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# Moving NRQCD: $B$ mesons at high velocities

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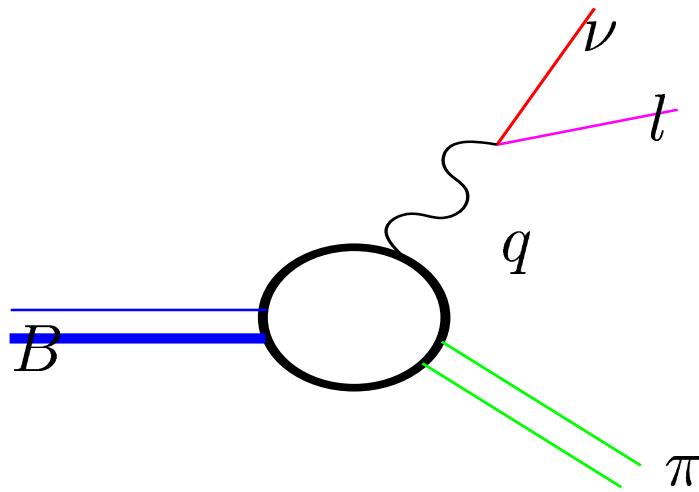
Christine Davies, Alex Dougall (next talk)

University of Glasgow

# *Moving NRQCD: Motivation*

## Exclusive Semileptonic Decays

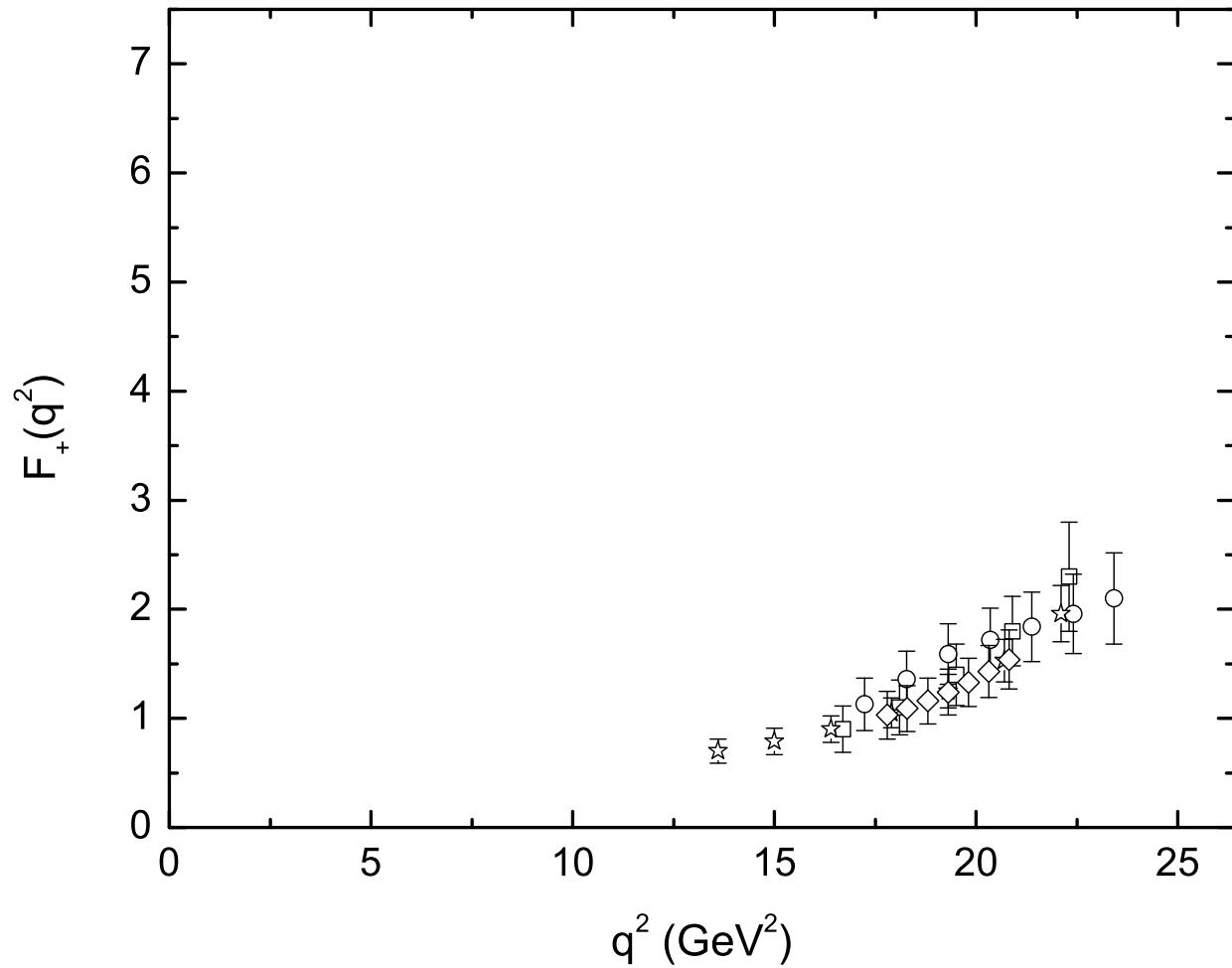
Form Factors needed for CKM elements



- $B \rightarrow \pi l \nu \Rightarrow V_{ub}$
- $D \rightarrow \pi l \nu \Rightarrow V_{cd}$
- $B \rightarrow D l \nu \Rightarrow V_{cb}$

large mass difference  $\Rightarrow$  large recoil momentum likely  
 $\Rightarrow$  high recoil region ( $q^2$  small)

# *Lattice Form Factor Results*



# *Why no large recoil values?*

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- Lattice calculation done in rest frame of initial meson
- for  $B \rightarrow \pi l \nu$ :

$$q^2 \approx 0 \quad \Rightarrow \quad p_\pi \approx 2.6 \text{ GeV}$$

- errors  $\approx (ap)^2$

$$p_\pi \approx 2.6 \text{ GeV} \quad \Rightarrow \quad a^{-1} \gg 3 \text{ GeV}$$

small  $a$  computationally unfeasible

# Better Reference Frame

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## (1) Light Quarks:

- better choice of lattice frame:  $p_\pi \approx \sqrt{\Lambda_{\text{QCD}} M_B}$
- share momentum between initial and final mesons for  $B \rightarrow \pi l \nu$ :

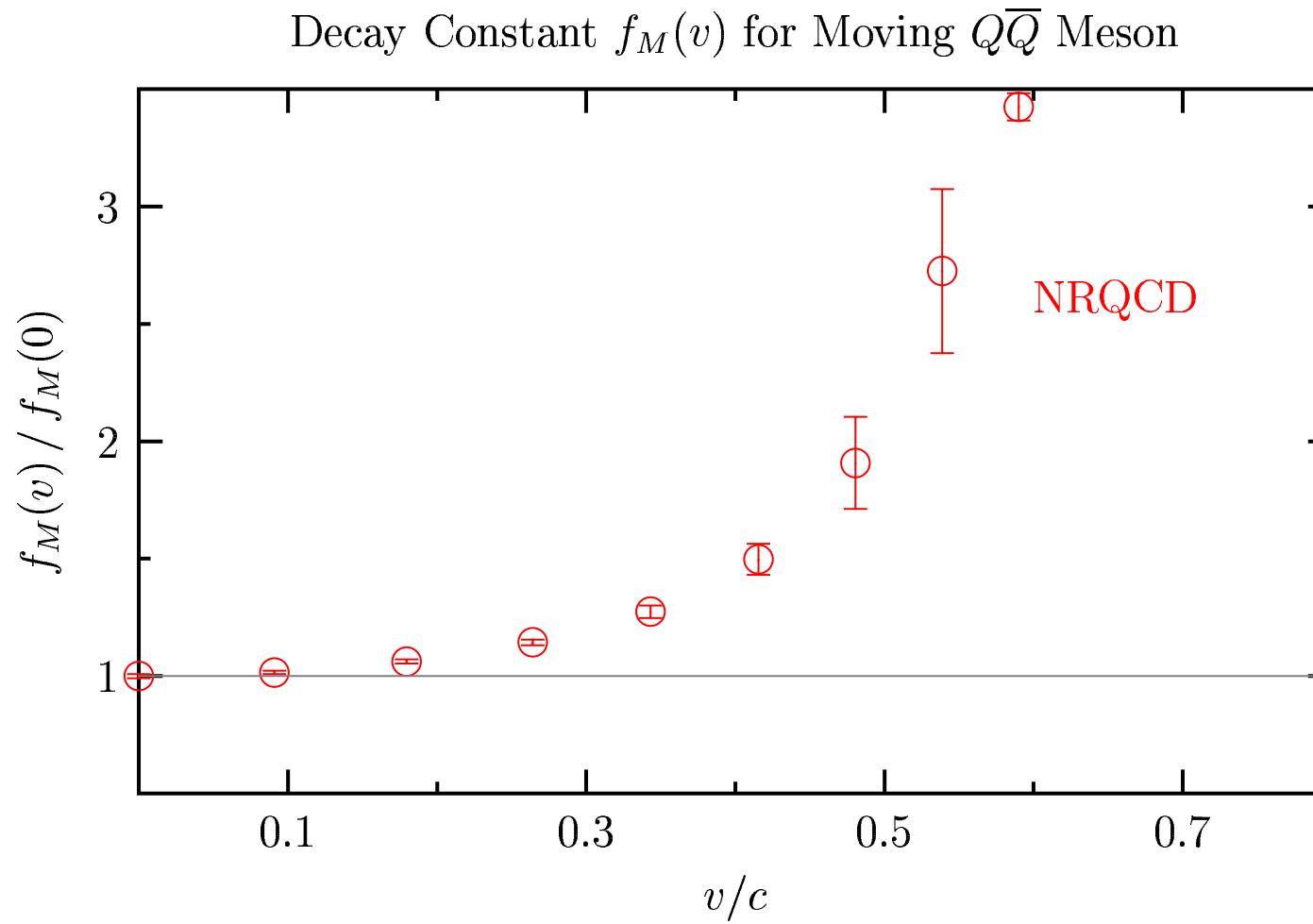
$$p_\pi \approx 800 \text{MeV} \quad \& \quad p_B \approx 8.0 \text{GeV}$$

- discretization errors of all light quarks under control

## (2) heavy quarks:

- $b$  quark has velocity  $v \approx 0.8c$

# Decay Constants: Limits of current methods



Heavy  $b$  Quark momentum:

$$P_b^\mu = M_b u^\mu + k^\mu$$

- $u^\mu$  = 4-velocity of B meson
- $\Rightarrow k^\mu \sim \Lambda_{QCD} u^\mu \ll M u^\mu$
- treat  $M_b u^\mu$  exactly
- discretize  $k^\mu$
- All quarks (heavy and light) have same size errors!

# *moving NRQCD Lagrangian*

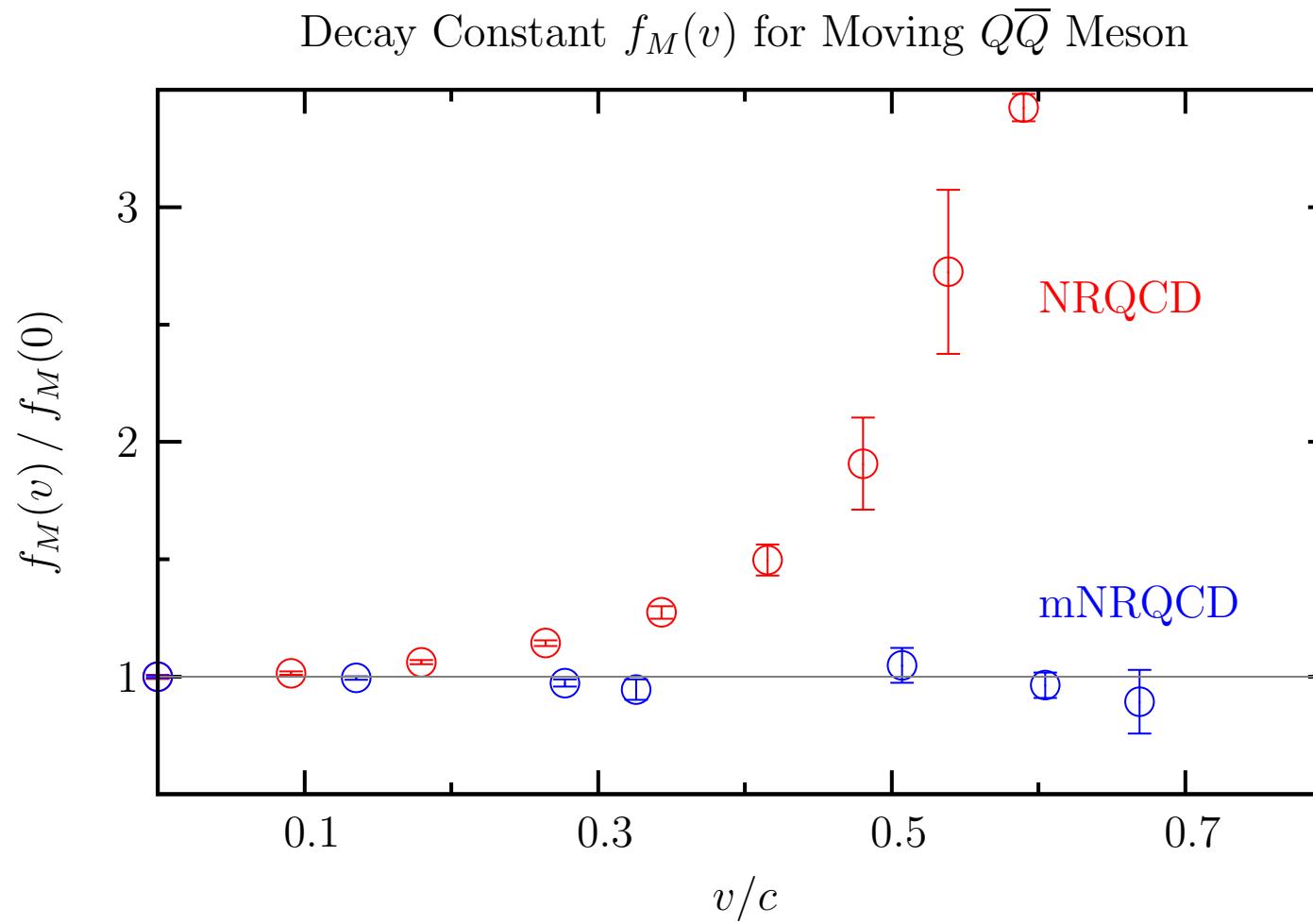
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$$\begin{aligned}
\mathcal{L}_{\text{quark}} &= \psi_Q^\dagger \left( iD_t + i\mathbf{v} \cdot \mathbf{D} + \frac{\mathbf{D}^2}{2\gamma m} - \frac{(\mathbf{v} \cdot \mathbf{D})^2}{2\gamma m} \right) \psi_Q + \mathcal{L}_{Q,K} + \mathcal{L}_{Q,E} + \mathcal{L}_{Q,B} \\
\mathcal{L}_{Q,K} &= \psi_Q^\dagger \left( \frac{i\{\mathbf{D}^2, \mathbf{v} \cdot \mathbf{D}\}}{4\gamma^2 m^2} - \frac{i(\mathbf{v} \cdot \mathbf{D})^3}{2\gamma^2 m^2} + \frac{\mathbf{D}^4}{8\gamma^3 m^3} - \frac{3\{\mathbf{D}^2, (\mathbf{v} \cdot \mathbf{D})^2\}}{8\gamma^3 m^3} + \frac{5(\mathbf{v} \cdot \mathbf{D})^4}{8\gamma^3 m^3} \right) \psi_Q \\
\mathcal{L}_{Q,E} &= \psi_Q^\dagger \left( \frac{[\mathbf{D} \cdot \mathbf{E}]}{8\gamma m^2} - \frac{2 - \mathbf{v}^2 [\mathbf{v} \cdot \mathbf{D}, \mathbf{v} \cdot \mathbf{E}]}{16m^2} - \frac{i\{\mathbf{v} \cdot \mathbf{D}, \boldsymbol{\sigma} \cdot \mathbf{v} \times \mathbf{E}'\}}{\gamma(\gamma+1)m^2} \right. \\
&\quad \left. + \frac{i\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E}' - \mathbf{E}' \times \mathbf{D})}{8\gamma m^2} \right) \psi_Q \\
\mathcal{L}_{Q,B} &= \psi_Q^\dagger \left( \frac{\boldsymbol{\sigma} \cdot \mathbf{B}'}{2\gamma m} + \frac{i\{\mathbf{v} \cdot \mathbf{D}, \boldsymbol{\sigma} \cdot \mathbf{B}'\}}{4\gamma^2 m^2} - \frac{\mathbf{v}^2 [\mathbf{v} \cdot \mathbf{D} \times \mathbf{B}]}{16m^2} \right) \psi_Q
\end{aligned}$$

- single first order time derivative  $\Rightarrow$  FAST

see also: Sloan; Hashimoto & Matsufuru; Mandula & Ogilvie

# Decay Constant: Comparison of *m*NRQCD & NRQCD



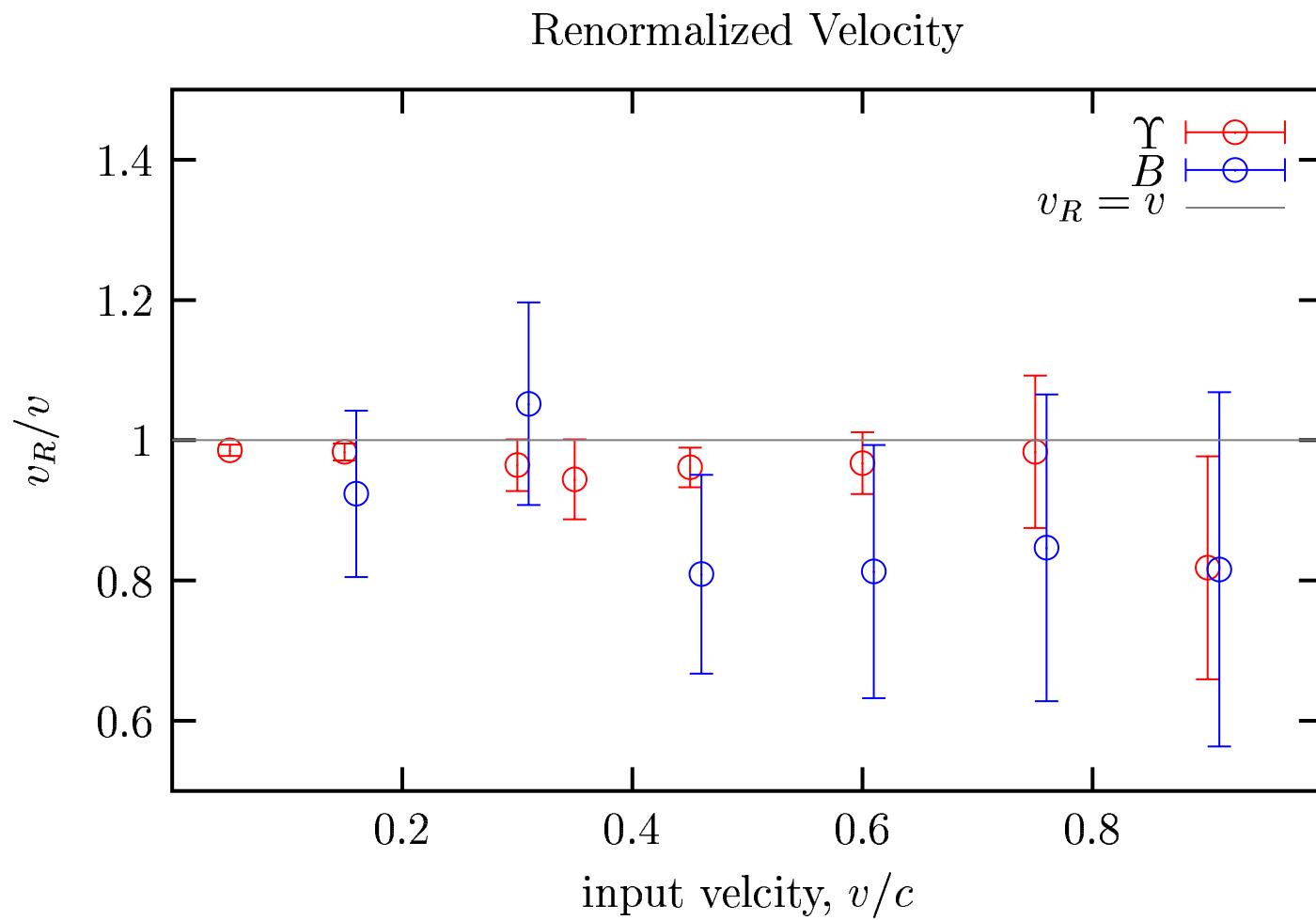
# *Velocity Renormalization*

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New feature of moving NRQCD

- input  $v$  not necessarily simulated meson  $v_R$   
    ⇒ due to breaking Lorentz invariance
- Must find renormalized velocity
  - perturbatively
  - non-perturbatively  
        same in both  $\Upsilon$  and  $B$  systems  
        use  $\Upsilon$  to tune

# Velocity Renormalization Results



# **Conclusions**

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Calculation of high recoil form factors now possible  
(add to list of high precision results)

- $B$  and  $\Upsilon$  mesons have been analyzed
  - velocity renormalization
  - kinetic energy
  - ground state energy
- 1-loop perturbative results (Next talk!)

In Progress:

- optimized “magic” smearing
  - ⇒ improve statistics at large  $v$
- Form factors at high recoil in near future!