Improving Algorithms to Compute All Elements of the Lattice Quark Propagator (II)

TrinLat Collaboration

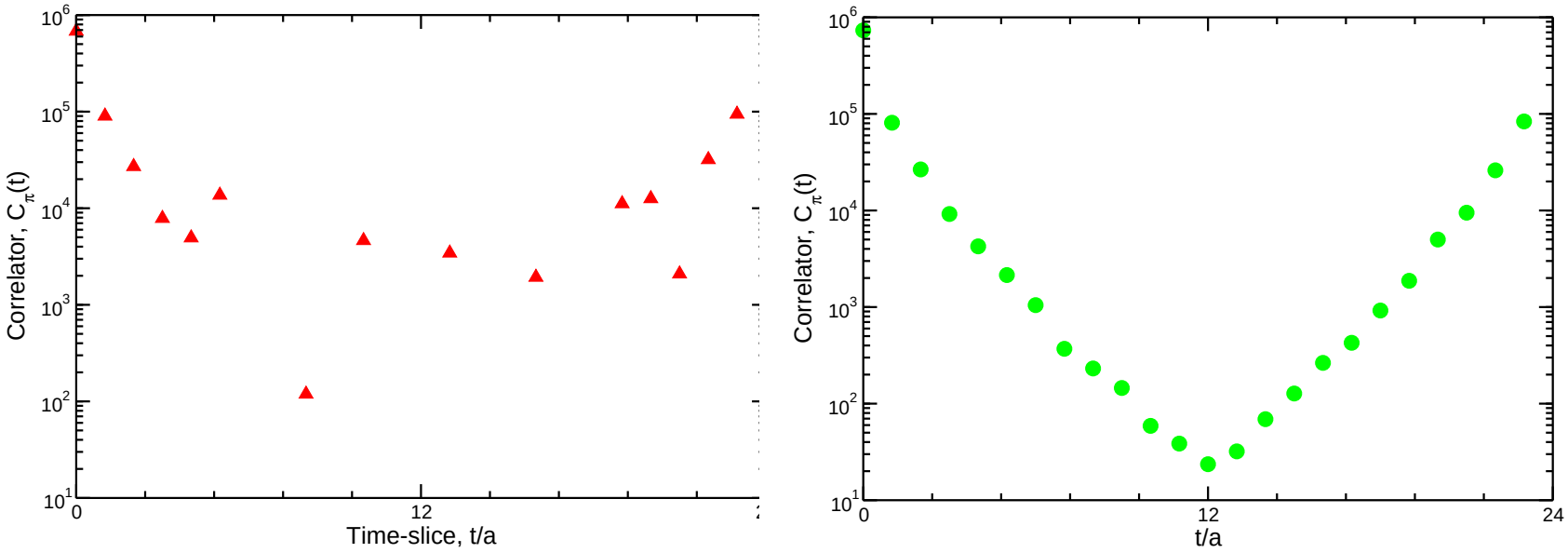
Alan Ó Cais, Jimmy Juge, Mike Peardon, Sinéad Ryan, Jon-Ivar Skullerud

Trinity College, Dublin

Outline

- Why should dilution work?
- Error behaviour for 2 quark masses
- Comparison with a conventional noise method
- Signal for other states
- Conclusions and outlook

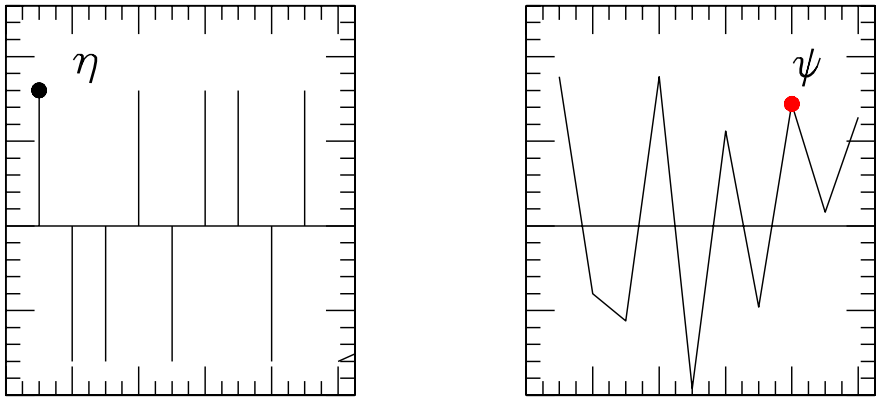
The effect of time dilution



The pion correlation function without, and then with time dilution.

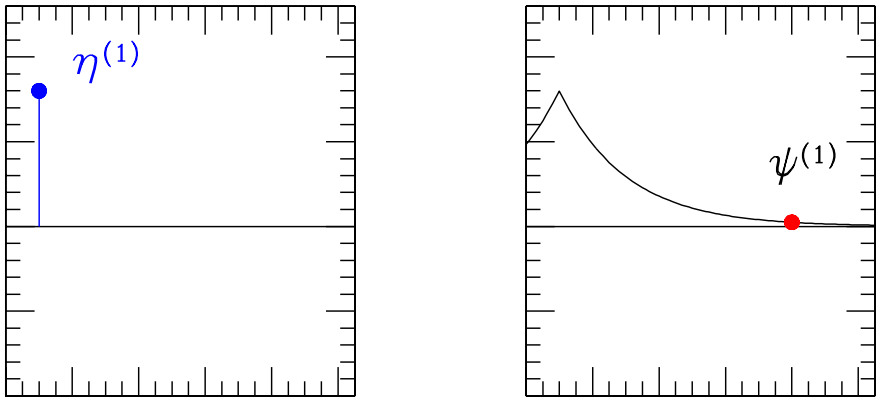
Motivation for dilution

- An undiluted noise vector, η has elements of $O(1)$
- The fermion matrix is diagonally dominant, so the solution to $M\psi = \eta$ is also $O(1)$
- The signal decays in t as $e^{-\lambda t}$, so we are attempting to extract a signal of $O(e^{-\lambda t})$ by cancellation (across our ensemble) of the contraction of elements of $O(1)$
- Time dilution of η means that $\psi(t)$ also decays in time from the dilution source timeslice. So, upon contraction the signal and the elements we extract it from are of comparable order



no dilution

diluted



Terminology and Parameters

- Dilution notation

  Dilution of η is its decomposition into subspaces

  according to the dilution method.

  We investigate dilution in

  $T = time$, $S = space$, $C = colour$ and $G = spin$

  The dilutions used in these degrees of freedom can be

  $f = full$, $e = even\_odd$ or $x = None$

- Equal Cost

  In this simulation equal computational cost is taken to mean an equal number of conjugate gradient inversions of the fermion matrix.

  Full spin dilution $= 4 \times \mathrm{N_T}$ CG inversions

  Equal cost to 4 time diluted noise sources

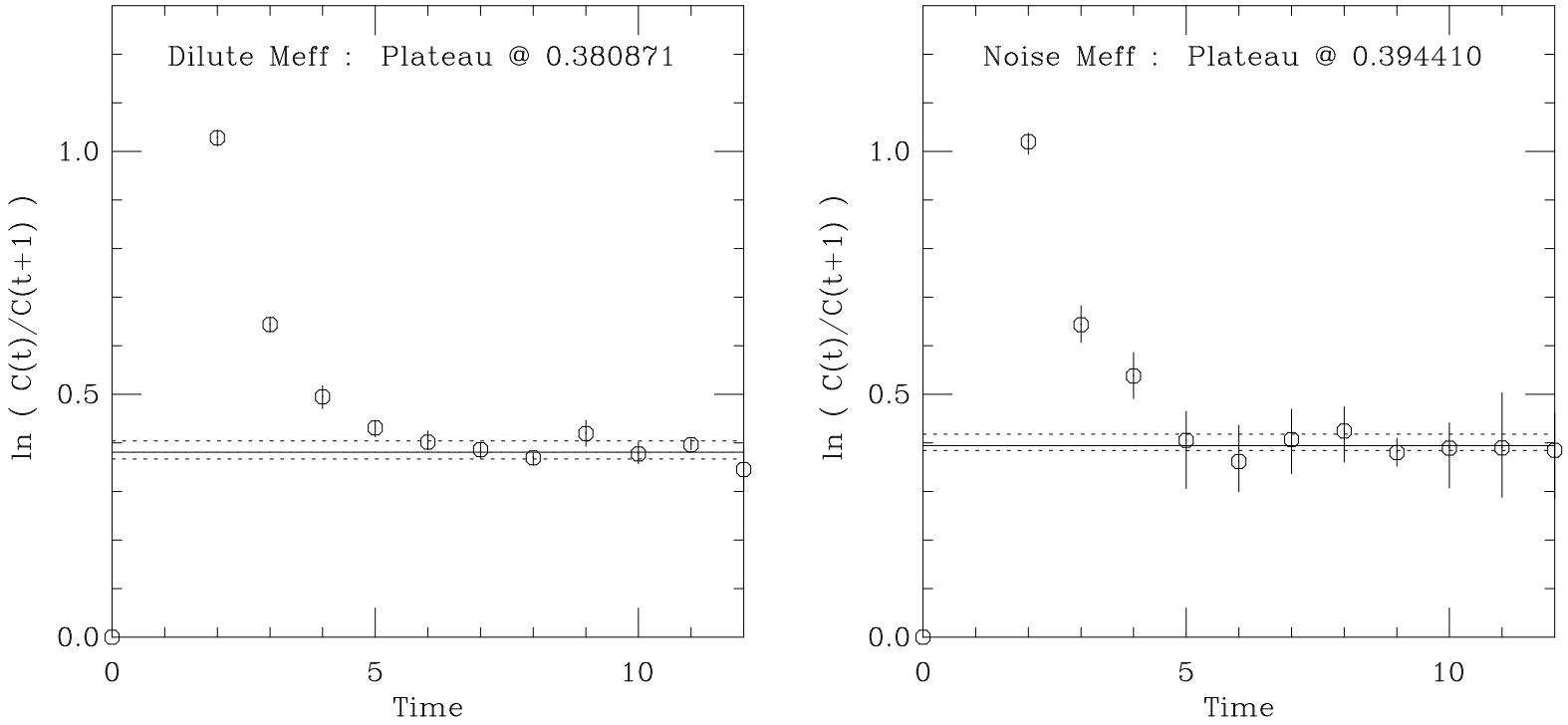
- Simulation Parameters

  Lattice Size $(12^3 * 24)$, 10 gauge configurations, Wilson action

  $\beta = 5.7$

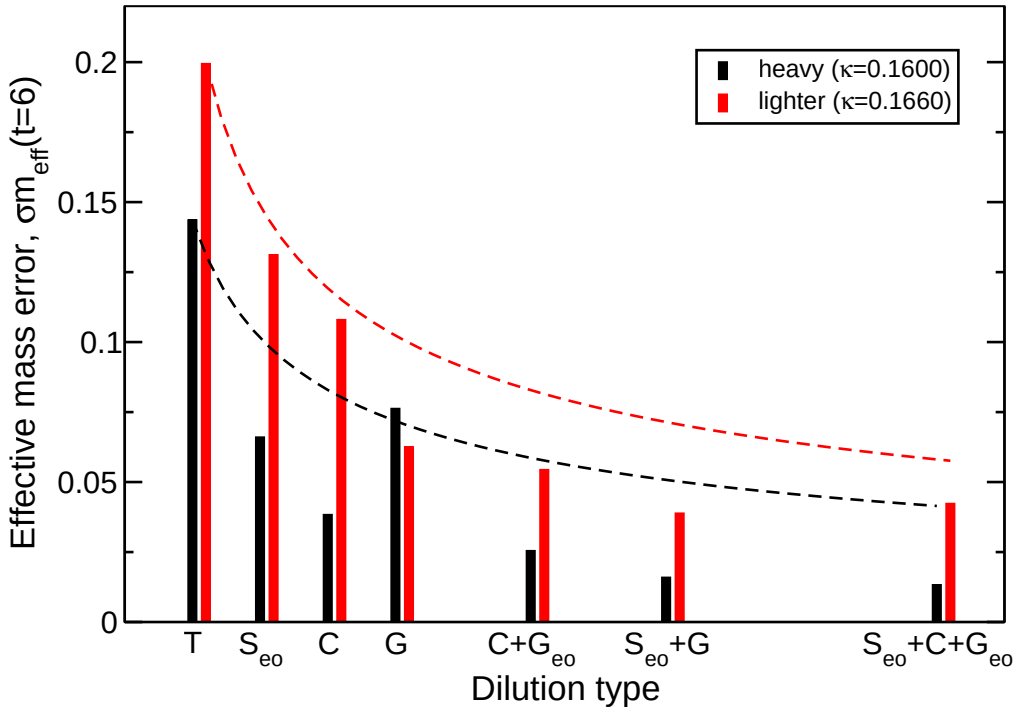
κ	M_π / M_ρ
0.1600	0.8625
0.1663	0.6129

Exploring further dilution



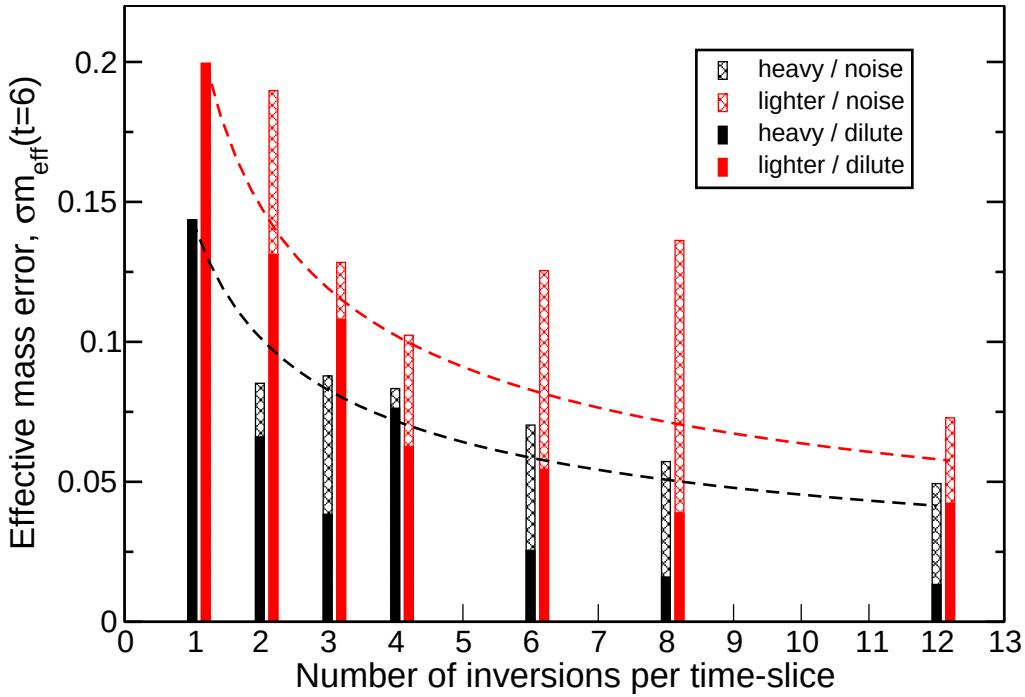
Pion effective mass plots for dilution in time, spin and even-odd in space, and for 8 time diluted noise vectors at $M_\pi/M_\rho = 0.6129$.

Error behaviour of Dilution



- Better than $1/\sqrt{N}$ behaviour
- No obvious dilution dependant behaviour

Equal Cost Comparison of Dilution



- Consistently better error behaviour for dilution
- Just time dilution gives worse than $1/\sqrt{N}$ behaviour

There are other particles besides pions ...

Particles which are difficult to get a good signal without all-to-all propagators

- P-waves (displace ψ and $\bar{\psi}$)
- hybrids ($1^{-+} \longrightarrow$ P-wave for heavy quarks)
- static-lights and f_B

  large gain in statistics

  f_B from point/smeared sources and sinks.

  (see also S. Ryan's talk)

Meson/Hybrid Operators

Quark Model

$P = (-1)^{L+1}$ Parity

$C = (-1)^{L+S}$ Charge Conjugation

1^{+-} $(^1\mathrm{P}_1)$ $(L = 1, S = 0)$

$$\mathcal{O}(\vec{x}, t) = \bar{\psi}(\vec{x}, t)\gamma_5 \vec{D}\psi(\vec{x}, t)$$

1^{++} $(^3\mathrm{P}_1)$ $(L = 1, S = 1)$

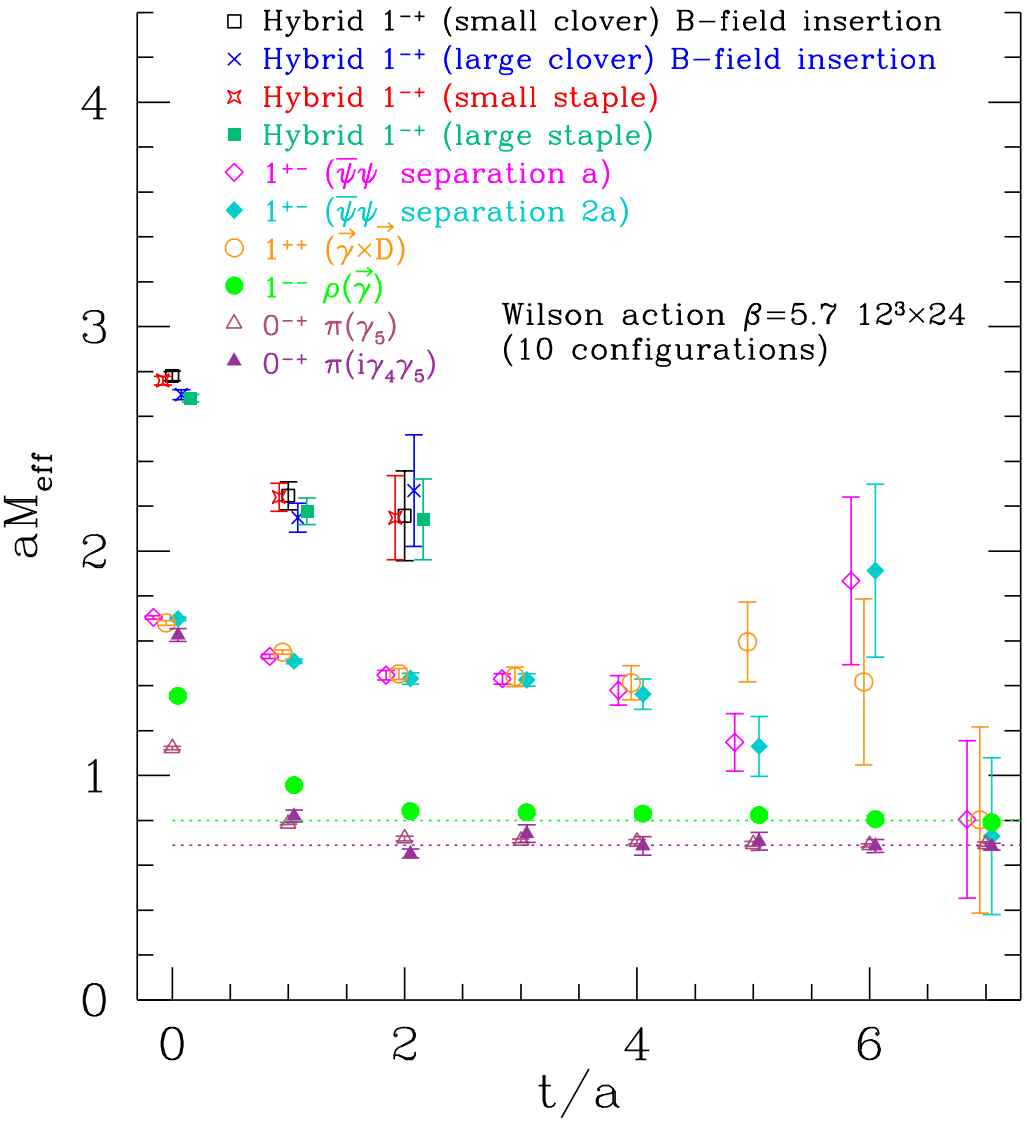
$$\mathcal{O}(\vec{x}, t) = \bar{\psi}(\vec{x}, t)\vec{\gamma} \times \vec{D}\psi(\vec{x}, t)$$

1^{-+} exotic *hybrid*

$$\mathcal{O}(\vec{x}, t) = \bar{\psi}(\vec{x}, t)\vec{\gamma} \times \vec{B}(\vec{x}, t)\psi(\vec{x}, t)$$

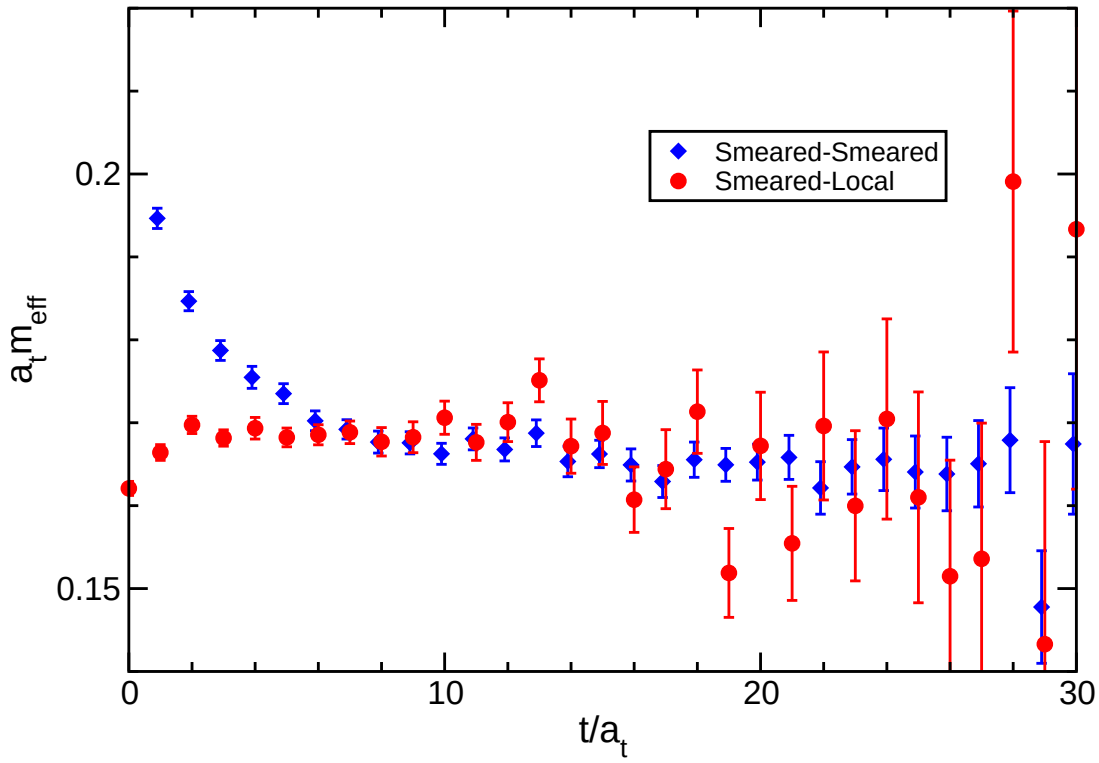
$$\mathcal{O}(\vec{x}, t) = \bar{\psi}(\vec{x} + \vec{R}, t)\vec{\gamma} \sum_i \mathrm{P_i}(\vec{x} + \vec{R}, \vec{x})\psi(\vec{x}, t)$$

where P is the "staple" path from ψ to $\bar{\psi}$ and B is the magnetic field


Hybrid 1⁻⁺ (small clover) B−field insertion
Hybrid 1⁻⁺ (large clover) B−field insertion
Hybrid 1⁻⁺ (small staple)
Hybrid 1⁻⁺ (large staple)
1⁺⁻ ($\bar{\psi}\psi$ separation a)
1⁺⁻ ($\bar{\psi}\psi$ separation 2a)
1⁺⁺ ($\vec{\gamma}\times D$)
1⁻⁻ $\rho(\vec{\gamma})$
0⁻⁺ $\pi(\gamma_5)$
0⁻⁺ $\pi(i\gamma_4\gamma_5)$
Wilson action β=5.7 $12^3\times24$
(10 configurations)
aM_{eff}
t/a
0
1
2
3
4
0
2
4
6

Static-Light Meson

Correlator has one quark propagator, this is noisy wih point source methods. Time dilution only (so far ...)



Noise Recycling

For pairs of noise/solution vectors

$$\eta_1, \eta_2, \eta_3 \ldots \tag{1}$$

$$\psi_1, \psi_2, \psi_3 \ldots \tag{2}$$

where $M\psi_i = \eta_i$, we contract a pion propagator via

$$\psi_a^* \gamma_5 \eta_b \qquad\qquad a \neq b \tag{3}$$

If all pairs are stored then we can contract all possible combinations of a, b $(a \neq b)$ with a gain of $N(N-1)/2 \sim N^2$ in statistics for N pairs (N large).

Conclusions

- This preliminary study suggests greater than $1/\sqrt{N}$ improvement of variance for increasing number of dilution vectors, N.
- Dynamical gauge configurations very expensive. Need method to obtain maximal information from these configurations
- This method also appears to give signals for conventionally noisy particles on just 10 configurations. It also increases the available spectrum of particles
- The method is easily implemented and expandable and is an almost ‘black-box‘ technique

Outlook

- Compare technique to other variance reduction methods
- Search for dilution dependence in signals
- Use of Symanzik improved actions
- A full-scale simulation on a greater number of gauge configurations.
- Increase the spectrum of particles.

  disconnected diagrams ...
- Couple with truncated eigenvector decomposition and noise-recycling techniques