SU(N) chiral gauge theories on the lattice

Maarten Golterman

San Francisco State University

Yigal Shamir

Tel-Aviv University

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Chiral gauge theories on the lattice:

Nielsen–Ninomiya theorem & the anomaly $\Rightarrow$

A: Give up chiral invariance, but enforce recovery in continuum limit.

B: Or: modify chiral symm. on the lattice, and aim for Wilsonian approach.

Nearly all attempts follow approach A:

$\rightarrow$ Long. gauge fields couple through fermions.
(good fermion det. for smooth gauge fields not enough!)
These couplings are relevant and due to momenta $\sim 1/a$. (Example: Wilson mass term.)

$\rightarrow$ Need perturbative control at critical point.

$\rightarrow$ Renormalizable gauge fixing.

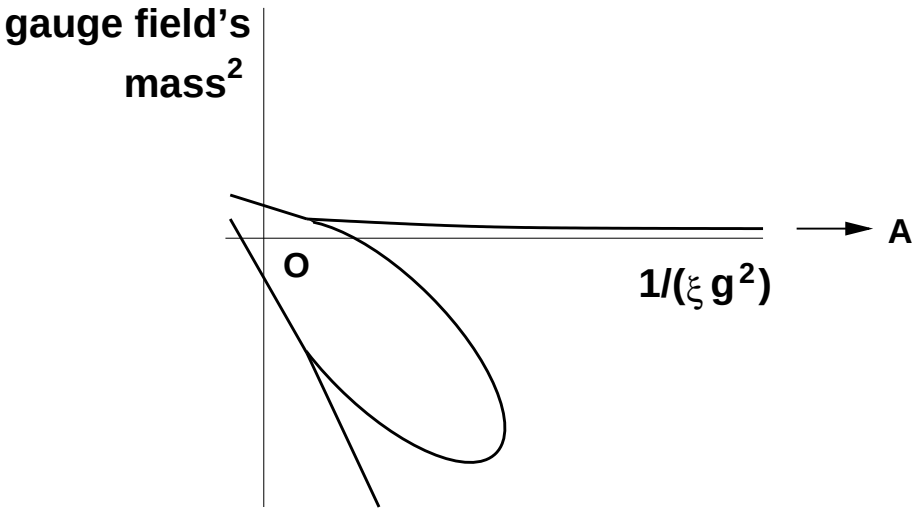
$\rightarrow$ Control counter terms.

("Rome" approach)

- No "automatic" recovery of gauge invariance!

(No miracles ...)

Non-perturbative gauge fixing: (MG & YS)



O: YM critical point: no control over long. modes

A: new critical point: pert. theory and no doublers.

Choose $S_{naive} \sim (\partial \cdot V)^2$ and

$$S_{gaugefix}(U) = \frac{1}{\xi g^2}\left(S_{naive}(U) + S_{irr}(U)\right)$$

such that $U = exp(iV) = 1$ is unique minimum $\Rightarrow$

- lattice pert. theory near critical point;
- no doublers if not present at tree level;
- power counting controls counter terms.

Need symm. principle to determine counter terms:

require target theory to be BRST invariant.

For non-abelian theories, this means ghosts

— how does BRST work non-perturbatively?

Start with $Z = \int dU \; Z_{gf}(U) \; e^{-S_{inv}(U)}$ $\qquad (SU(2))$

insert $Z_{gf} = \int d\phi \; db \; dc \; d\overline{c} \; e^{-S_{gf}(U^{\phi}, c, \overline{c}, b)}$ $\qquad$ (FP trick)

$$S_{gf} = 2t \left[-i \mathrm{tr}(bF(U)) + \mathrm{tr}(\overline{c}\mathcal{M}(U)c) \right] + \xi g^2 \, \mathrm{tr}(b^2)$$

invariant under BRST for all t, ξ.

Good news:

$dZ_{gf}/dt = 0$ because $t\,[\ldots] = \delta_{BRST}\,(2\mathrm{tr}(\overline{c}F(U)))$

$\Rightarrow$ $Z_{gf}(t)$ independent of gauge field!

Bad news: (Neuberger)

$$Z_{gf}(1) = Z_{gf}(0) = \int d\phi \; db \; \underbrace{dc \; d\overline{c}}_{\text{6 Grassmanns}} \; e^{-\xi g^2 \, \mathrm{tr}(b^2)} = 0$$

(Mechanism: $U(1)$ circles in gauge group (Testa))

Solution: (Schaden, MG & YS)

1) gauge fix only $SU(2)/U(1)$ $(SU(N)/U(1)^{N-1})$
   (compare maximal abelian gauge)
2) fix remaining $U(1)$'s without ghosts

1) New BRST (equivariant BRST):

- $V = \frac{1}{2} A\, \tau_3 + W^+ \tau_+ + W^- \tau_-$,
  $F(V) = \partial \cdot W + i[A, W]$
- ghosts only for coset $SU(2)/U(1)$: $C^\pm$, $\overline{C}^\pm$
- $sC = (-iC^2)_{coset} \equiv -iC^2 + X$,
  $s\overline{C} = -ib$
- $s^2 C = -i[X, C] = \delta_{U(1)} C$
  $\Rightarrow$ need $s^2 \overline{C} = -i[X, \overline{C}]$
  $\Rightarrow$ need $sb = is^2 \overline{C} = [X, \overline{C}] \neq 0$

eBRST is (still) nil-potent on any $U(1)$ invariant operator.

Now $\xi g^2 \,\mathrm{tr}(b^2)$ no longer eBRST invariant:
Replace by

$$s\left(i\xi g^2 \,\mathrm{tr}(\overline{C} b)\right) = \xi g^2 \,\mathrm{tr}(b^2) + 4\xi g^2\, \overline{C}^+ C^+ \overline{C}^- C^-$$

Only good news:
still $dZ_{gf}/dt = 0 \Rightarrow Z_{gf}(t)$ independent of gauge field, but now

$$Z_{gf}(1) = Z_{gf}(0) = \int d\phi\, db\, \underbrace{dC\, d\overline{C}}_{\text{4 Grassmanns}}\, e^{-\xi g^2 \left[\mathrm{tr}(b^2) + 4\overline{C}^+ C^+ \overline{C}^- C^-\right]} \neq 0$$

2) Fix $U(1)$ with lattice version of $(\partial \cdot A)^2$.

- This breaks eBRST (already in cont. pert. theory), but modified Slavnov–Taylor id^{s} still sufficient to prove unitarity.

  Reason:
  $Z_{unfixed} \equiv Z_{eBRST}$, same physical sectors
  $\Rightarrow$ gauge fixing either side should not change this.

- Lattice version of $(\partial \cdot A)^2$ is non-linear $\rightarrow$ exact BRST invariance would lead to coupling to abelian ghosts through irrel. operators.

  No abelian ghosts $\Rightarrow$ no exact BRST on lattice
  $\Rightarrow$ need counter terms.

  No new price to pay: fermions break (e)BRST anyway!

What we have:

- Locality

- Pert. theory near critical point at which the target continuum theory is defined
- No fermion doublers
  (control over longitudinal modes!)
- Lattice accounts for fermion-number violation
  (take fermions gauge anomaly-free) (MG & YS)
- Universality: works for any lattice fermions!

(Basak & De)

What we do not have:

- Exact gauge invariance on the lattice
  (need finite number of counter terms)
- Manifest unitarity — but recover unitarity to all orders in pert. theory

Claim: If a certain chiral gauge theory exists, we provide a valid, non-perturbative lattice regularization.

Compare to Lüscher's program:

- Approach B:
  Modify chiral symmetry on the lattice
  — need Ginsparg–Wilson fermions:
  left-handed fermions are defined with

$$\hat{\gamma}_5(U) = \gamma_5 \left(1 - aD_{GW}(U)\right) \ .$$

- Maintain exact gauge invariance
  $\rightarrow$ gauge-field dependent fermionic measure!
  $\Rightarrow$ integrability condition on configuration space:
  - solved for abelian case (admissible fields);
  - solved perturbatively for non-abelian case: construction of measure amounts to adjustment of infinite number of irrelevant counter terms;
  - no non-perturbative solution known (recover Witten anomaly (Bär & Campos)).

No gauge fixing — no control unless gauge invariance is exact!