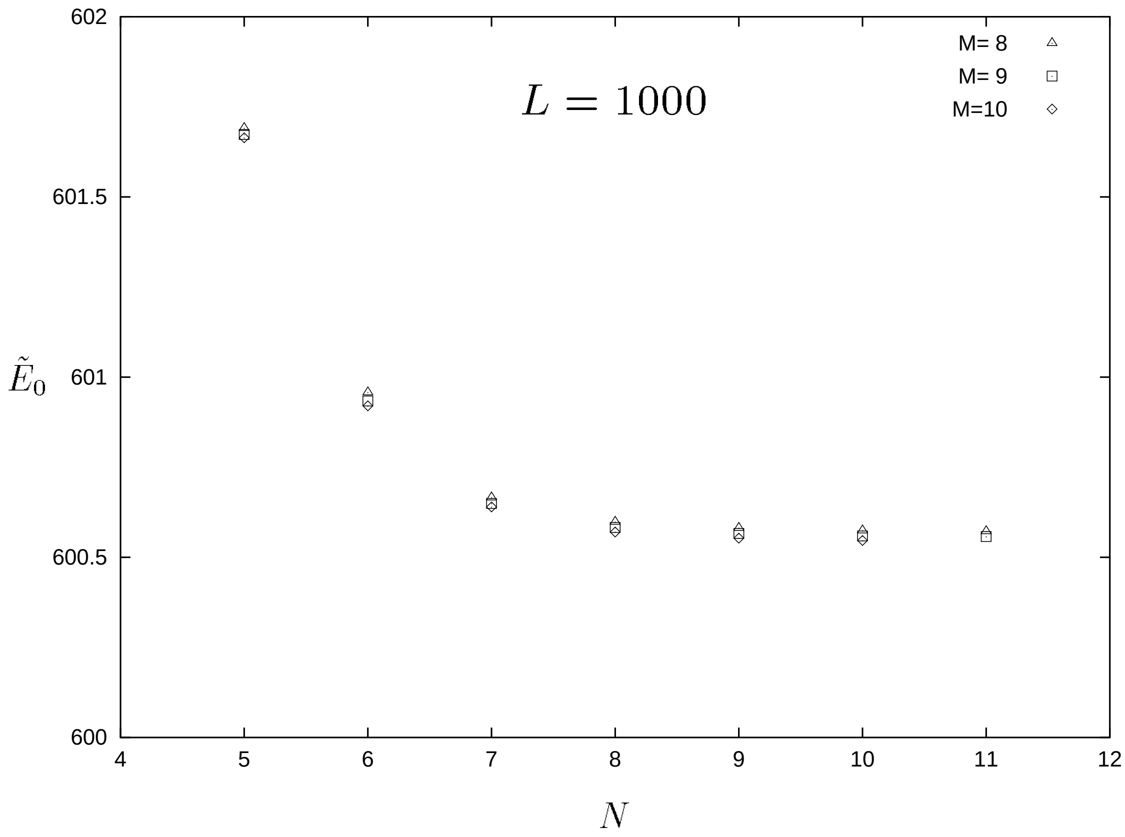
DMRG sweep



$$|\Psi\rangle = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{M} \Psi_{ijk} |u_i^{(\mathrm{L})}\rangle |j^{(n)}\rangle |v_k^{(\mathrm{R})}\rangle$$

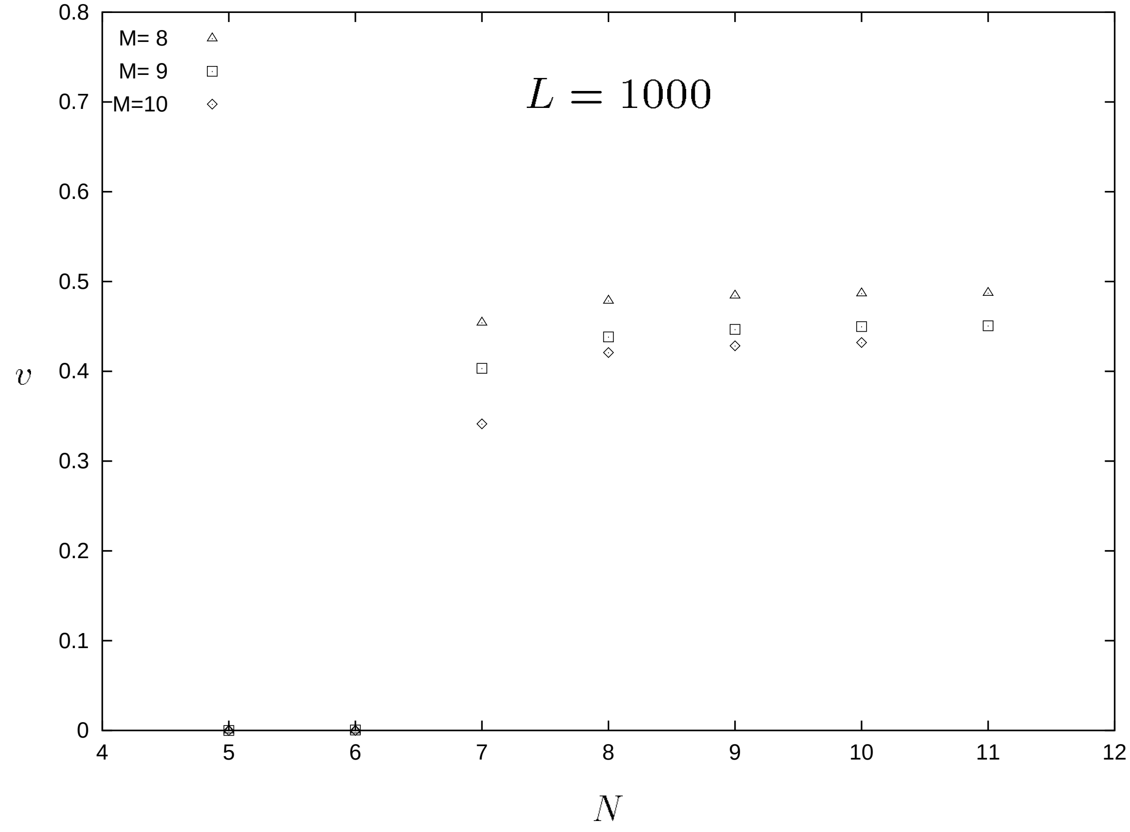
$$|j^{(n)}\rangle \equiv \frac{1}{\sqrt{(j-1)!}} (a_n^\dagger)^{j-1} |0\rangle$$

Vacuum energy



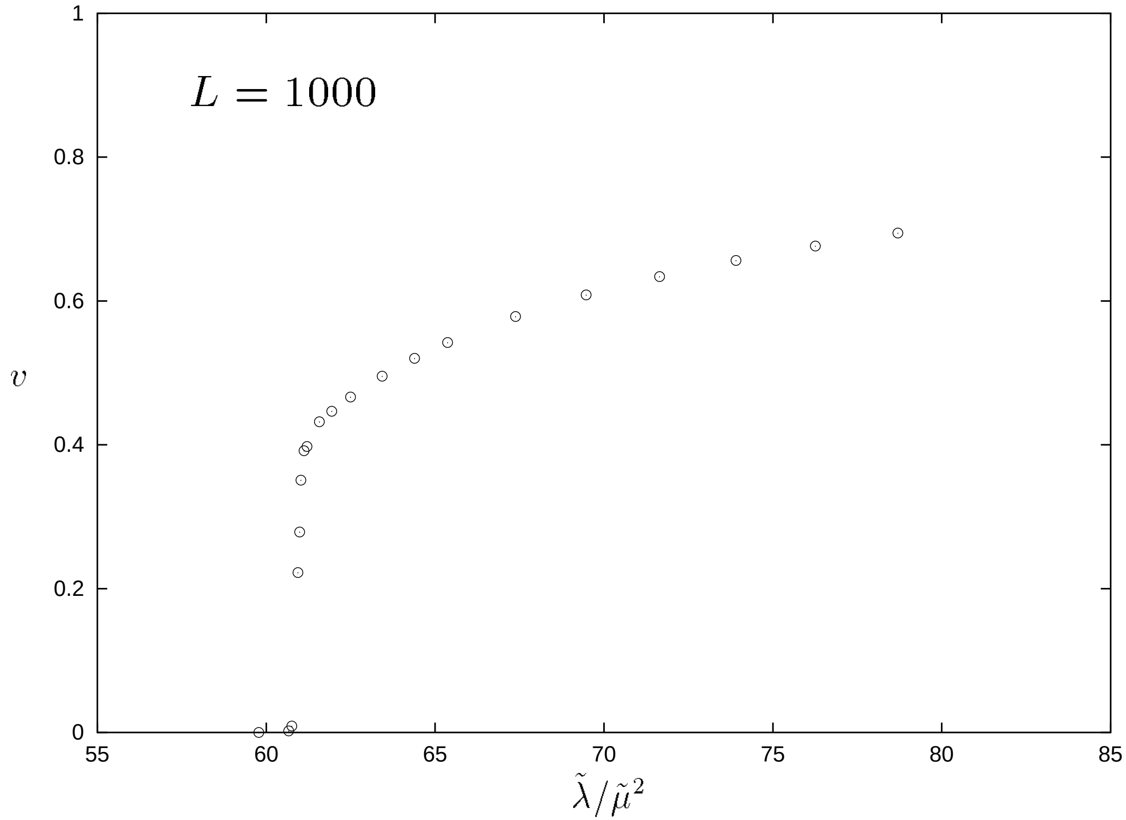
Four digit convergence.

Vacuum expectation value



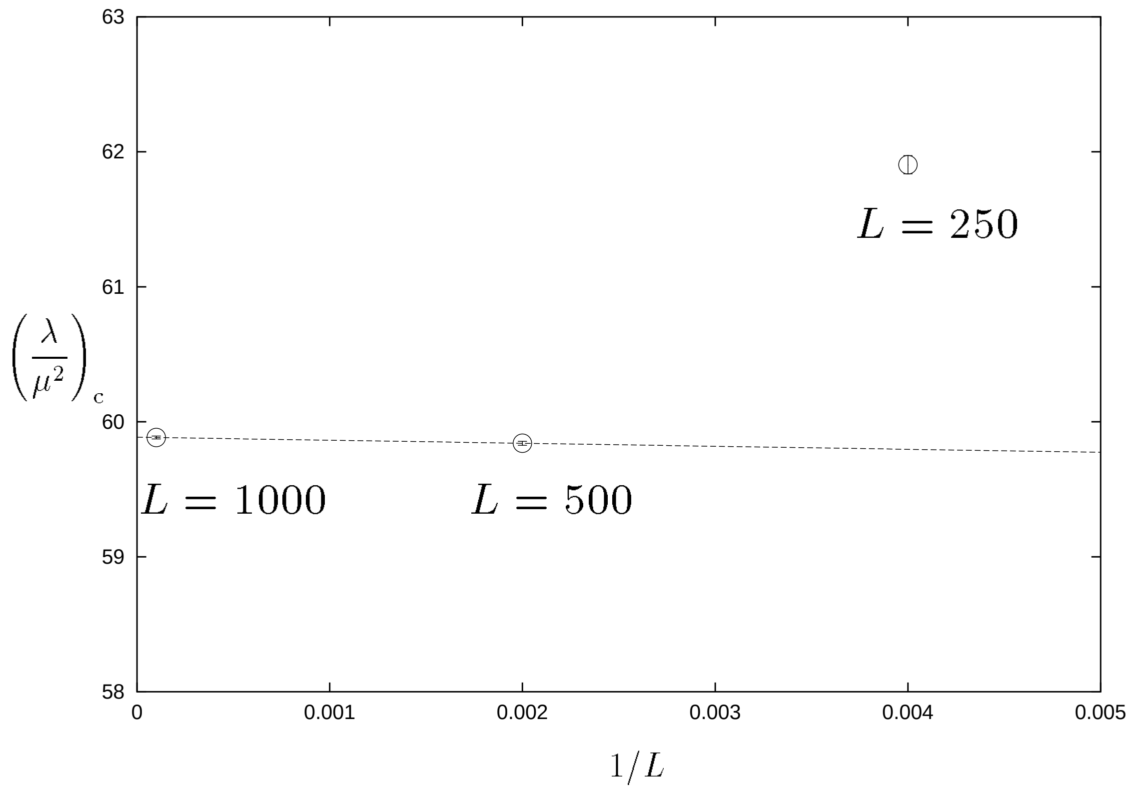
$(M, N) = (10, 10)$ is used.

Critical values



$$v = A\left[\frac{\lambda}{\mu^2} - \left(\frac{\lambda}{\mu^2}\right)_{\rm c}\right]^{\beta}$$

Critical coupling

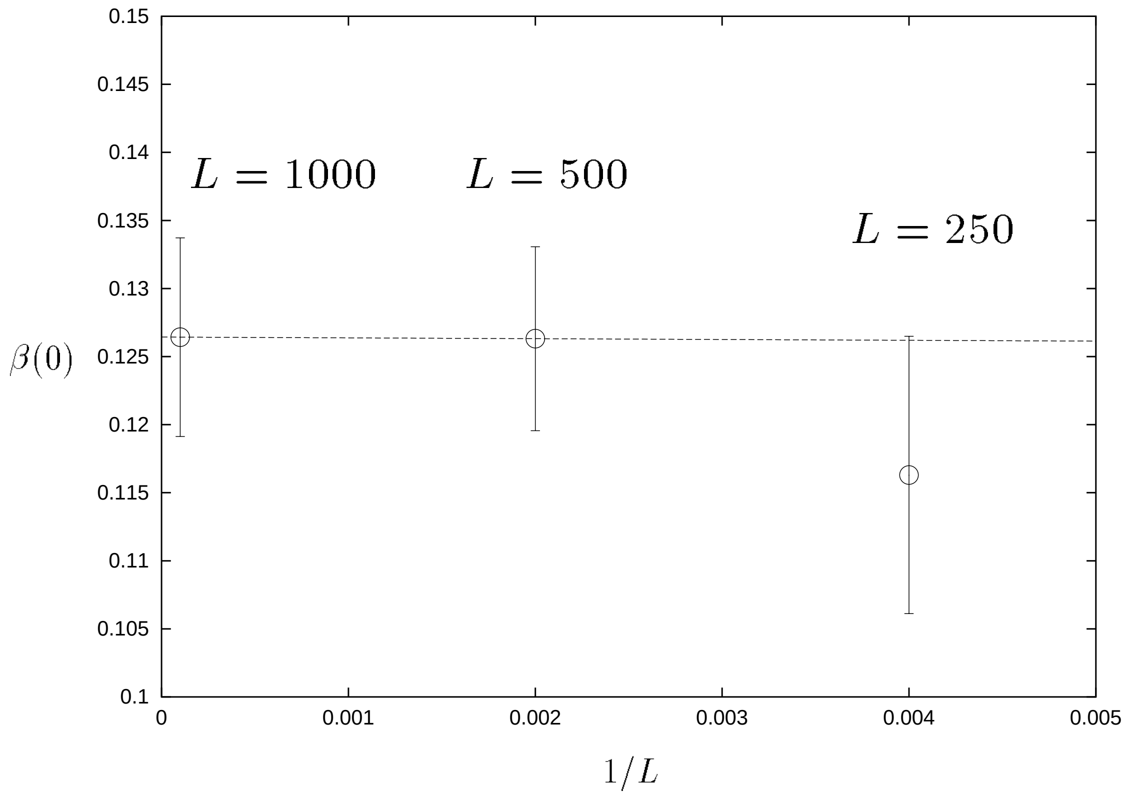


$$\left(\frac{\lambda}{\mu^2}\right)_{\mathrm{c}} = 59.89 \pm 0.01$$

Critical coupling

Method	Result
DMRG ($L = 1000$)	59.89 ± 0.01
Monte Carlo ($L = 512$)	$61.56^{+0.48}_{-0.24}$
Gaussian effective potential	61.266
Gaussian effective potential	61.632
Connected Green function	58.704
Coupled cluster expansion	$22.8 < (\lambda/\mu^2)_{\mathrm{c}} < 51.6$
Non-Gaussian variational	41.28
Discretized light cone 1	43.896, 33.000
Discretized light cone 2	42.948, 46.26

Critical exponent β



DMRG: $\beta = 0.1264 \pm 0.0073$

Exact: $\beta = 0.125$

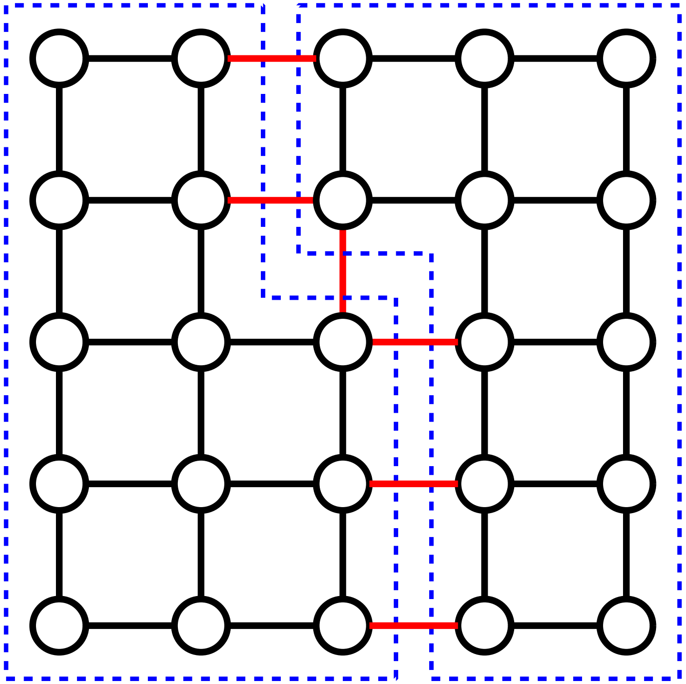
Conclusion

- DMRG works well also in the bosonic model.
- $\left(\frac{\lambda}{\mu^2}\right)_{\rm c} = 59.89 \pm 0.01$
- $\beta = 0.1264 \pm 0.0073$
- $L = 500$ is sufficiently close to the limit $L \to \infty$.

Application of DMRG to SU(2) Yang-Mills Hamiltonian.

Extension to higher dimensions

2D Hubbard



Large amount of memory is necessary.