The B_{B} parameter in the static approximation, without mixings, using tm-Wilson fermions.

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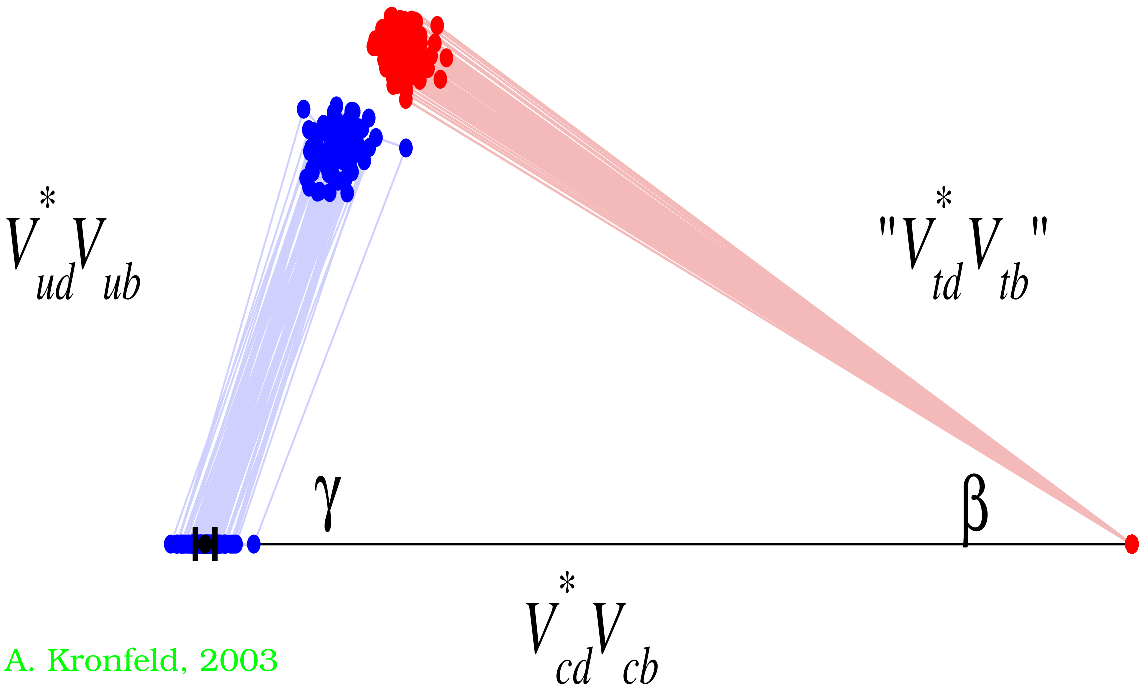
$B_{\rm B}$ parametrizes B_q^0–$\overline{B}_q^0$ oscillations in the EWH and enters the unitarity triangle analysis.

$$\langle \overline{B}_q | (\bar{b}q)_{\rm V-A} (\bar{b}q)_{\rm V-A} | B_q \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 B_{B_q} \ ,$$



The side $V_{td}^* V_{tb}$ can be obtained from the frequency of $B_q^0 - \overline{B}_q^0$ mixings ($q = s, d$).

Heavy quarks on the lattice

Static approximation:

$$S_{\rm stat} = \sum_n \left\{ \bar{h}^{(+)}(n) \left[h^{(+)}(n) - U_0(n-\hat{0})^\dagger h^{(+)}(n-\hat{0}) \right] - \bar{h}^{(-)}(n) \left[U_0(n) h^{(-)}(n+\hat{0}) - h^{(-)}(n) \right] \right\} ,$$

among the symmetries of the action (gauge, cubic and parity invariance, local heavy flavor conservation), the most relevant one in the following is Heavy Quark Spin Symmetry (HQS)

$$h^{(\pm)}(x) \rightarrow \frac{1}{2}\epsilon_{ijk}\gamma_j\gamma_k h^{(\pm)}(x) ,$$
$$\bar{h}^{(\pm)}(x) \rightarrow -\bar{h}^{(\pm)}(x)\frac{1}{2}\epsilon_{ijk}\gamma_j\gamma_k , \quad (i = 1,2,3);$$

The correlation function relevant for $B_{\rm B}$ is (q=light quark)

$$C_{\mathcal{B}\mathcal{O}\mathcal{B}}(x,y) = \langle (\bar{q}\gamma_5 h^{(+)})(x) \mathcal{O}^{\Delta b=2}_{\rm VV+AA}(0) (\bar{q}\gamma_5 h^{(-)})(y) \rangle ,$$

with

$$\mathcal{O}^{\Delta b=2}_{\rm VV+AA} = (\bar{h}^{(+)}\gamma_\mu q)(\bar{h}^{(-)}\gamma_\mu q) + (\bar{h}^{(+)}\gamma_\mu\gamma_5 q)(\bar{h}^{(-)}\gamma_\mu\gamma_5 q) .$$

What about the action for the light quarks ?

Wilson or GW (Overlap) fermions :

Becirevic and Reyes, 2003

Basis of P-conserving $\Delta b = 2$ operators

$$\mathcal{O}_{\Gamma\Gamma} \in \{\mathcal{O}_{VV+AA}, \mathcal{O}_{SS+PP}, \mathcal{O}_{VV-AA}, \mathcal{O}_{SS-PP}\} \; .$$

- Wilson fermions (O(3), HQS)

$$Z = \begin{pmatrix} Z_{11} & 0 & Z_{13} & 2\,Z_{13} \\ \frac{-Z_{11}+Z_{22}}{4} & Z_{22} & Z_{23} & -Z_{13} - 2\,Z_{23} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ \frac{2\,Z_{31}-Z_{32}}{4} & \frac{-Z_{32}}{2} & \frac{Z_{34}}{4} & Z_{33} \end{pmatrix}$$

- GW fermions (O(3), HQS, χS)

$$Z = \begin{pmatrix} Z_{11} & 0 & 0 & 0 \\ \frac{-Z_{11}+Z_{22}}{4} & Z_{22} & 0 & 0 \\ 0 & 0 & Z_{33} & Z_{34} \\ 0 & 0 & \frac{Z_{34}}{4} & Z_{33} \end{pmatrix} .$$

$\Rightarrow$ with Wilson fermions $\mathcal{O}_{VV+AA}$ mixes with $\mathcal{O}_{VV-AA}$ and $\mathcal{O}_{SS-PP}$

How's the situation with tmQCD ?

$$S_{\rm tm}(\bar\psi,\psi,U,r,m_q) = a^4 \sum_{\rm x} \bar\psi(x) \left[\frac{1}{2}\gamma_\mu(\nabla^*_\mu + \nabla_\mu) + \right.$$
$$\left. i\gamma_5\tau_3 \left(r\frac{a}{2}\nabla^*_\mu\nabla_\mu - M_{\rm cr}(r)\right) + m_q\right]\psi(x) \ .$$

$\psi = \begin{pmatrix} q \\ q' \end{pmatrix}$, physical basis, $\omega = \pi/2$,

(R. Frezzotti and G. Rossi, 2003)

It is a legal discretization of 2-flavor QCD (Frezzotti, Grassi, Sint and Weisz, 2001).

Recalling: $M_{\rm cr}(r) = -M_{\rm cr}(-r)$ (Aoki '84, FR '03)

$$S_{\rm tm}(\bar\psi,\psi,U,1,m_q) = a^4 \sum_{\rm x} \bar q(x) \left[\frac{1}{2}\gamma_\mu(\nabla^*_\mu + \nabla_\mu) + \right.$$
$$\left. i\gamma_5 \left(\frac{a}{2}\nabla^*_\mu\nabla_\mu - M_{\rm cr}(1)\right) + m_q\right] q(x) + a^4 \sum_{\rm x} \bar q'(x)$$
$$\left[\frac{1}{2}\gamma_\mu(\nabla^*_\mu + \nabla_\mu) + i\gamma_5 \left(-\frac{a}{2}\nabla^*_\mu\nabla_\mu - M_{\rm cr}(-1)\right) + m_q\right] q'(x)$$

hint 1 (motivating me to consider this approach) : it is as if the flavor q were regularized by $r=1$ and the flavor q' by $r=-1$. In operators symmetrical under $q \leftrightarrow q'$ the leading chirality breaking effects (introduced by the Wilson term) might cancel . . .

hint 2: $S_{\rm tm}(\bar\psi,\psi,U,1,m_q)$ @ $\omega=\pi/2$ for massless quarks is invariant under:

$$\psi \longrightarrow i\gamma_5\tau_1\psi \qquad \text{or} \quad \tau_1 \longrightarrow \tau_2$$
$$\overline{\psi} \longrightarrow i\overline{\psi}\gamma_5\tau_1 \qquad \text{or} \quad \tau_1 \longrightarrow \tau_2$$

chiral rotations of angle $\alpha=\pi$, in the massive case the mass term changes sign.

? We only did a simple modi£cation of the Wilson action (rotating the Wilson term). Is there a violation of the "no free lunch" theorem ? No

massles Wilson	massless tmQCD @ $\omega=\pi/2$
vector transformations τ_1,τ_2,τ_3 exactly conserved	only vector transformation τ_3 exactly conserved
all the axial transformations broken by O(a)	axial transformations τ_1 and τ_2 exactly conserved

$\Rightarrow$ exactly conserved vector/axial transformations 3:3.

The residual chiral invariance in the case of tmQCD @ $\omega=\pi/2$ is an advantage concerning renormalization (on the other hand the breaking of ¤avor symmetry, at £nite a, might have negative consequences).

$B_{\rm B}$ without mixings

Previous examples: $B_{\rm K}$ (Guagnelli *et al.*, 2001 and FR, EUnet meeting Orsay, 2004)

The second can be more easily extended to other quantities, e.g. $B_{\rm B}$ with static quarks.

We propose to use:

$$C_{\mathcal{B}'\mathcal{Q}\mathcal{B}}(x,y) = \langle (\bar{q}'\gamma_5 h^{(+)})(x)\mathcal{Q}^{\Delta b=2}_{\rm VV+AA}(0)(\bar{q}\gamma_5 h^{(-)})(y)\rangle \ ,$$

with

$$\begin{aligned}\mathcal{Q}^{\Delta b=2}_{\rm VV+AA} &= (\bar{h}^{(+)}\gamma_\mu q)(\bar{h}^{(-)}\gamma_\mu q') + (\bar{h}^{(+)}\gamma_\mu\gamma_5 q)(\bar{h}^{(-)}\gamma_\mu\gamma_5 q') \\ &+ (\bar{h}^{(+)}\gamma_\mu q')(\bar{h}^{(-)}\gamma_\mu q) + (\bar{h}^{(+)}\gamma_\mu\gamma_5 q')(\bar{h}^{(-)}\gamma_\mu\gamma_5 q) \ .\end{aligned}$$

In the continuum limit it is equivalent to using (Wick's theorem):

$$C_{\mathcal{B}\mathcal{O}\mathcal{B}}(x,y) = \langle (\bar{q}\gamma_5 h^{(+)})(x)\mathcal{O}^{\Delta b=2}_{\rm VV+AA}(0)(\bar{q}\gamma_5 h^{(-)})(y)\rangle \ ,$$

with

$$\mathcal{O}^{\Delta b=2}_{\rm VV+AA} = (\bar{h}^{(+)}\gamma_\mu q)(\bar{h}^{(-)}\gamma_\mu q) + (\bar{h}^{(+)}\gamma_\mu\gamma_5 q)(\bar{h}^{(-)}\gamma_\mu\gamma_5 q) \ .$$

q and q' are degenerate in the c.l. (where the choice for r is irrelevant)

Since O(3) and HQS are conserved by tmQCD @ $\omega = \pi/2$ + static quarks, the starting point is as for Wilson fermions. We want to rule out mixings of $\mathcal{Q}_{\mathrm{VV+AA}}$ with $\mathcal{Q}_{\mathrm{VV-AA}}$ and $\mathcal{Q}_{\mathrm{SS-PP}}$ + mixings induced by P-breaking in tmQCD ($\mathcal{Q}_{\mathrm{VA\pm AV}}$ and $\mathcal{Q}_{\mathrm{SP\pm PS}}$). We de£ne:

- Ex_5

$$
\begin{aligned}
q &\to -i\gamma_5 q' \,, \qquad & \bar{q} &\to -i\bar{q}'\gamma_5 \\
q' &\to +i\gamma_5 q \,, \qquad & \bar{q}' &\to +i\bar{q}\gamma_5
\end{aligned}
$$

  changes the sign of m_q.

- $\mathcal{P}_{\pi/2}$ ($x_P = (-\mathbf{x}, x_0)$)

$$
\begin{aligned}
U_0(x) &\to U_0(x_P) \,, \qquad & U_{\mathrm{k}}(x) &\to U_{\mathrm{k}}^{\dagger}(x_P - a\hat{k}) \\
\bar{q}(x) &\to i\bar{q}(x_P)\gamma_0\gamma_5 \,, \qquad & q(x) &\to i\gamma_5\gamma_0 q(x_P) \\
\bar{q}'(x) &\to i\bar{q}'(x_P)\gamma_0\gamma_5 \,, \qquad & q'(x) &\to i\gamma_5\gamma_0 q'(x_P) \\
\bar{h}^{(\pm)}(x) &\to \bar{h}^{(\pm)}(x_P)\gamma_0\gamma_5 \,, \qquad & h^{(\pm)}(x) &\to \gamma_5\gamma_0 h^{(\pm)}(x_P)
\end{aligned}
$$

  again it changes the sign of the mass term.

- $\mathcal{P}'_{\pi/2}$, same as $\mathcal{P}_{\pi/2}$ except

$$
\bar{h}^{(\pm)}(x) \to \bar{h}^{(\pm)}(x_P)\gamma_0 \,, \qquad h^{(\pm)}(x) \to \gamma_0 h^{(\pm)}(x_P),
$$

$\mathcal{Q}$-operators have de£nite parities wrt those transformations ($Ex_5^2 = \mathcal{P}_{\pi/2}^2 = \mathcal{P}'^2_{\pi/2} = 1$).

The complete action is invariant under $Ex_5 \times \mathcal{P}_{\pi/2}$ and $Ex_5 \times \mathcal{P}'_{\pi/2}$.

	Ex_5	$\mathcal{P}_{\pi/2}$	$\mathcal{P}'_{\pi/2}$	$Ex_5 \times \mathcal{P}_{\pi/2}$	$Ex_5 \times \mathcal{P}'_{\pi/2}$
$\mathcal{Q}_{\mathrm{VV+AA}}$	even	odd	odd	odd	odd
$\mathcal{Q}_{\mathrm{VV-AA}}$	odd	odd	even	even	odd
$\mathcal{Q}_{\mathrm{SS-PP}}$	odd	odd	even	even	odd
$\mathcal{Q}_{\mathrm{AV+VA}}$	even	even	even	even	even
$\mathcal{Q}_{\mathrm{AV-VA}}$	odd	even	odd	odd	even
$\mathcal{Q}_{\mathrm{SP+PS}}$	even	even	even	even	even
$\mathcal{Q}_{\mathrm{SP-PS}}$	odd	even	odd	odd	even

Conclusions:

- tmQCD is useful in simplifying the mixings of composite operators also when used together with heavy (static) quarks.
- The main drawback of the approach is the breaking of isospin symmetry. Theoretical investigations, e.g, χPT at £nite a for tmQCD (Münster *et al.*) and numerical studies (χ_{L}F Collab.), are ongoing.