Optimization of baryonic sources using irreducible representations of the octahedral group

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I. Introduction

Matrix of correlation functions using baryon interpolating operators $\overline{B}_m(\vec{x},t)$ is

$$C_{lm}(t) = \sum_{\vec{x}} \langle 0| B_l(\vec{x},t)\overline{B}_m(\vec{0},0)\,|0\rangle$$

1. Find operators $\overline{B}_m$ that transform irreduciblly under rotations of double octahedral group. (*See also Dr. C. Morningstar*)
2. Show $C_{lm}(t)$ is *block-diagonal* in the basis of irreducible operators.
3. Find optimized linear combinations of IR operators by variational method. (*Will be discussed later by Dr. S. Basak*)

$$\tilde{\overline{B}}^{(\alpha)} = \sum_m c_m^{(\alpha)} \overline{B}_m$$

II. Double octahedral group, O^D

There are 24 distinct ways to rotate a *vectorial* object on cubic lattice. *Spinorial* degree of freedom doubles the number of rotations: they form a group known as double octahedral group.

spatial rotations: $\pm\pi/2, \pi$ about x, y, z axis
$\pm 2\pi/3$ about four body-diagonal axis
π about six face-diagonal axis
identity

24 -spatial rotations
$\times 2$ -spinor
$= 48$ **group elements**

The reduction of the double octahedral group:

		IR (dim)	J				
		$G_1(2)$	1/2			7/2	...
O^D	$\Longrightarrow$	$G_2(2)$			5/2	7/2	...
		$H\ (4)$		3/2	5/2	7/2	...

III. Local sources (point or smeared)

$$\sum_{c's} \varepsilon_{c_1 c_2 c_3}\, \overline{q}^{f_1 c_1}_{\mu_1}(\vec{x},t)\, \overline{q}^{f_2 c_2}_{\mu_2}(\vec{x},t)\, \overline{q}^{f_3 c_3}_{\mu_3}(\vec{x},t)$$

c's are the color indices, f's are the types of flavor, and μ's are the four-component Dirac indices.

a) Isospin 1/2 baryons

Choose baryonic isospin $I = I_z = 1/2$
(mixed antisymmetric)

$$N^* \text{ family} \Rightarrow \overline{N}_{\mu_1\mu_2\mu_3} \equiv \left(\overline{u}_{\mu_1}\overline{d}_{\mu_2} - \overline{d}_{\mu_1}\overline{u}_{\mu_2}\right)\overline{u}_{\mu_3}$$

Overall Antisym. ←by Pauli Principle

↙ ↓ ↘

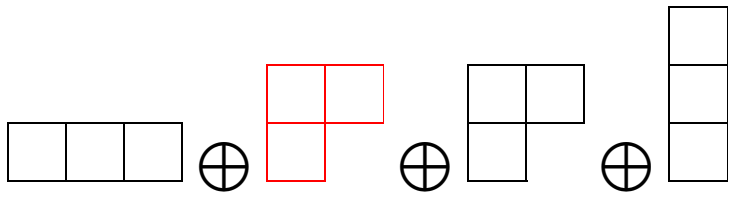
Color Antisym. | Isospin M. A. | Dirac spin M. A.

Symmetry of Dirac spin indices

Three quarks, Four components

$\Downarrow$

can create $4^3 = 64$ states



20	**20**	**20**	**4**
S.	M.A.	M.S.	A.

Young Tableau for Dirac indices.

$\Rightarrow$ **20 local operators for N^***

ρ-spin labels

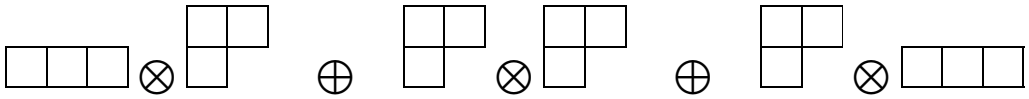
$$\gamma^i = i\rho^2 \otimes \sigma^i, \quad \gamma^4 = \rho^3 \otimes \mathbf{1}$$ (in Dirac-Pauli notation)

Table 1: Translation of Dirac index into s and ρ.

$s = +$ is taken along z-axis and $\rho = +$ is taken for upper two-component.

μ	1	2	3	4
ρ	+	+	-	-
s	+	-	+	-

ρ-spin and s-spin combinations for nucleon:



S. ρ-spin	M.A. s-spin	M.S. ρ-spin	M.A. s-spin	M.A. ρ-spin	S. s-spin
$4 \times 2 = 8$		$2 \times 2 = 4$		$2 \times 4 = 8$	
$G_1(2)$		$G_1(2)$		$H(4)$	

Direct products of spin and ρ-spin Young tableau for the $I = 1/2, I_z = 1/2$ operators $N^{\rho_1\rho_2\rho_3}_{s_1\,s_2\,s_3}$.

G_1 ($\rho = 3/2, s = 1/2$) operators

ρ_3 ($\mathcal{P}$)	$S_3 = 1/2$ (row 1)	$S_3 = -1/2$ (row 2)
$3/2$ $(+)$	$2^{-1/2}\overline{N}^{+++}_{+-+}$	$2^{-1/2}\overline{N}^{+++}_{+--}$
$1/2$ $(-)$	$6^{-1/2}(\overline{N}^{++-}_{+-+} + \overline{N}^{+-+}_{+-+} + \overline{N}^{-++}_{+-+})$	$6^{-1/2}(\overline{N}^{++-}_{+--} + \overline{N}^{+-+}_{+--} + \overline{N}^{-++}_{+--})$
$-1/2$ $(+)$	$6^{-1/2}(\overline{N}^{+--}_{+-+} + \overline{N}^{-+-}_{+-+} + \overline{N}^{--+}_{+-+})$	$6^{-1/2}(\overline{N}^{+--}_{+--} + \overline{N}^{-+-}_{+--} + \overline{N}^{--+}_{+--})$
$-3/2$ $(-)$	$2^{-1/2}\overline{N}^{---}_{+-+}$	$2^{-1/2}\overline{N}^{---}_{+--}$

G_1 ($\rho = 1/2, s = 1/2$) operators

ρ_3 ($\mathcal{P}$)	$S_3 = 1/2$ (row 1)	$S_3 = -1/2$ (row 2)
$1/2$ $(-)$	$(12)^{-1/2}(2\overline{N}^{+-+}_{++-} - \overline{N}^{+-+}_{+-+} - \overline{N}^{+-+}_{-++})$	$-(12)^{-1/2}(2\overline{N}^{+-+}_{--+} - \overline{N}^{+-+}_{+--} - \overline{N}^{+-+}_{-+-})$
$-1/2$ $(+)$	$(12)^{-1/2}(2\overline{N}^{+--}_{++-} - \overline{N}^{+--}_{+-+} - \overline{N}^{+--}_{-++})$	$-(12)^{-1/2}(2\overline{N}^{+--}_{--+} - \overline{N}^{+--}_{+--} - \overline{N}^{+--}_{-+-})$

H ($\rho = 1/2, s = 3/2$) operators

ρ_3 ($\mathcal{P}$)	$S_3 = 3/2$ (row 1)	$S_3 = 1/2$ (row 2)	
$1/2$ $(-)$	$2^{-1/2}\overline{N}^{+-+}_{+++}$	$6^{-1/2}(\overline{N}^{+-+}_{++-} + \overline{N}^{+-+}_{+-+} + \overline{N}^{+-+}_{-++})$	$\swarrow$
$-1/2$ $(+)$	$2^{-1/2}\overline{N}^{+--}_{+++}$	$6^{-1/2}(\overline{N}^{+--}_{++-} + \overline{N}^{+--}_{+-+} + \overline{N}^{+--}_{-++})$	$\swarrow$

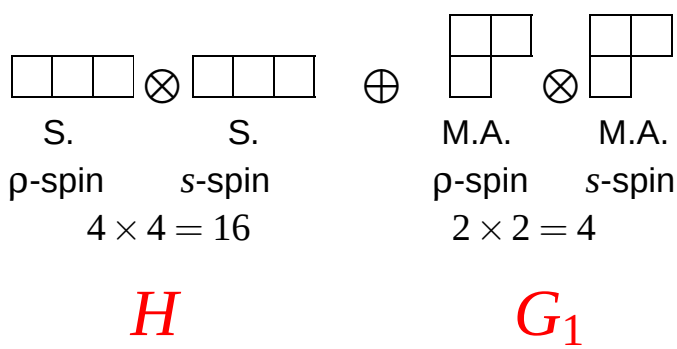
	$S_3 = -1/2$ (row 3)	$S_3 = -3/2$ (row 4)
$\cdots$	$6^{-1/2}(\overline{N}^{+-+}_{+--} + \overline{N}^{+-+}_{-+-} + \overline{N}^{+-+}_{--+})$	$2^{-1/2}\overline{N}^{+-+}_{---}$
$\cdots$	$6^{-1/2}(\overline{N}^{+--}_{+--} + \overline{N}^{+--}_{-+-} + \overline{N}^{+--}_{--+})$	$2^{-1/2}\overline{N}^{+--}_{---}$

b) Isospin 3/2 baryons

For Δ^{++} baryons local operator is

$$\overline{\Delta^{++}}_{\mu_1\mu_2\mu_3} = \overline{u}_{\mu_1}\overline{u}_{\mu_2}\overline{u}_{\mu_3}$$

$I_z = 1/2, -1/2, -3/2$ operators can be obtained by applying isospin lowering operator.



Direct products of spin and ρ-spin Young tableau for the $I = 3/2, I_z = 3/2$ operators $(\overline{\Delta^{++}})^{\rho_1\rho_2\rho_3}_{s_1 s_2 s_3}$.

Using $\mu_1\mu_2\mu_3$ labels (denote $\{\mu_1\mu_2\mu_3\}$ as totally symmetric combination),

H ($\rho = 3/2, s = 3/2$) operators

$\rho_3\ (\mathcal{P})$	$S_3 = 3/2$ (row 1)	$S_3 = 1/2$ (row 2)	$S_3 = -1/2$ (row 3)	$S_3 = -3/2$ (row 4)
$3/2\ (+)$	111	$3^{-1/2}\{112\}$	$3^{-1}\{112\}$	222
$1/2\ (-)$	$3^{-1/2}\{113\}$	$3^{-1}\{114\}+\{123\}$	$3^{-1}\{124\}+\{223\}$	$3^{-1/2}\{224\}$
$-1/2\ (+)$	$3^{-1/2}\{133\}$	$3^{-1}\{134\}+\{233\}$	$3^{-1}\{144\}+\{234\}$	$3^{-1/2}\{244\}$
$-3/2\ (-)$	333	$3^{-1/2}\{334\}$	$3^{-1}\{344\}$	444

G_1 ($\rho = 1/2, s = 1/2$) operators

$\rho_3\ (\mathcal{P})$	$S_3 = 1/2$ (row 1)	$S_3 = -1/2$ (row 2)
$1/2\ (-)$	$2^{-1}(141-231-321+411)$	$2^{-1}(142-232-322+412)$
$-1/2\ (+)$	$2^{-1}(143-233-323+413)$	$2^{-1}(144-234-324+414)$

IV. One-link sources

Consider the simplest nonlocal operator: 3rd quark displaced.

$$\overline{O}_i \equiv \sum_{c's} \varepsilon_{c_1 c_2 c_3}\, \overline{q}^{f_1 c_1}_{\mu_1}(\vec{x},t)\, \overline{q}^{f_2 c_2}_{\mu_2}(\vec{x},t)\, \overline{q}^{f_3 c_3'}_{\mu_3}(\vec{x}+a\hat{e}_i,t) U_i^{\dagger c_3', c_3}(\vec{x}).$$

(3 spinors) $\times$ (1 spatial)

$\longrightarrow$ $\{\overline{O}_x, \overline{O}_y, \overline{O}_z, \overline{O}_{-x}, \overline{O}_{-y}, \overline{O}_{-z}\}$ transform amongst themselves under group rotations.

The spatial part has six degrees of freedom. They can be reduced into three irreducible representations known as $A_1(1), T_1(3)$, and $E(2)$. This reduction provides new basis operators for the spatial part.

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \end{pmatrix} \equiv \begin{pmatrix} \frac{1}{\sqrt{6}}(\overline{O}_x + \overline{O}_y + \overline{O}_z + \overline{O}_{-x} + \overline{O}_{-y} + \overline{O}_{-z}) \\ \frac{i}{2\sqrt{2}a}[(\overline{O}_x - \overline{O}_{-x}) + i(\overline{O}_y - \overline{O}_{-y})] \\ \frac{-i}{2\sqrt{2}a}[(\overline{O}_x - \overline{O}_{-x}) - i(\overline{O}_y - \overline{O}_{-y})] \\ \frac{-i}{2a}[\overline{O}_z - \overline{O}_{-z}] \\ \frac{1}{\sqrt{6}a^2}[2(\overline{O}_z + \overline{O}_{-z}) - (\overline{O}_x + \overline{O}_{-x}) - (\overline{O}_y + \overline{O}_{-y})] \\ \frac{1}{\sqrt{2}}[(\overline{O}_x + \overline{O}_{-x}) - (\overline{O}_y + \overline{O}_{-y})] \end{pmatrix}$$

$\longrightarrow$ **Now,** $\{L_1\}$, $\{L_2, L_3, L_4\}$, $\{L_5, L_6\}$ **transform** *irreduciblly* **under group rotations.**

Note that the form of L_i's corresponds to spherical basis.

basis operators	IR	spherical harmonics	wave
L_1	A_1	Y_{00}	S
L_2, L_3, L_4	T_1	$Y_{1\pm1},\ \ Y_{10}$	P
L_5, L_6	E	$Y_{20},\ \ (Y_{22}+Y_{2-2})$	D

Define **derivative operators** as follows.

$$D_+\overline{B} \equiv L_2,\ \ D_-\overline{B} \equiv L_3,\ \ D_0\overline{B} \equiv L_4.$$

These are the operators with covariant first-derivative acting on the third quark field.

$$E_0\overline{B} \equiv L_5,\ \ E_2\overline{B} \equiv L_6.$$

These are the operators with covariant second-derivative acting on the third quark field, *i.e.*, $L_5 \leftrightarrow 2D_z^2 - D_x^2 - D_y^2,\ \ L_6 \leftrightarrow D_x^2 - D_y^2$.

Denote spinorial part of operator $\overline{B}$ as ϕ_{S,S_z}, and the spatial part as Y_{lm}. In order to reduce the set of derivative operators, one needs to find the coefficient $C(S,S_z;l,m)$,

$$\overline{B}^{(IR)} \leftrightarrow \sum_{S_z,m} C(S,S_z;l,m)\ \phi_{S,S_z}\ Y_{lm},$$

such that the resultants are the eigenvectors of **group theoretical projection operator:**

$$P^{(IR)} = \frac{d^{(IR)}}{n_g} \sum_a \chi_a^{(IR)}\ R(G_a).$$

where $d^{(IR)}$ = dim of IR, n_g = number of group elements, $\chi_a^{(IR)}$ = character of IR for group element G_a, and $R(G_a)$ = group representation.

1) **Ambiguity of projection operator**: all embeddings and rows are degenerate w.r.t. projection.

2) **Convention**: use of **Clebsch-Gordan coefficients** on discretized spatial basis, Y_{lm} (not always possible).

a) $\rightarrow$ can create most of IR operators

b) $\rightarrow$ remainders are orthogonal to the obtained IR operators.

H first-derivative sources $J = 3/2,\ 5/2, \ldots$

basis operators	$\mathcal{P}$	J_z
$D_+ \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	$\mp$	$3/2$
$\frac{1}{\sqrt{3}} D_+ \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2} + \sqrt{\frac{2}{3}} D_0 \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	$\mp$	$1/2$
$\frac{1}{\sqrt{3}} D_- \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2} + \sqrt{\frac{2}{3}} D_0 \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	$\mp$	$-1/2$
$D_- \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	$\mp$	$-3/2$
$-\sqrt{\frac{2}{5}} D_+ \overline{\Psi}^{(H^\pm)}_{3/2,1/2} + \sqrt{\frac{3}{5}} D_0 \overline{\Psi}^{(H^\pm)}_{3/2,3/2}$	$\mp$	$3/2$
$-\sqrt{\frac{8}{15}} D_+ \overline{\Psi}^{(H^\pm)}_{3/2,-1/2} + \sqrt{\frac{2}{5}} D_- \overline{\Psi}^{(H^\pm)}_{3/2,3/2} + \frac{1}{\sqrt{15}} D_0 \overline{\Psi}^{(H^\pm)}_{3/2,1/2}$	$\mp$	$1/2$
$-\sqrt{\frac{2}{5}} D_+ \overline{\Psi}^{(H^\pm)}_{3/2,-3/2} + \sqrt{\frac{8}{15}} D_- \overline{\Psi}^{(H^\pm)}_{3/2,1/2} - \frac{1}{\sqrt{15}} D_0 \overline{\Psi}^{(H^\pm)}_{3/2,-1/2}$	$\mp$	$-1/2$
$\sqrt{\frac{2}{5}} D_- \overline{\Psi}^{(H^\pm)}_{3/2,-1/2} - \sqrt{\frac{3}{5}} D_0 \overline{\Psi}^{(H^\pm)}_{3/2,-3/2}$	$\mp$	$-3/2$
$\frac{1}{\sqrt{10}} D_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,1/2} + \sqrt{\frac{5}{6}} D_- \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2} + \frac{1}{\sqrt{15}} D_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2}$	$\mp$	
$\sqrt{\frac{3}{10}} D_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2} + \frac{1}{\sqrt{10}} D_- \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2} + \sqrt{\frac{3}{5}} D_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,1/2}$	$\mp$	
$\frac{1}{\sqrt{10}} D_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2} + \sqrt{\frac{3}{10}} D_- \overline{\Psi}^{(H^\pm)}_{3/2,1/2} + \sqrt{\frac{3}{5}} D_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2}$	$\mp$	
$\sqrt{\frac{5}{6}} D_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2} + \frac{1}{\sqrt{10}} D_- \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2} + \frac{1}{\sqrt{15}} D_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2}$	$\mp$	

$$D_m \overline{\Psi}^{(IR^{\mathcal{P}})}_{S,S_z} = D_m \overline{N}^{(IR^{\mathcal{P}})}_{S,S_z}, \quad D_m \overline{\Delta^{++}}^{(IR^{\mathcal{P}})}_{S,S_z}, \ \ldots$$

G_1 first-derivative sources $J = 1/2$

basis operators	$\mathcal{P}$	J_z
$-\sqrt{\frac{2}{3}}D_+\overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2} + \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	$\mp$	$1/2$
$\sqrt{\frac{2}{3}}D_-\overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2} - \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	$\mp$	$-1/2$
$\frac{1}{\sqrt{2}}D_-\overline{\Psi}^{(H^\pm)}_{3/2,3/2} + \frac{1}{\sqrt{6}}D_+\overline{\Psi}^{(H^\pm)}_{3/2,-1/2} - \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(H^\pm)}_{3/2,1/2}$	$\mp$	$1/2$
$-\frac{1}{\sqrt{2}}D_+\overline{\Psi}^{(H^\pm)}_{3/2,-3/2} - \frac{1}{\sqrt{6}}D_-\overline{\Psi}^{(H^\pm)}_{3/2,1/2} + \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(H^\pm)}_{3/2,-1/2}$	$\mp$	$-1/2$

G_2 first-derivative sources $J = 5/2,\ 7/2, \ldots$

basis operators	$\mathcal{P}$	J_z
$-\frac{1}{\sqrt{6}}D_+\overline{\Psi}^{(H^\pm)}_{3/2,3/2} + \frac{1}{\sqrt{2}}D_-\overline{\Psi}^{(H^\pm)}_{3/2,-1/2} + \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(H^\pm)}_{3/2,-3/2}$	$\mp$	
$\frac{1}{\sqrt{6}}D_-\overline{\Psi}^{(H^\pm)}_{3/2,-3/2} - \frac{1}{\sqrt{2}}D_+\overline{\Psi}^{(H^\pm)}_{3/2,1/2} - \frac{1}{\sqrt{3}}D_0\overline{\Psi}^{(H^\pm)}_{3/2,3/2}$	$\mp$	

** No G_2 operators for local operators!*

Second-derivative one-link sources

$$E_0\overline{q}(\vec{x},t) \equiv 6^{-1/2}a^{-2}\{2D_z^2 q(\vec{x},t) - \left[D_x^2\overline{q}(\vec{x},t) + D_y^2\overline{q}(\vec{x},t)\right]\}$$

$$E_2\overline{q}(\vec{x},t) \equiv 2^{-1/2}a^{-2}\left[D_x^2\overline{q}(\vec{x},t) - D_y^2\overline{q}(\vec{x},t)\right]$$

basis operators	IR	$\mathcal{P}$	J	J_z	Type
$E_2\overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	H	$\pm$	$3/2$	$3/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	H	$\pm$	$3/2$	$1/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	H	$\pm$	$3/2$	$-1/2$	$d_{3/2}$
$E_2\overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	H	$\pm$	$3/2$	$-3/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,3/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,-1/2}$	H	$\pm$	$3/2$	$3/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,1/2} - E_2\overline{\Psi}^{(H^\pm)}_{3/2,-3/2}$	H	$\pm$	$3/2$	$1/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,-1/2} - E_2\overline{\Psi}^{(H^\pm)}_{3/2,3/2}$	H	$\pm$	$3/2$	$-1/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,-3/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,1/2}$	H	$\pm$	$3/2$	$-3/2$	$d_{3/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,1/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,-3/2}$	G_1	$\pm$	$1/2$	$1/2$	$d_{1/2}$
$E_0\overline{\Psi}^{(H^\pm)}_{3/2,-1/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,3/2}$	G_1	$\pm$	$1/2$	$-1/2$	$d_{1/2}$
$-E_0\overline{\Psi}^{(H^\pm)}_{3/2,3/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,-1/2}$	G_2	$\pm$	$5/2$	$3/2$	$d_{5/2}$
$-E_0\overline{\Psi}^{(H^\pm)}_{3/2,-3/2} + E_2\overline{\Psi}^{(H^\pm)}_{3/2,1/2}$	G_2	$\pm$	$5/2$	$3/2$	$d_{5/2}$

V. Numerical check of orthogonality

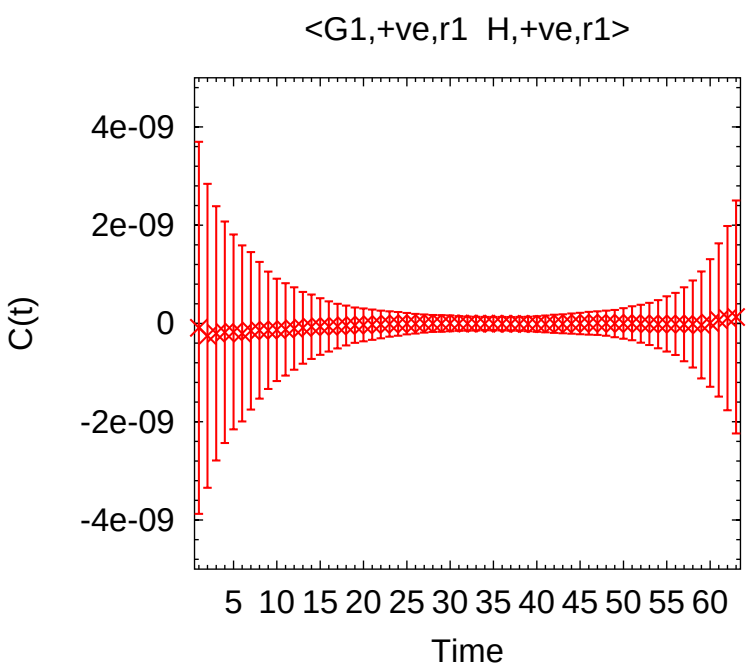
Orthogonality relation:

$$\sum_{\vec{x}} \langle 0| B^{(IR',\mathcal{P}',row')} \; B^{\dagger(IR,\mathcal{P},row)} |0\rangle \;\sim\; \delta_{IR'IR}\delta_{\mathcal{P}'\mathcal{P}}\delta_{row'row}$$

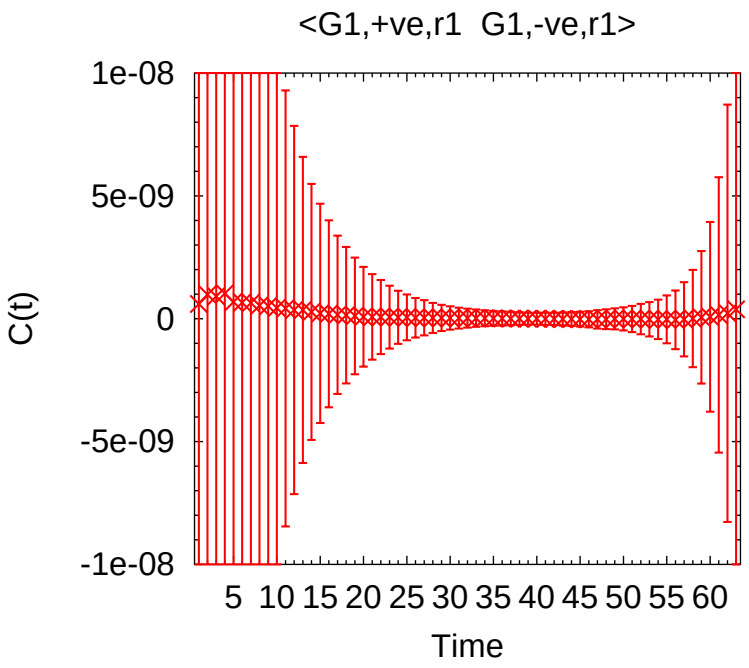
Using 20 local, and 60 first-derivative operators, 80×80 matrix of correlators have been obtained, for both isospin 1/2 and 3/2 operators. The orthogonality relation is obtained; correlation functions between different IRs, parities, or rows (corresponding to S_z) are zero within one errorbar.

Representative graph of correlation functions

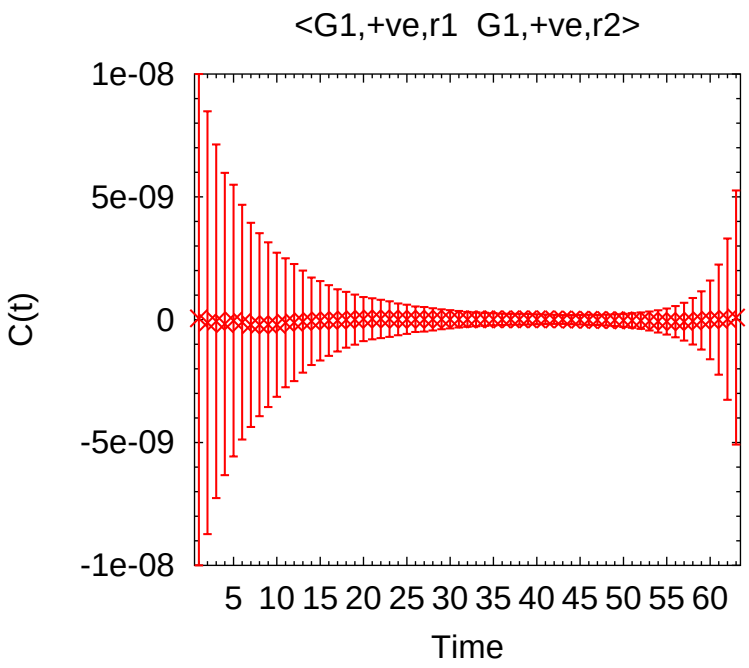
Different IRs



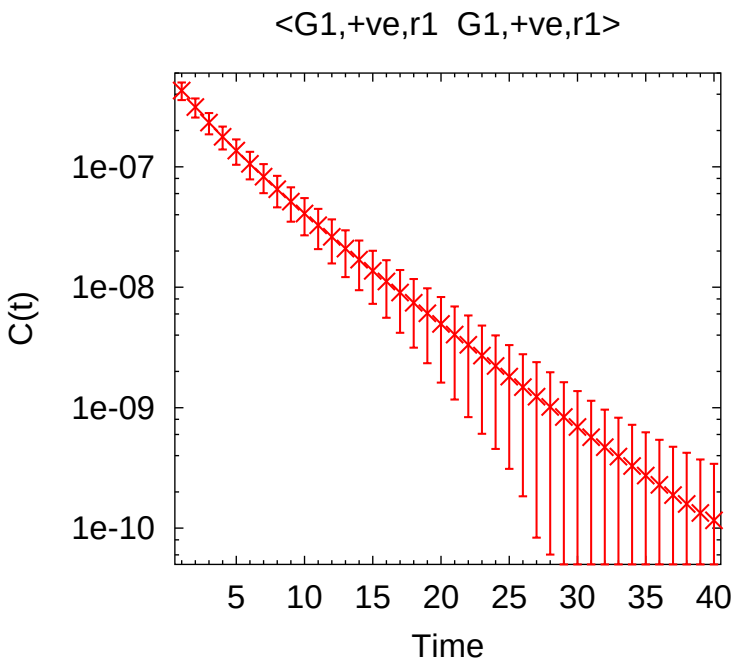
Different parities



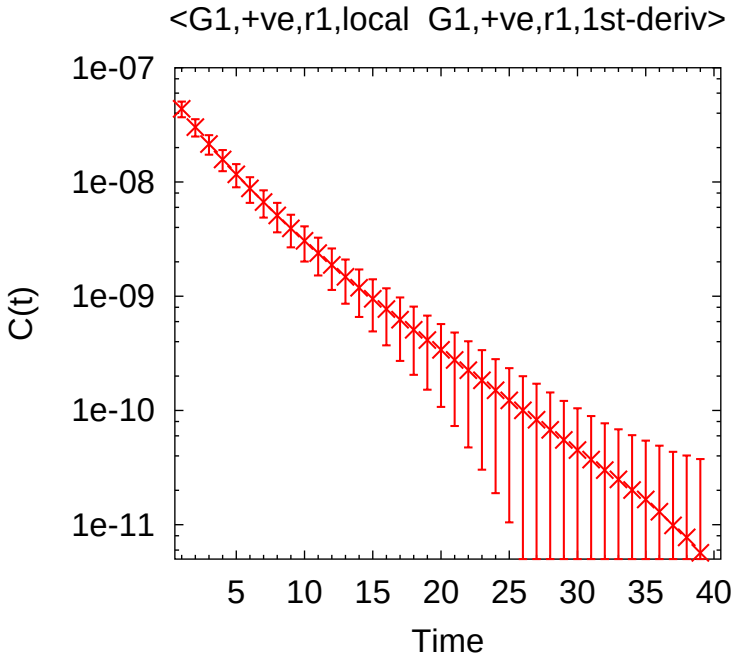
Different rows



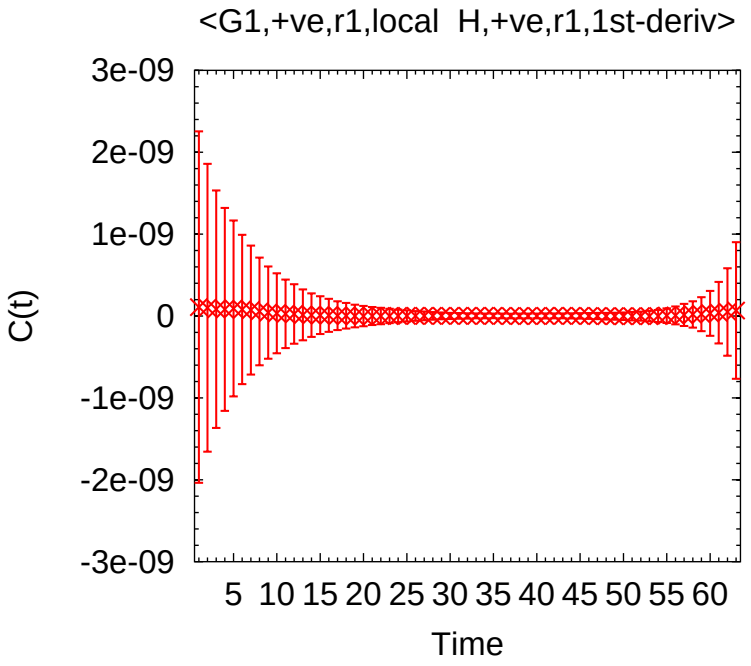
same source and sink (local)



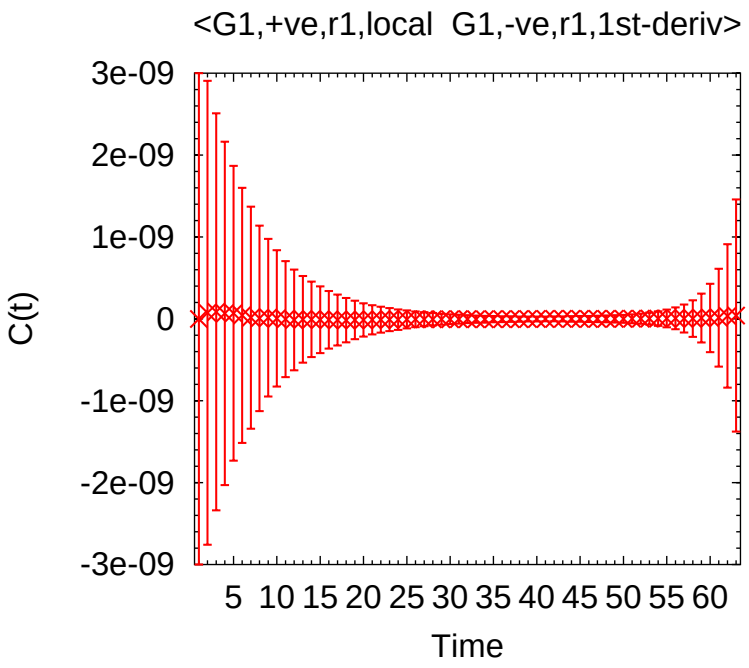
same IR, parity, row ＜local nonlocal＞



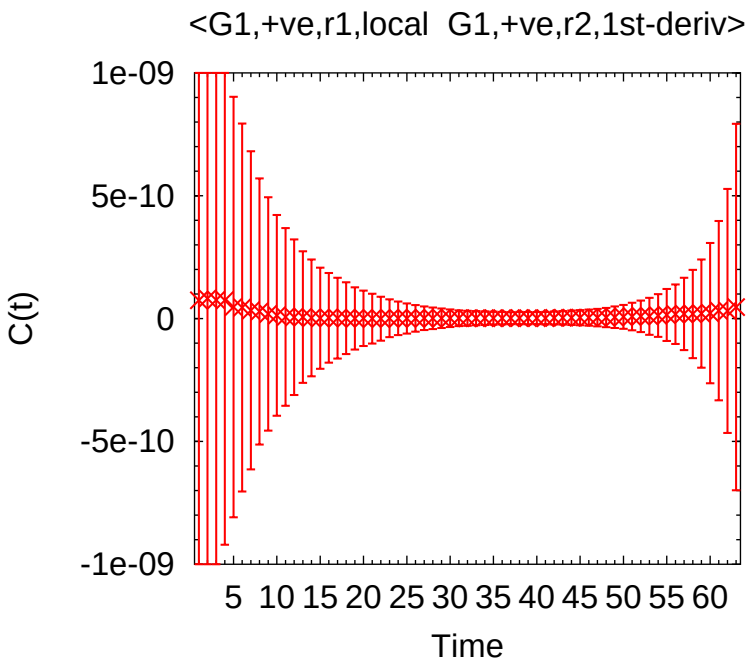
Different IRs ＜local nonlocal＞



Different parities ＜local nonlocal＞



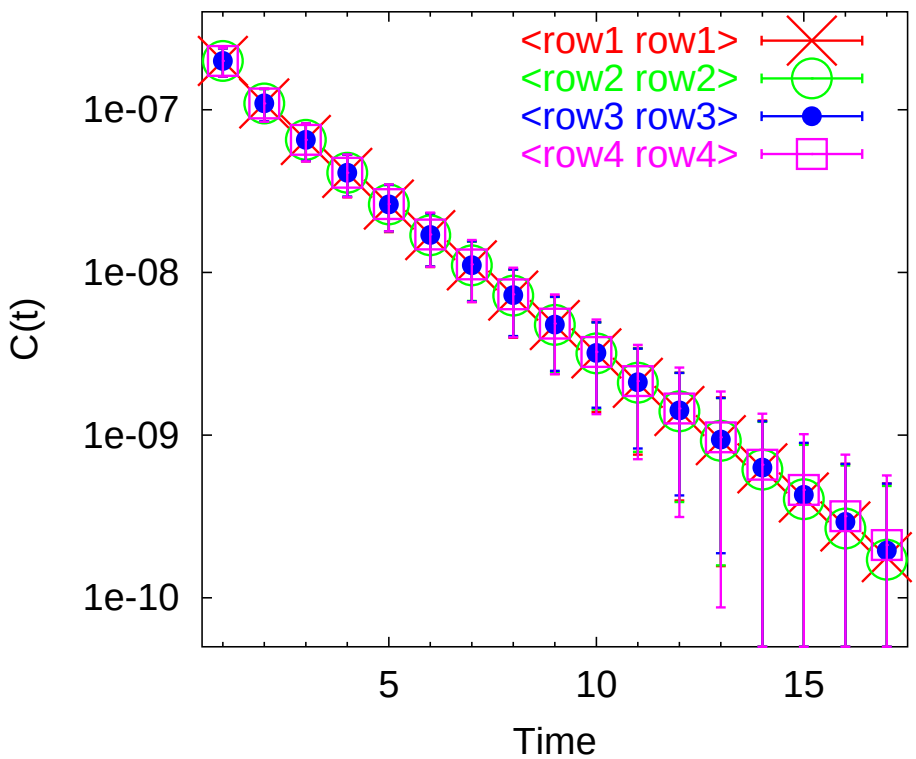
Different rows ＜local nonlocal＞



VI. Technique to increase statistics

The dimension of H is four (row 1, row 2, etc.). Operators from different rows of the same embedding of an IR give on average equal correlation functions, $< B^{(IR,rowA)} B^{\dagger(IR,rowA)} > = < B^{(IR,rowB)} B^{\dagger(IR,rowB)} >$.

diagonal correlation functions of local, H, $I = I_z = 1/2$



Correlation functions of different rows within the same IR can be averaged configuration by configuration to have better statistical ensemble.

VII. Summary and outlook

- Lots of irreducible operators for baryons (local, one-link) have been constructed.
- Local IR operators are simple based on the symmetries of ρ spin and two-component s spin.
- Nonlocal operators are reduced into three IRs, $A_1(1)$, $T_1(3)$, and $E(2)$, corresponding to S, P, and D.
- Orthogonality relations are verified numerically.

$$\sum_{\vec{x}} \langle 0| B^{(IR',\mathcal{P}',row')} B^{\dagger(IR,\mathcal{P},row)} |0\rangle \sim \delta_{IR'IR}\delta_{\mathcal{P}'\mathcal{P}}\delta_{row'row}$$

- The equality of correlation functions of different "rows" is checked numerically. They can be averaged configuration by configuration for better statistical ensemble.
- Using the basis of IR operators of double octahedral group, matrix of correlation functions can be obtained. By the variational method, optimized linear combinations of basis operators can be found. (*Will be discussed in Dr. S. Basak's talk.*)

- Obtained IR operators can be used for Λ (nucleon type), and Σ^+, Ξ^0, and Ω^- (delta type) baryons with changing flavor(s) of quark(s).
- More complicated spatial distribution of valence quarks needs to be considered to have richer angular distribution of baryon. (*Will be discussed in Dr. C. Morningstar's talk.*)
- Radial excitation using different smearing widths will enrich the spectrum.
- Similar group theoretical construction of operators are applicable to hybrids and pentaquarks.