# *Rough Gauge Fields, Smearing and Domain Wall Fermions*

Meifeng Lin

`mflin@phys.columbia.edu`

Columbia University

# Outline

- Motivation

  lattice dislocations, chiral symmetry breaking, dynamical simulations, smearing...
- Residual chiral symmetry breaking in DWF

  a brief review
- Gauge field smearing: attempts to reduce $m_{\mathrm{res}}$

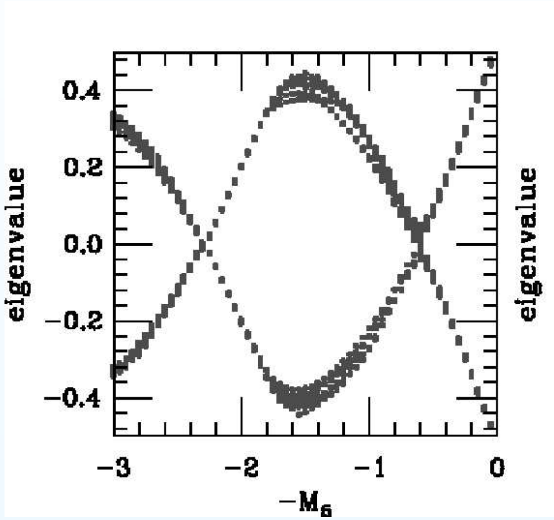
  smearing schemes, lattice scales, mass normalization...
- $L_s$ dependence of $m_{\mathrm{res}}$ on unsmeared and smeared lattices

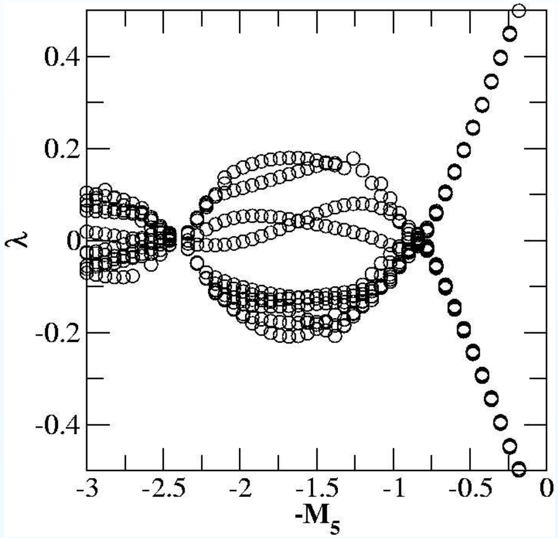
  do we improve chiral properties by smearing?
- Conclusion

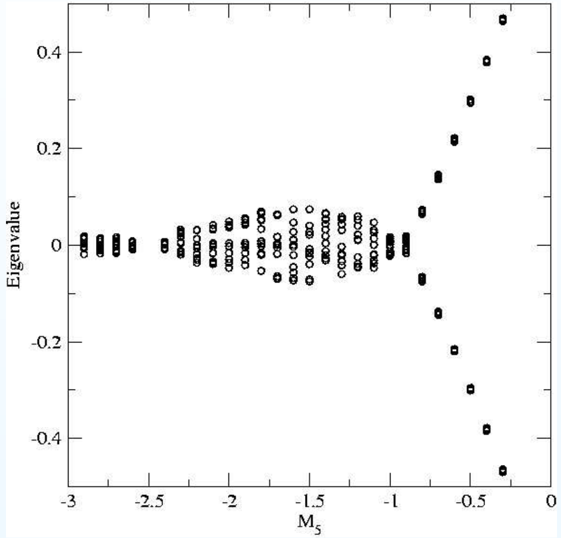
# Motivation

- Topological lattice dislocations on a gauge field give rise to a non-vanishing density of near-zero modes of Hermitian Wilson-Dirac operator.
- These zero modes lead to large chiral symmetry breaking in DWF with a finite extent of the 5th dimension.
- With appropriate choices of gauge actions,e.g.DBW2 action, we are able to suppress the creation of these dislocations and have reasonably small residual masses in quenched simulations.
- However, adding more flavors of fermions into dynamical simulations increases the roughness of the gauge fields even with DBW2 gauge action. As a result, the residual chiral symmetry breaking in DWF, consequently the residual mass $m_{\mathrm{res}}$, is larger at a comparable lattice scale.(See some representative spectral flows on next slide).
- Ways to reduce lattice dislocations? By smearing the gauge links while keeping the lattice scale unchanged, we hope the gauge fields will be smoother and hence the residual chiral symmetry breaking may be reduced.

# Some representative spectral flows




| quenched | $N_f = 2$, DBW2 + DWF | $N_f = 3$, DBW2 + DWF |
|---|---|---|
| $\beta = 1.04$,$16^3 \times 32$, $a^{-1}\sim$ 2 GeV | $\beta = 0.80$,$16^3 \times 32$, $a^{-1}\sim$ 1.7 GeV | $\beta = 0.72$,$16^3 \times 32$, $a^{-1}\sim$ 1.7 GeV (*est.*) |
| Y.Aoki,*et al.*, Phys.Rev.D **69** (2004) 074504 | L.Levkova and R.Mawhinney, Nucl.Phys.B (Proc.Suppl.) **129** (2004) 399 | RBC Collaboration |

# Residual chiral symmetry breaking in DWF

- The quantity we use to measure the residual chiral symmetry breaking is

$$R(t) = \frac{\sum_{\vec{x}} \langle J^a_{5q}(\vec{x},t) J^a_5(\vec{0},0)\rangle}{\sum_{\vec{x}} \langle J^a_5(\vec{x},t) J^a_5(\vec{0},0)\rangle}$$

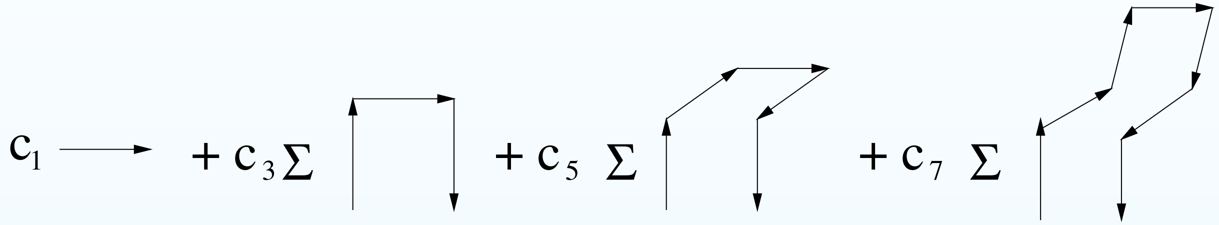
- And the residual mass is defined as

$$m_{\mathrm{res}} = \frac{1}{N_t} \sum_{t \geq t_{\mathrm{min}}} R(t)$$

- What happens when $t < t_{\mathrm{min}}$?
  - When $t < t_{\mathrm{min}}$,$R(t)$ could be either larger or smaller than $m_{\mathrm{res}}$
  - Precise behavior depends on relative contributions to "mid-point" correlators from–
    - in the Aoki phase, non-locality (M.Golterman & Y.Shamir)
    - in and outside the Aoki phase, non-locality and heavy doubler modes

# Smearing-general

- **Construction of new links** $\longrightarrow$un-unitarized :



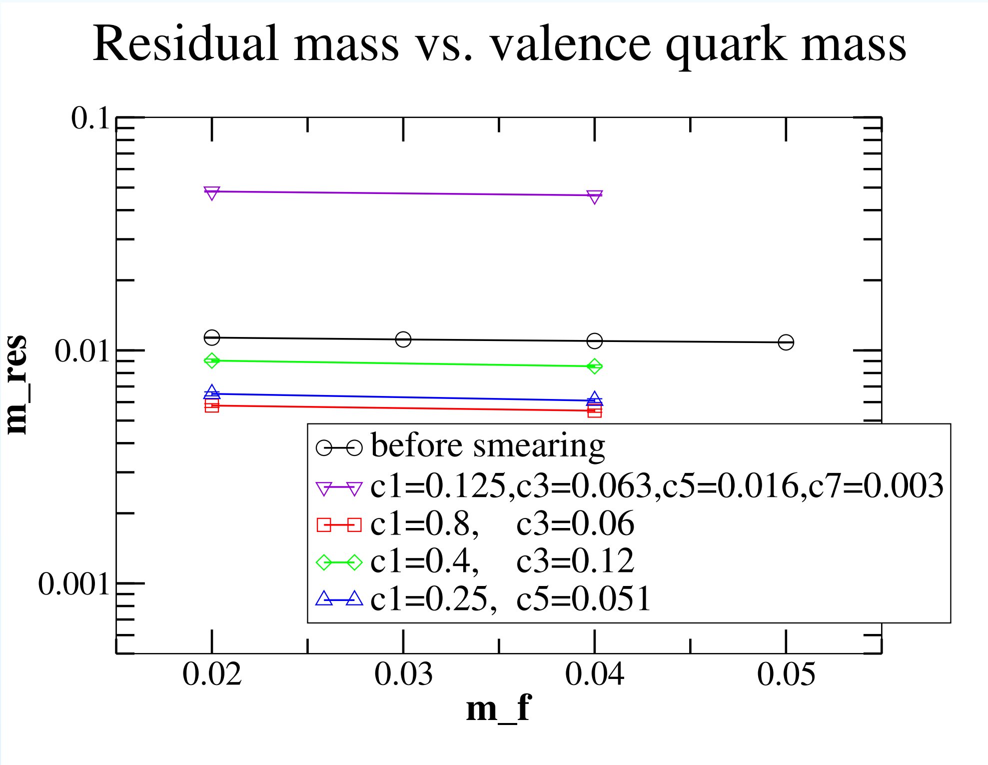

  weak-coupling limit: $c_1 + 6c_3 + 24c_5 + 48c_7 = 1$.
  "Tadpole-improved": $c_1 + 6c_3 u_0^2 + 24c_5 u_0^4 + 48c_7 u_0^6 = 1$, where $u_0$ is quartic root of average plaquette.

- **Coefficients studied**:
  1. $c_1 = 0.125, c_3 = 0.063, c_5 = 0.016, c_7 = 0.003$ –normalized to 1 (Fat7,MILC)
  2. $c_1 = 0.800, c_3 = 0.060, c_5 = 0.000, c_7 = 0.000$ –normalized to 1.16
  3. $c_1 = 0.400, c_3 = 0.012, c_5 = 0.000, c_7 = 0.000$ –normalized to 1.12
  4. $c_1 = 0.250, c_3 = 0.000, c_5 = 0.051, c_7 = 0.000$ –Tadpole-improved

NOTE: Following studies use gauge configurations:
$N_f = 3, \beta = 0.72, 16^3 \times 32$,$m_f = 0.04$,DBW2+DWF, generated by RBC Collobaration.

# Smearing—residual masses

Residual masses for different smearing coefficients. All the measurements were done with $L_s = 8$ and $M_5 = 1.8$ except the one with seven link, which has $M_5 = 2.2$. The residual masses for unsmeared lattices are from 84 configurations; others are all from 30 configurations.



*The residual masses have dropped down*

*by a factor of 2*

*except the one with*

$$(c_1, c_3, c_5, c_7) = (0.125, 0063, 0.016, 0.003)$$

*Further, need to consider* $\Longrightarrow$

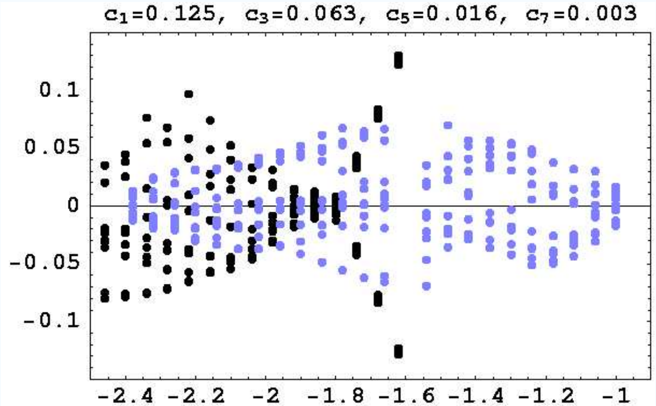
*possible change of lattice spacings, mass renormalization...*

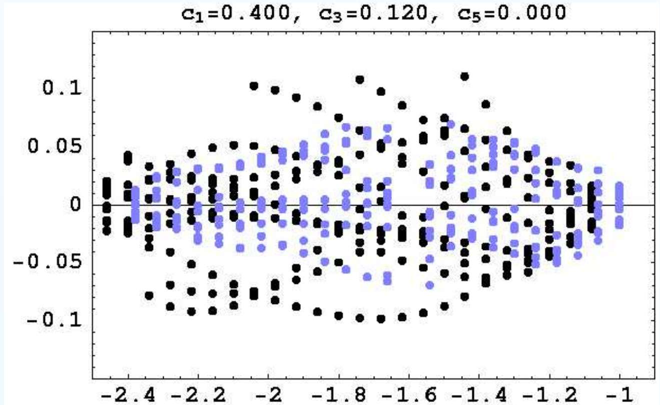
# Spectral flows for different smearing coefficients**

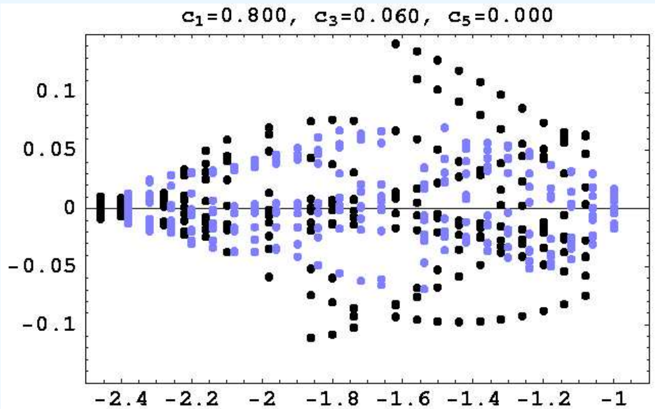
blue dots–unsmeared lattices; black dots–smeared lattices

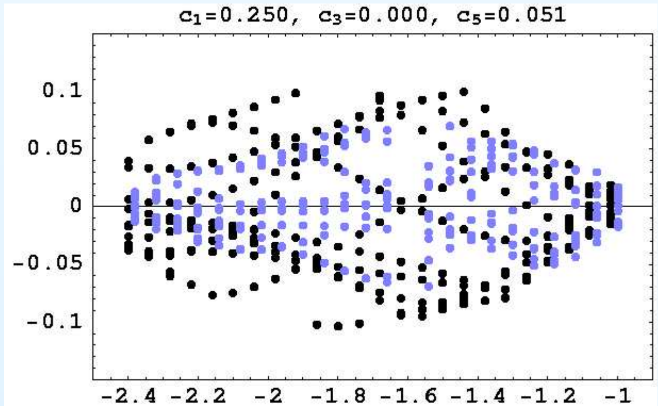
Note that the "pinch point" of the upper left graph has moved dramatically from 0.9 to 1.8.

This is due to mass normalization induced by smearing.



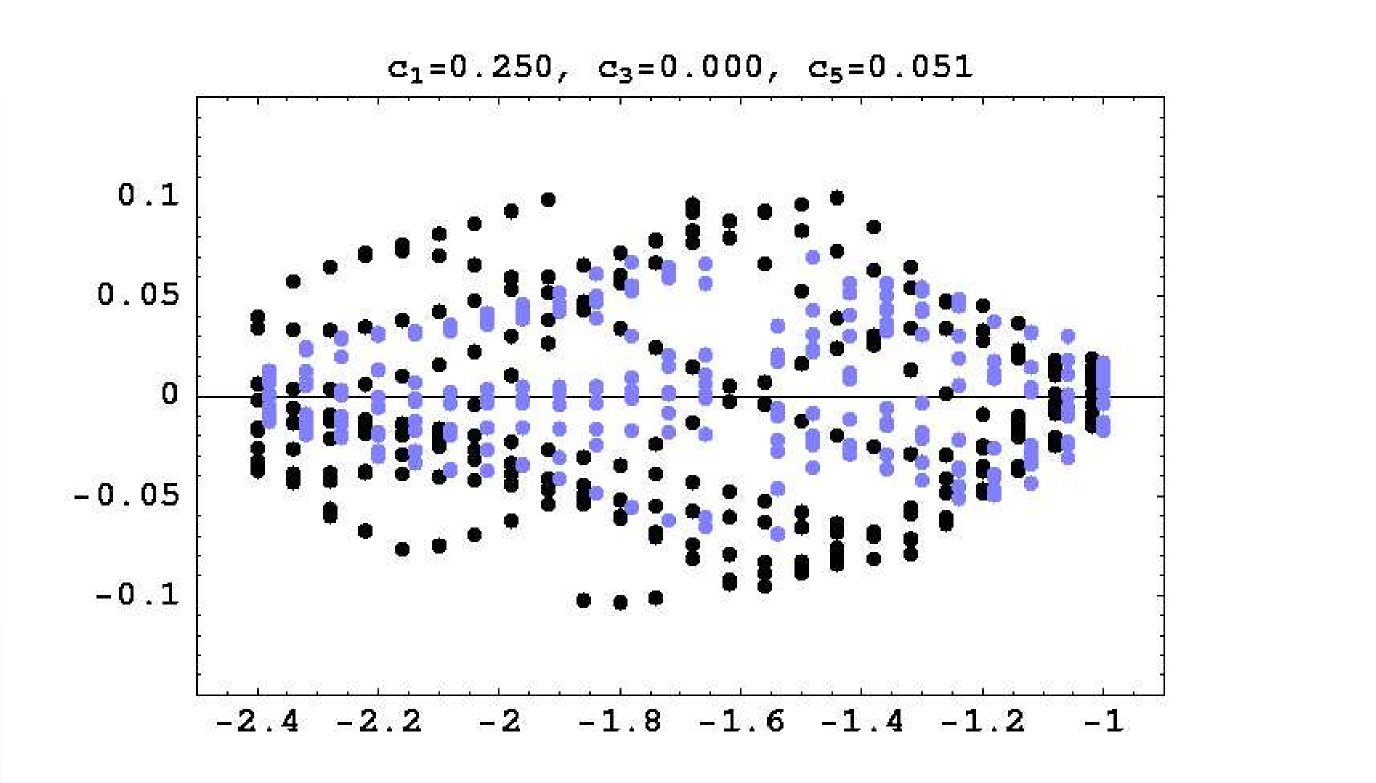




***Thanks to Saul Cohen*

# A Closer Look

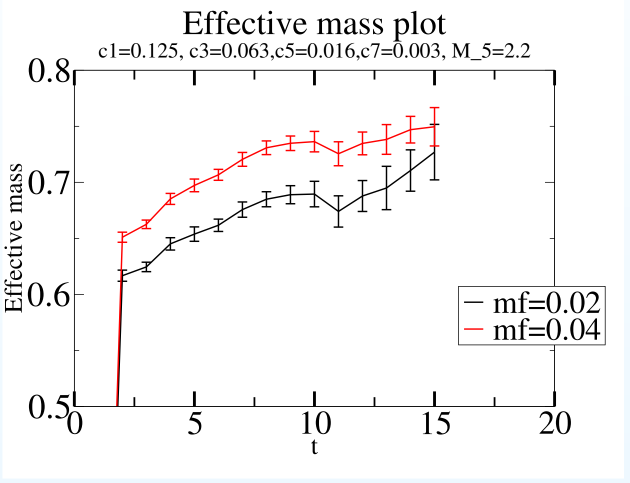


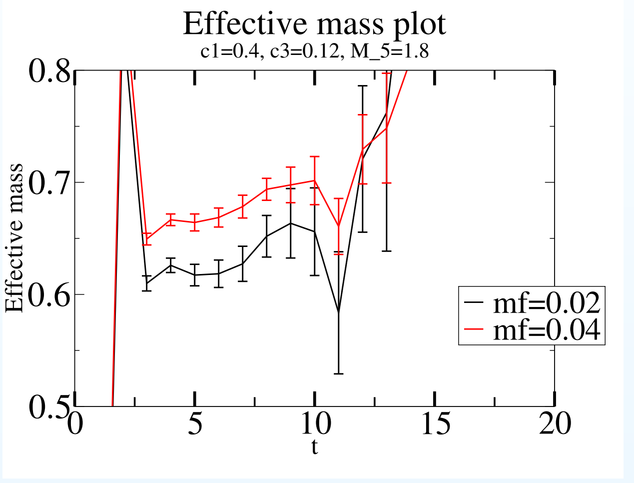

After smearing$\Longrightarrow$

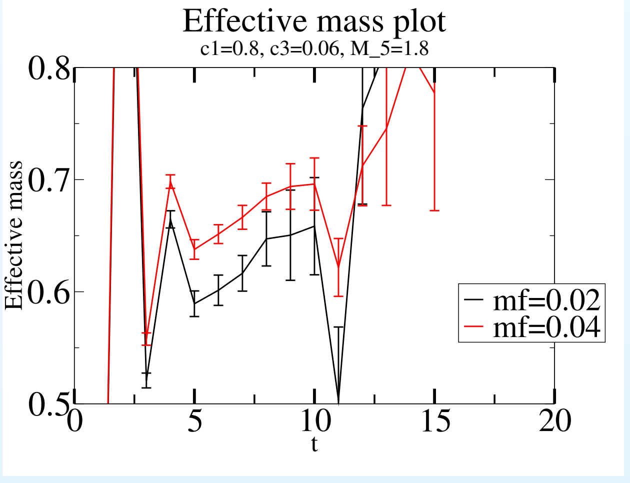
- less condensation of near-zero modes
- larger gap at $M_5 \sim 1.8$
- however,major crossings do not change

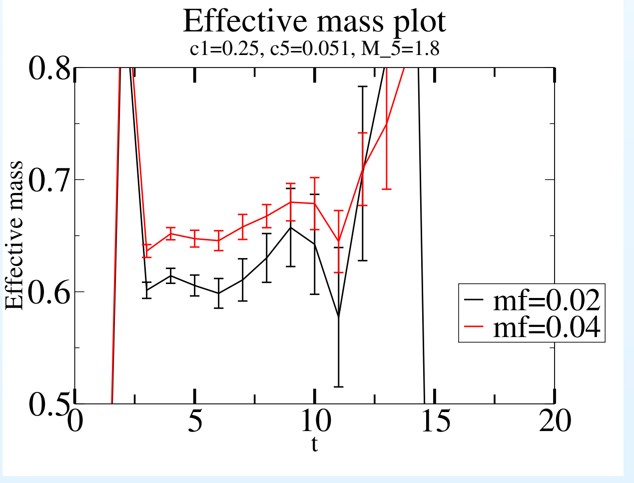
# Smearing—lattice scales

- Effective rho mass plots:($m_\rho$ fit from $t = 6$ to $t = 10$)

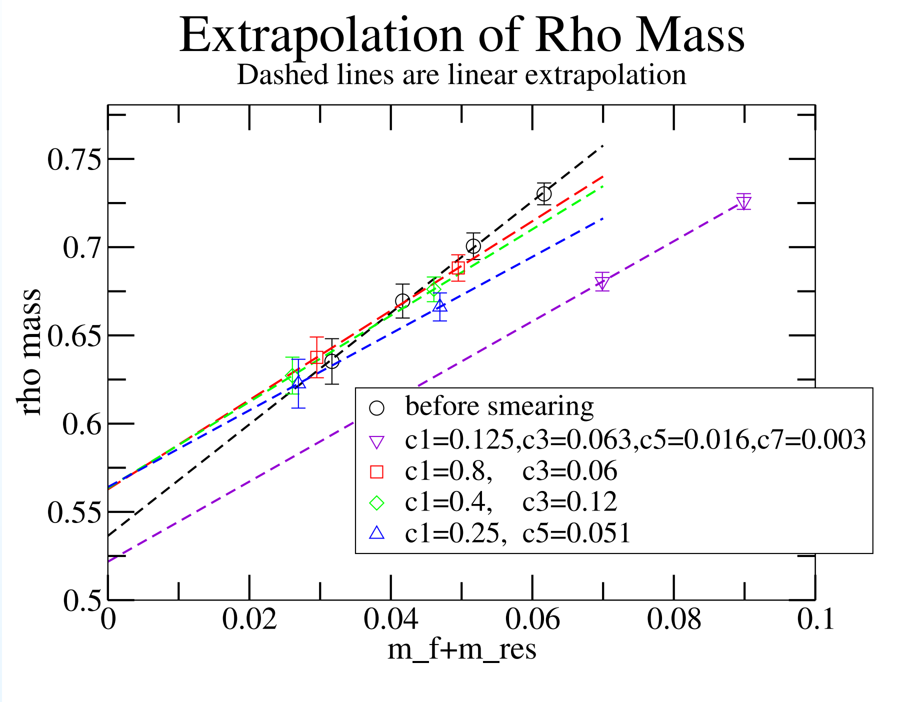


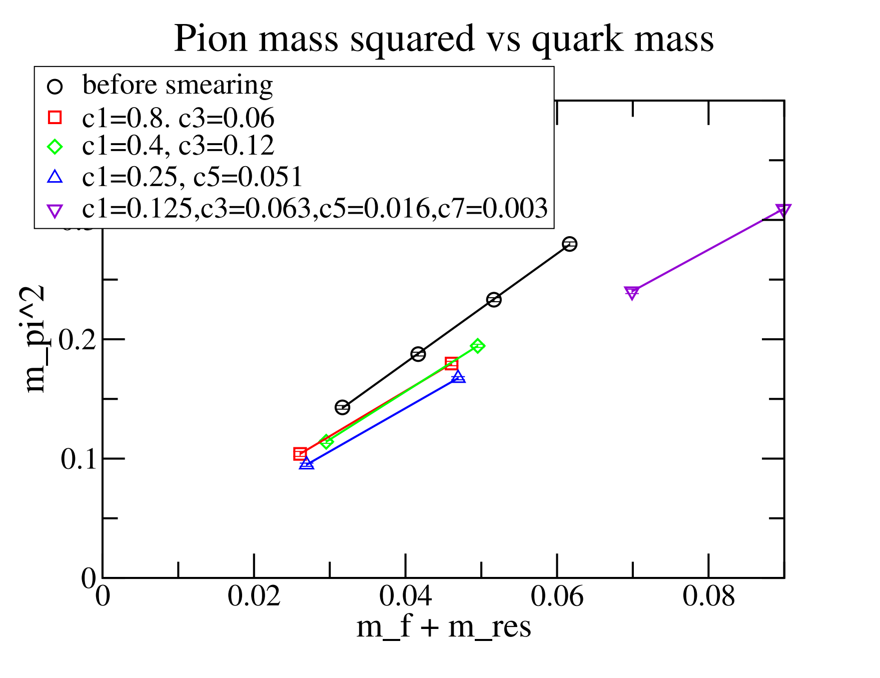
# Smearing—lattice scales(Cont'd)




| $c_1$ | $c_3$ | $c_5$ | $c_7$ | $m_\rho$ |
|---|---|---|---|---|
| 1.0 | | | | 0.539(4) |
| 0.125 | 0.063 | 0.016 | 0.003 | 0.522 |
| 0.8 | 0.06 | | | 0.563 |
| 0.4 | 0.12 | | | 0.563 |
| 0.25 | | 0.051 | | 0.564 |

# Smearing—mass normalization

If we define $m^{eff} = Z_m m_f$, $m^{eff}_{\rm res} = Z_m m_{\rm res}$, then we can extract $Z_m$ from $m_\pi^2$ vs. $m_f + m_{\rm res}$ plots.

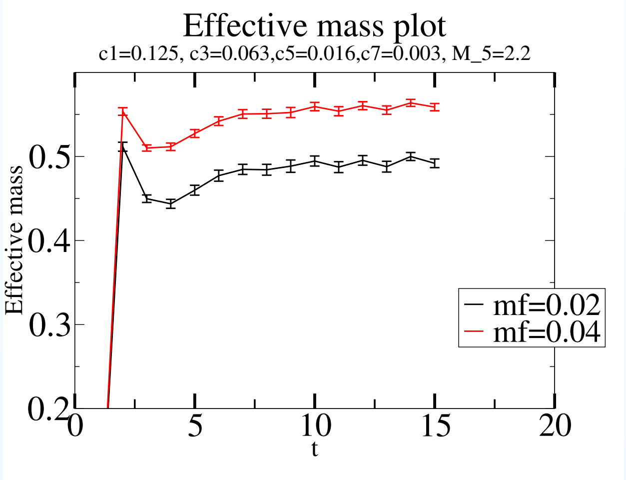


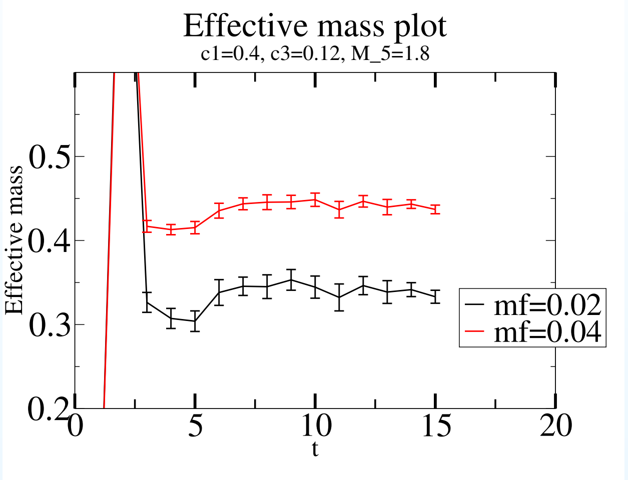

| $c_1$ | $c_3$ | $c_5$ | $c_7$ | $Z_m$ |
|---|---|---|---|---|
| 1.0 | | | | 1 |
| 0.125 | 0.063 | 0.016 | 0.003 | 0.76 |
| 0.8 | 0.06 | | | 0.83 |
| 0.4 | 0.12 | | | 0.88 |
| 0.25 | | 0.051 | | 0.79 |

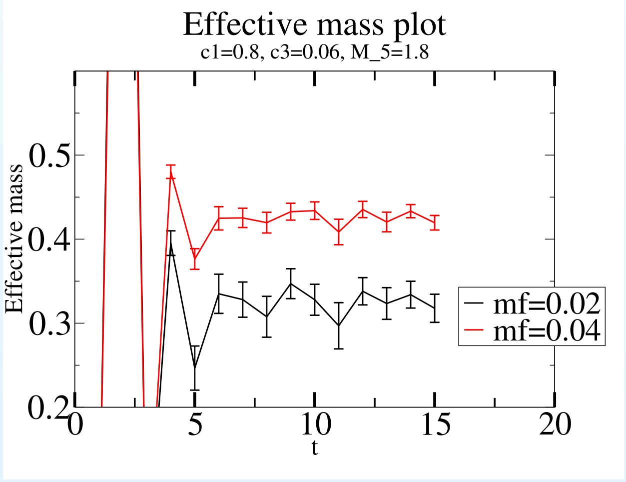
Taking these normalization factors into account, the effective residual masses should be even smaller than the measured values.

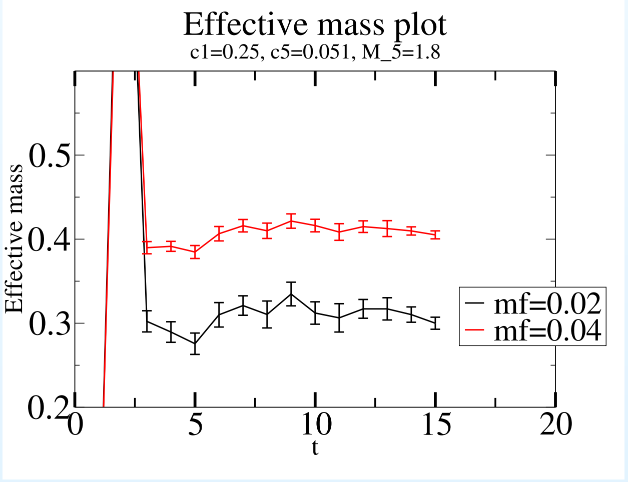
# Smearing—mass normalization(Cont'd)

- Effective pion mass plots:($m_\pi$ fit from $t = 7$ to $t = 16$)



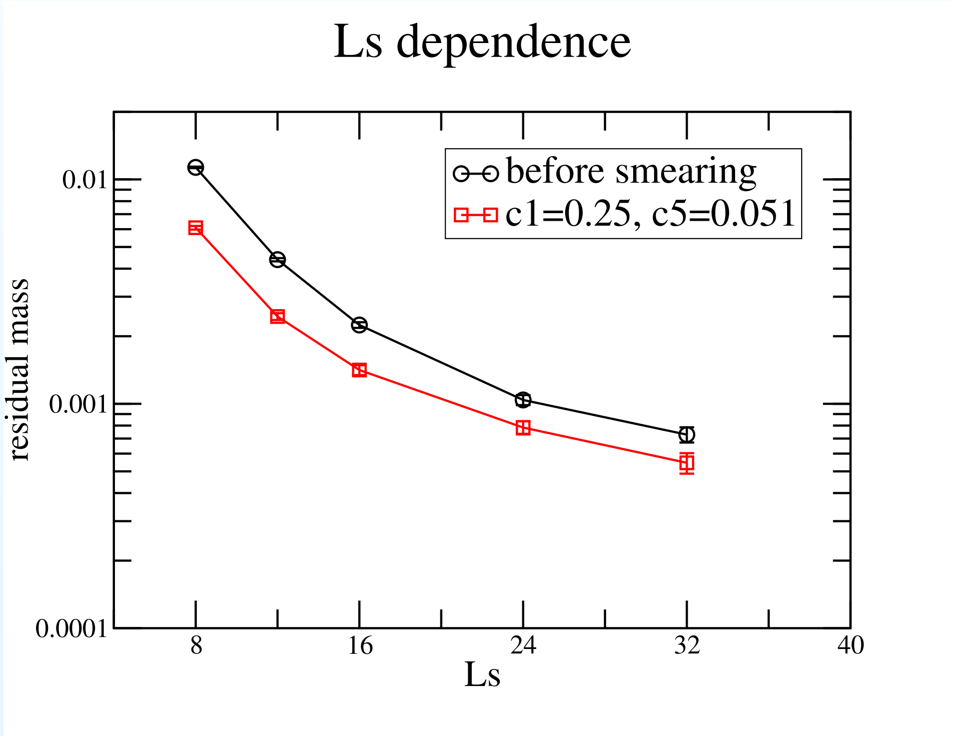



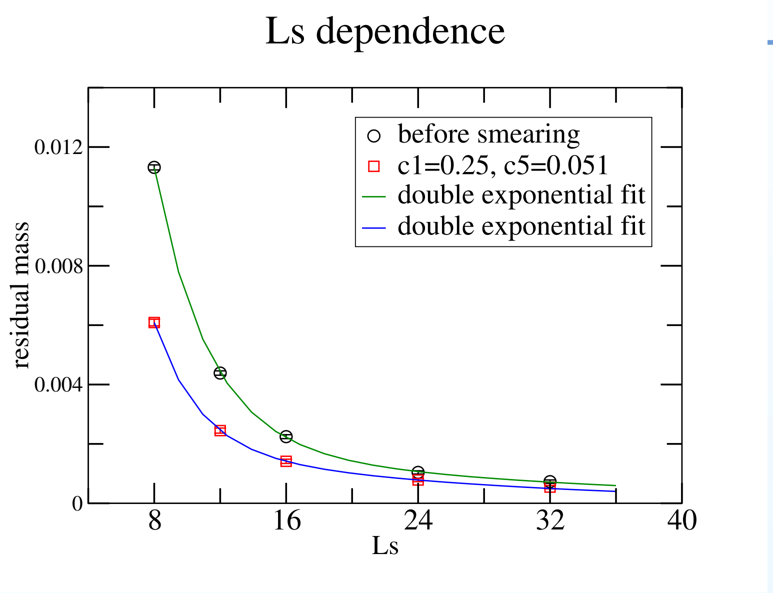
# Smearing—$L_s$ dependence of residual masses

Also interested in $L_s$ dependence of the residual masses after smearing.
$c_1 = 0.25, c_5 = 0.051$ was investigated.




⇒ NOT simple exponential decay,both unsmeared and smeared.
⇒ similar behaviors in quenched Wilson and $N_f = 2$ DBW2 simulations.(See R.Mawhinney's talk)

# Smearing—$L_s$ dependence of residual masses (Cont'd)




⇒ fit well into double exponential forms (also seen in quenched simulations with Wilson gauge action,T.Blum *et al.*, Phys. Rev. D 69 (2004) 074502)

–before smearing

$$m_{\mathrm{res}}(L_s) = 0.109\exp(-0.307L_s) + 0.00283\exp(-0.0436L_s)$$

–smeared with $c_1 = 0.25$,$c_5 = 0.051$

$$m_{\mathrm{res}}(L_s) = 0.0826\exp(-0.371L_s) + 0.00288\exp(-0.0548L_s)$$

## Conclusion

- Smearing doesn't change lattice scales but introduces a mass normalization factor.
- Smearing reduces the residual masses by a factor of 2.
- Large $L_s$ decay of $m_{\rm res}$ doesn't seem to improve by smearing.
- We don't really change the exceptional configurations with topological dislocations by smearing. Chiral symmetry is not improved a lot.
- At $a^{-1} \geq$2 GeV, $N_f = 3$ DWF gives $m_{\rm res}$ of a few $MeV$, which is reasonably small for dynamical simulations.