Results for light pseudoscalars from three-flavor simulations

MILC Collaboration

$+$ (for part) HPQCD, UKQCD Collaborations

Collaborators

MILC Collaboration:

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Motivation

- Compute decay constants f_π and f_K: check of lattice methods, staggered fermions, $\sqrt[4]{\mathrm{Det}}$.
- If methods ok, then f_K or f_K/f_π can be used to find V_{us}.
- With perturbative mass renormalization, can compute m_s and $\hat{m} \equiv (m_u + m_d)/2$.
    - Ratio $m_s/\hat{m}$ independent of renormalization, but, at our level of errors, requires some control of electromagnetic (EM) and isospin-violating effects.
    - m_u/m_d can also be extracted, but is sensitive to EM effects (biggest error). Can answer question: Is $m_u = 0$ solution to strong CP problem viable?
- Several $\mathcal{O}(p^4)$ low energy constants (Gasser-Leutwyler constants) can be extracted: L_5, L_4, $2L_8 - L_5$, $2L_6 - L_4$. These constants important for phenomenology. $2L_8 - L_5$ provides another way of getting at m_u.

Lattice Data: $N_F = 3$ Improved Staggered

$a\hat{m}'$ / am'_s	$10/g^2$	*dims.*	# *lats.*	m_π/m_ρ
0.03 / 0.05	6.81	$20^3 \times 64$	262	0.37787(18)
0.02 / 0.05	6.79	$20^3 \times 64$	485	0.31125(16)
0.01 / 0.05	6.76	$20^3 \times 64$	608	0.22447(17)
0.007 / 0.05	6.76	$20^3 \times 64$	447	0.18891(20)
0.005 / 0.05	6.76	$24^3 \times 64$	137	0.15971(20)
0.00124 / 0.031	7.11	$28^3 \times 96$	531	0.20635(18)
0.00062 / 0.031	7.09	$28^3 \times 96$	583	0.14789(18)

Parameters of the coarse ($a \approx 0.125$ fm) and fine ($a \approx 0.09$ fm) lattices. m'_s, $\hat{m}' \Rightarrow$ simulation masses. Physical values are m_s, $\hat{m}$. $m'_s/m_s = 1.09$–1.28 (coarse) and 1.07–1.14 (fine). Volumes are all $\approx (2.5\,\mathrm{fm})^3$, except for $\approx (3.0\,\mathrm{fm})^3$ on coarse .005/.05 run.

Lattice Data

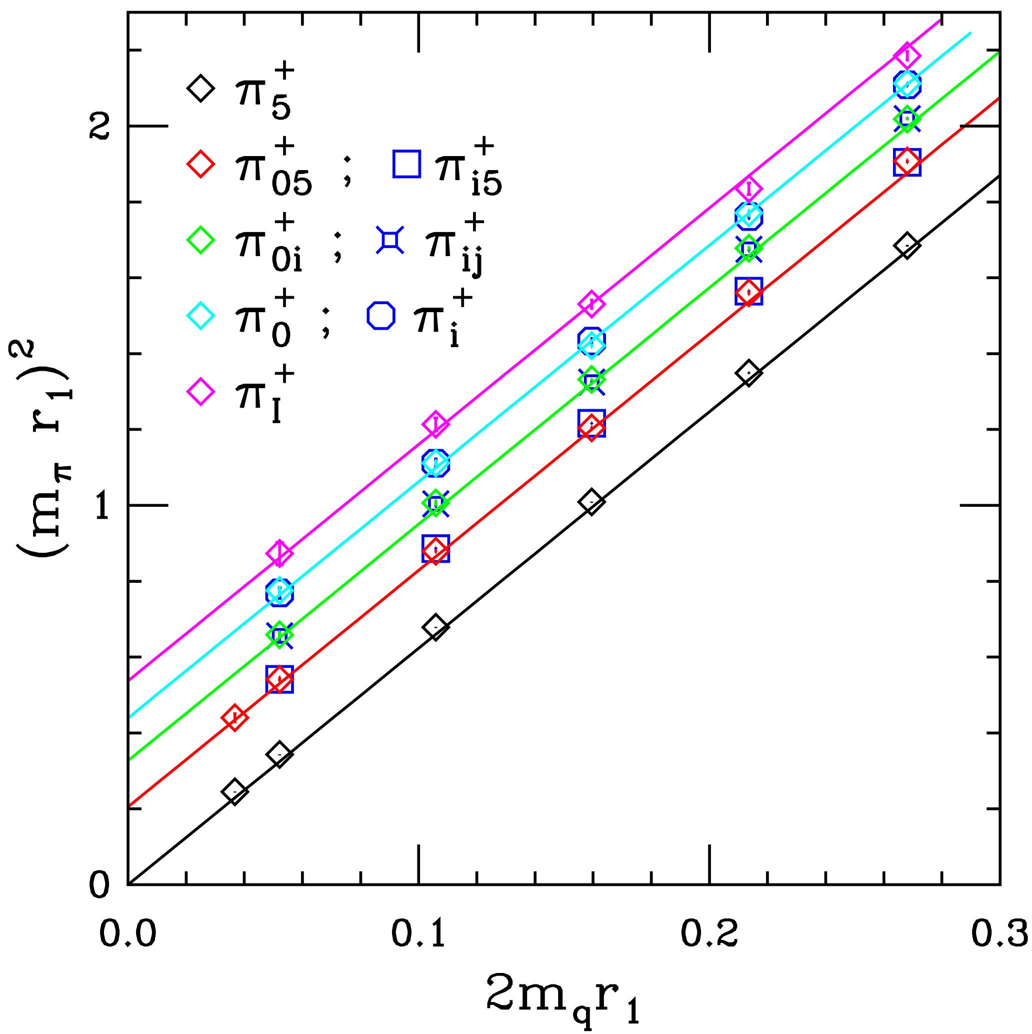
- For Goldstone (taste ξ_5) masses and decay constants have extensive partially quenched data:
  - Coarse: all combos of 9 valence masses between $0.1m_s'$ and m_s'.
  - Fine: all combos of 8 valence masses between $0.14m_s'$ and m_s'.
- For other tastes, have most full QCD pion masses and a few full QCD kaon masses, but no decay constants and no partially quenched data.

Chiral fits

- For precise results, and to extract L_i, need to include chiral logs at NLO.
- Large taste violations at finite $a \Rightarrow$ must use “staggered chiral perturbation theory” (SχPT) to control chiral & continuum extrap [Lee & Sharpe, Aubin and CB].
- Our data set and existing SχPT calculations $\Rightarrow$ chiral log fits for Goldstone masses and decay constants only.
  - Require, for input into chiral loops, the masses of mesons with other tastes.
  - Can get those splittings from tree level SχPT fit to full QCD data for pions of all tastes.
  - $\Rightarrow$ NNLO error, estimated to be well under 1%.

Tree level (LO) SχPT fit

- For coarse lattice, biggest taste violations are $\gtrsim 100\%$ at lowest masses.



Fit looks good, but has terrible confidence level (CL), since statistical errors tiny.

Still, gets squared masses usually within 2%, and no worse than 7% (for lighest Goldstone pions).

Data Subsets

- To get good fits to SχPT forms, need to place upper limit on valence quark masses (m_x, m_y).
- Consider 3 data subsets:
  - Subset I: $m_x + m_y \leq 0.40 m_s'$ (coarse);
    $m_x + m_y \leq 0.54 m_s'$ (fine). 94 data points.
  - Subset II: $m_x + m_y \leq 0.70 m_s'$ (coarse);
    $m_x + m_y \leq 0.80 m_s'$ (fine). 240 data points.
  - Subset III: $m_x + m_y \leq 1.10 m_s'$ (coarse);
    $m_x + m_y \leq 1.14 m_s'$ (fine). 416 data points.
- Can tolerate heavier valence masses (compared to m_s') on fine lattices, since m_s'/m_s is smaller and contributions to meson masses from taste splittings are smaller.
- Can't similarly limit sea quark masses: m_s' fixed on coarse or fine, and is not small. $\Rightarrow$ adjusting $m_s' \to m_s$ gives approx half of total chiral extrap/interp error.

Chiral Log Fits

- On subset I, maximum valence-valence Goldstone mass is $\approx 350\,\mathrm{MeV}$.
- Adding on average taste splitting gives $\approx 500\,\mathrm{MeV}$. (Maximum taste splitting gives $\approx 580\,\mathrm{MeV}$.)
- Expect errors of NLO SχPT to be of order:

$$\left(\frac{(500\,\mathrm{MeV})^2}{8\pi^2 f_\pi^2}\right)^2 \approx 3.5\%$$

- Statistical errors of data: 0.1% to 0.7% (squared masses); 0.1% to 0.4% (decay constants)
- $\Rightarrow$ NNLO needed.
- NNLO SχPT logs unknown. But for high masses, NNLO logs should be smoothly varying, well approximated by NNLO analytic terms

Chiral Log Fits

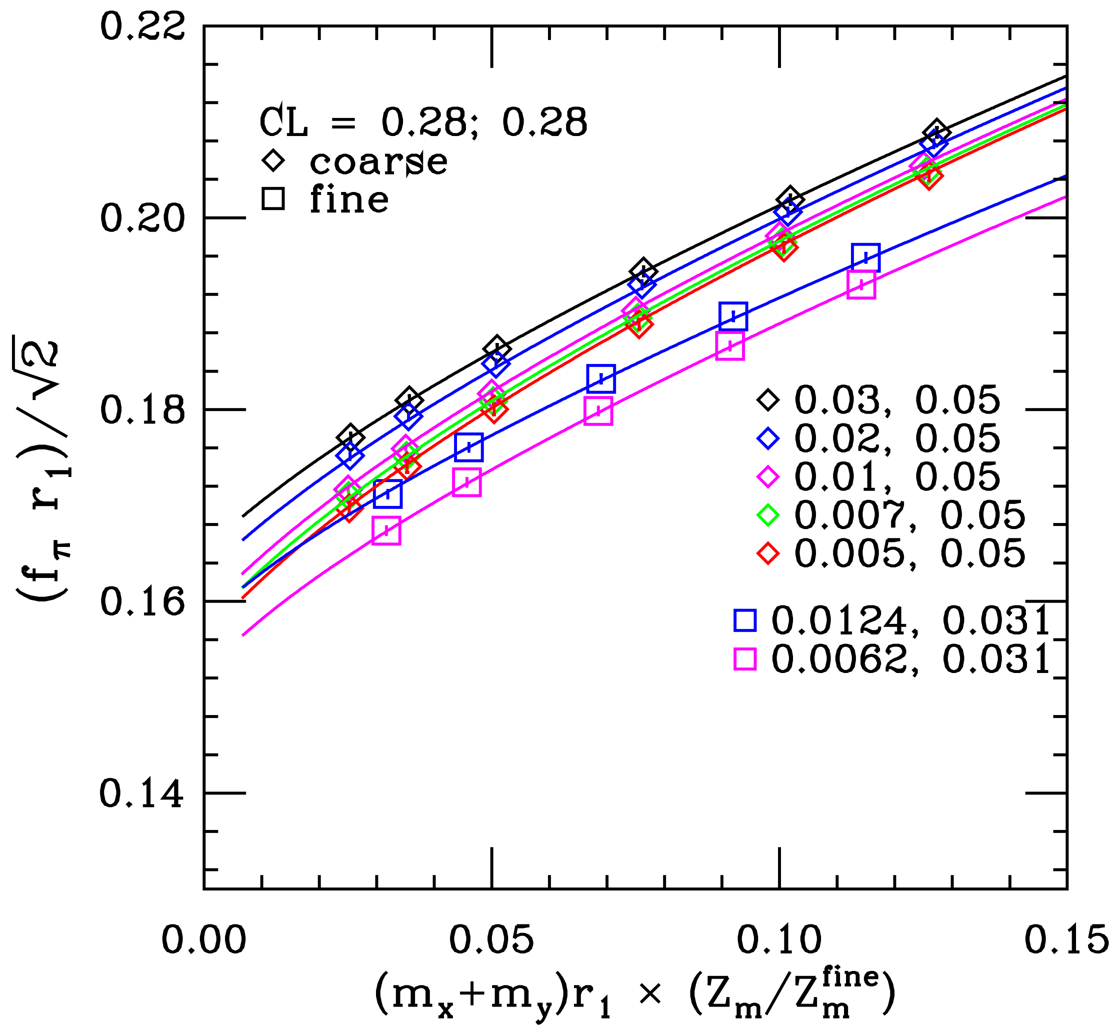
- Fit decay constants and masses together; include all correlations.
- Fit coarse and fine lattices together.
- NNLO fit has 20 unconstrained params:
  - 2 (LO)
  - 4 (physical NLO: L_i)
  - 4 (taste violating NLO: $\mathcal{O}(a^2)$)
  - 10 (NNLO analytic)
- Additional 16 tightly constrained params allow for variation of physical params with a ($\sim \alpha_S a^2 \Lambda^2_{\rm QCD} \approx 2\%$)
- Add 4 more tightly constrained params to allow scale determinations to vary within statistical errors
- Total of 40 params; corresponding "continuum NNLO fit" has 36.

Chiral Log Fits

- Get good NNLO fits for subsets I and II.
- Used for finding L_i.
- In subset III, even NNLO fits break down.
- But want subset III to interpolate around m_s .
- $\Rightarrow$ in subset III, fix LO and NLO terms from lower mass fits; then add on *ad hoc* additional higher order terms to get good interpolation around m_s.
- Use such fits in subset III for central values of quark masses & decay constants; results of subsets I, II are included in systematic error estimates.

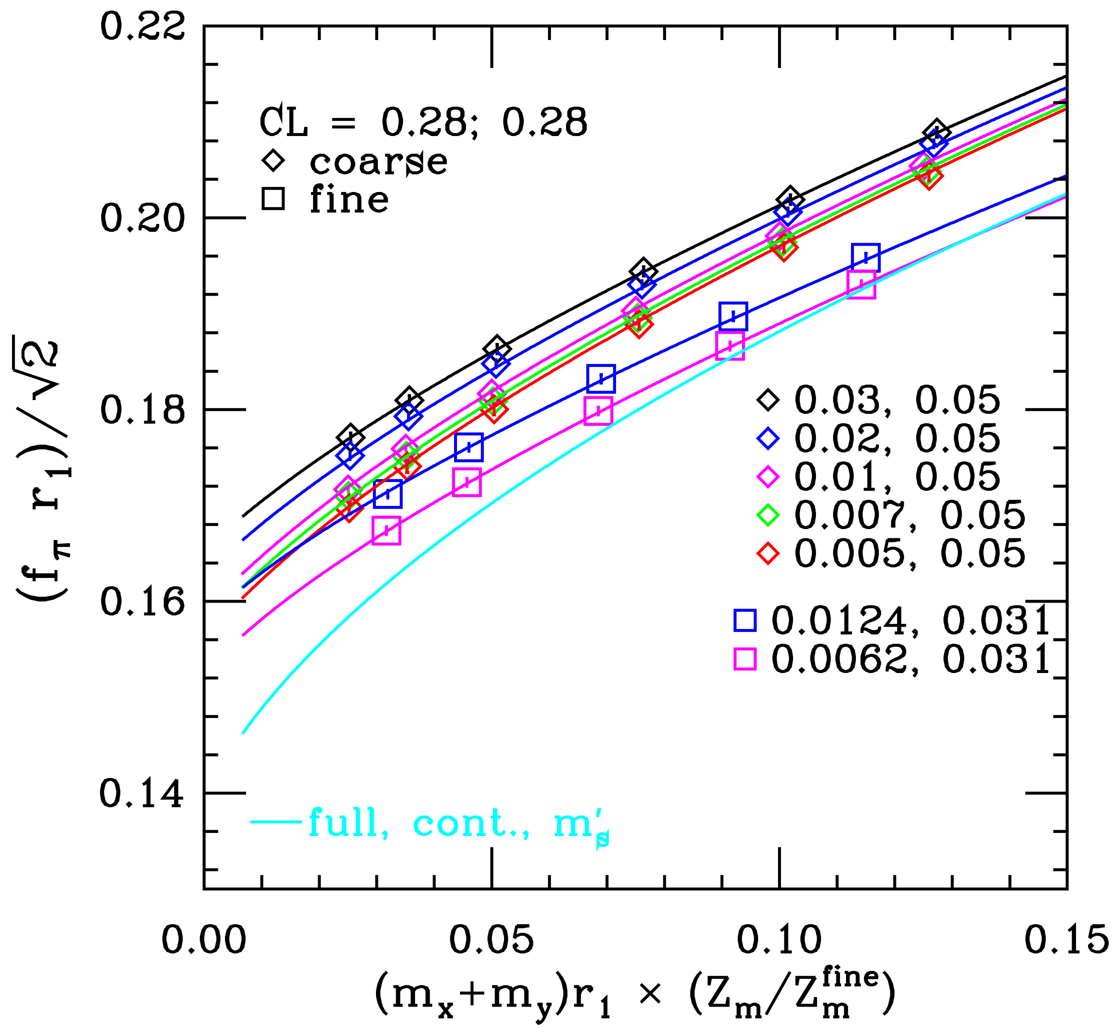
Fit of f_π

- Fit partially quenched f_π with taste violations.



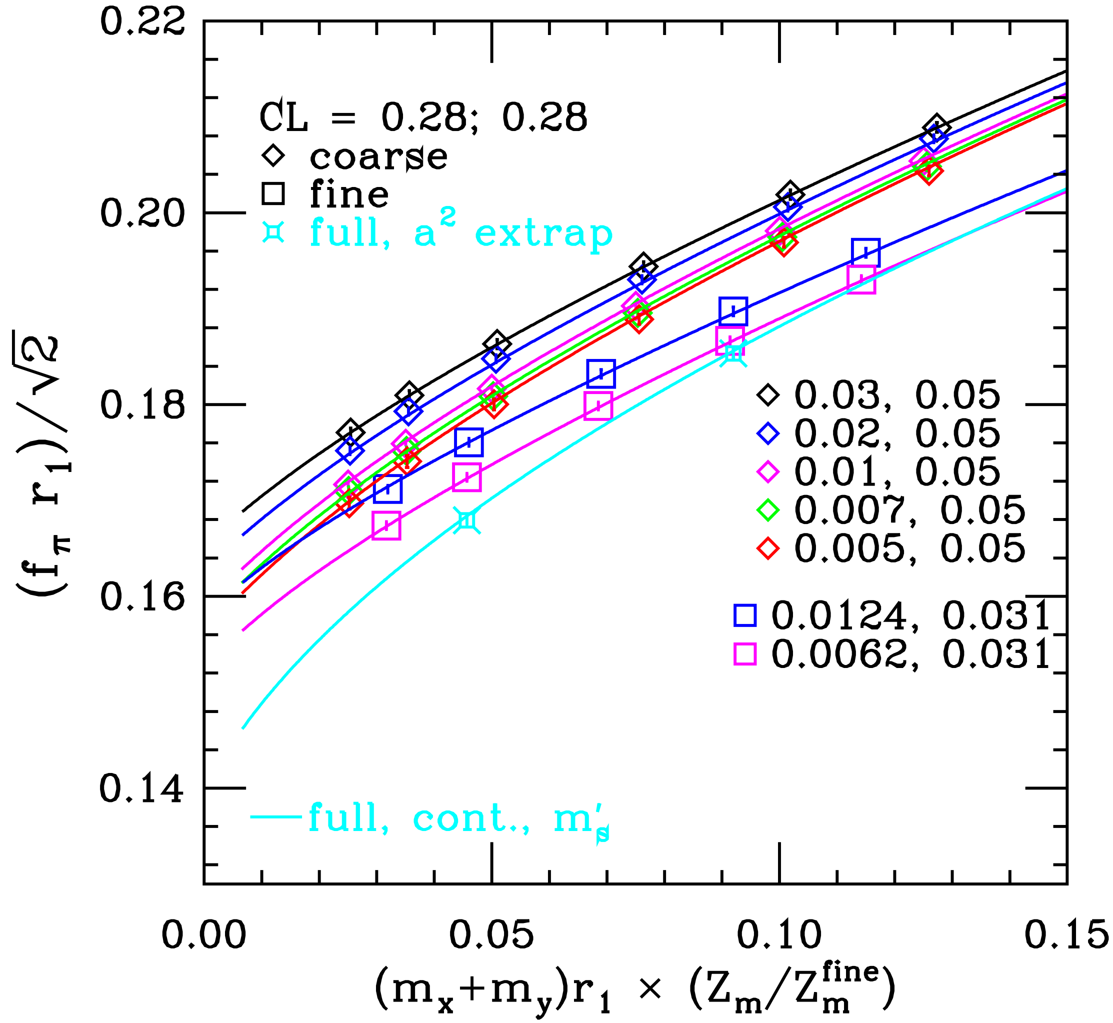
Fit of f_π

- Extrapolate fit params to continuum
- Go to "full QCD:" Set $\hat{m}'_{sea} = \hat{m}'_{val}$ and plot a function of $\hat{m}'_{val}$: ———



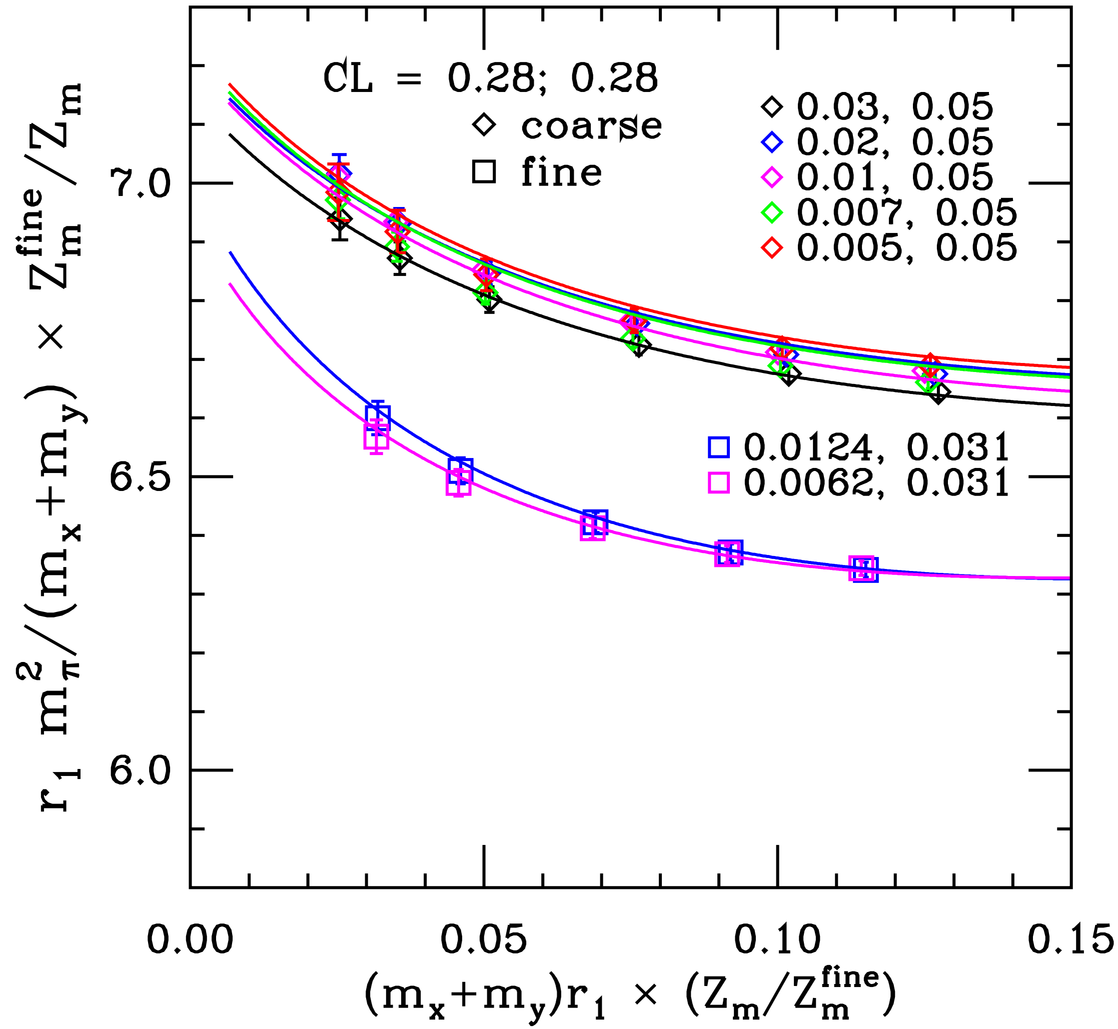
Fit of f_π

- Consistency check: extrapolate points with sea masses = valence masses to continuum at fixed quark mass.



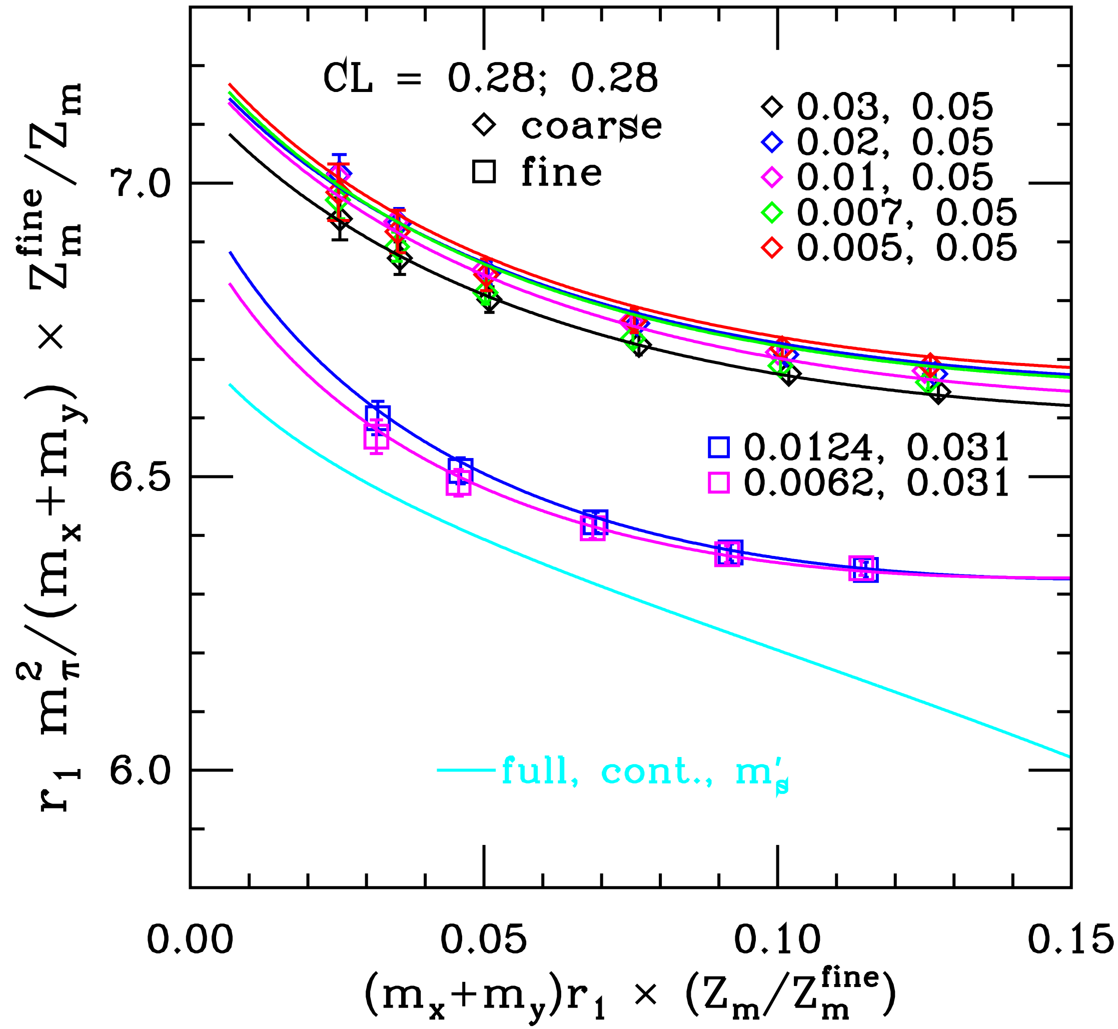
Fit of $m_\pi^2/(m_x + m_y)$

- Fit partially quenched $m_\pi^2/(m_x + m_y)$ with taste violations.



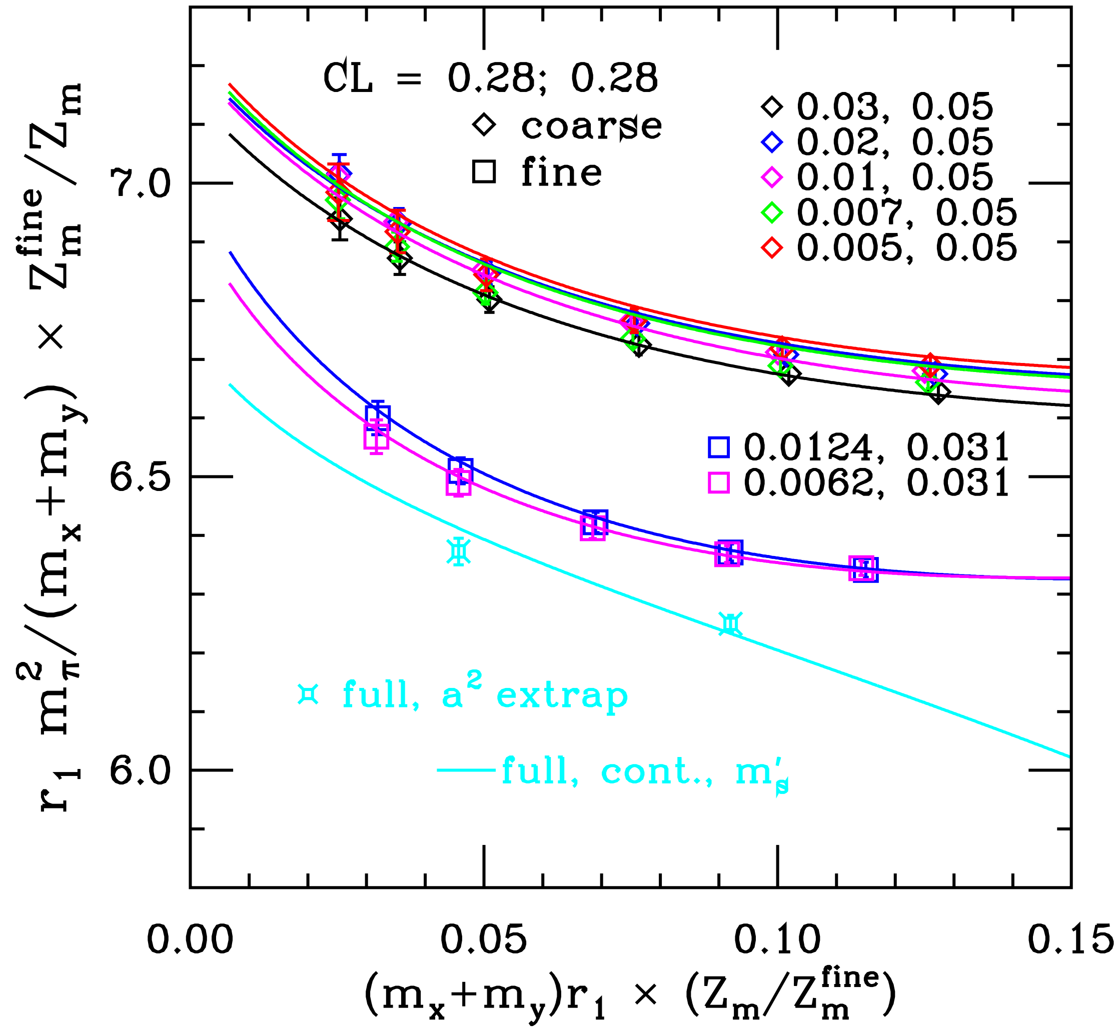
Fit of $m_\pi^2/(m_x+m_y)$

- Extrapolate fit params to continuum
- Go to "full QCD:" Set $\hat{m}'_{sea} = \hat{m}'_{val}$ and plot a function of $\hat{m}'_{val}$: ———



Fit of $m_\pi^2/(m_x + m_y)$

- Consistency check: extrapolate points with sea masses = valence masses to continuum at fixed quark mass.



Electromagnetism & Isospin Violations

- Now find physical quark masses by extrapolating to physical meson masses.
- Some control of electromagnetic (EM) and isospin-violating effects is necessary at the precision of the current calculation.
- Distinguish among meson masses with & without these effects:
  - Experimental masses:
    $m_{\pi^0}^{\text{expt}}$, $m_{\pi^+}^{\text{expt}}$, $m_{K^0}^{\text{expt}}$, $m_{K^+}^{\text{expt}}$
  - Masses with EM effects turned off:
    $m_{\pi^0}^{\text{QCD}}$, $m_{\pi^+}^{\text{QCD}}$, $m_{K^0}^{\text{QCD}}$, $m_{K^+}^{\text{QCD}}$
  - Masses with EM effects turned off and $m_u = m_d = \hat{m}$:
    $m_{\hat{\pi}}$, $m_{\hat{K}}$

Electromagnetism & Isospin Violations

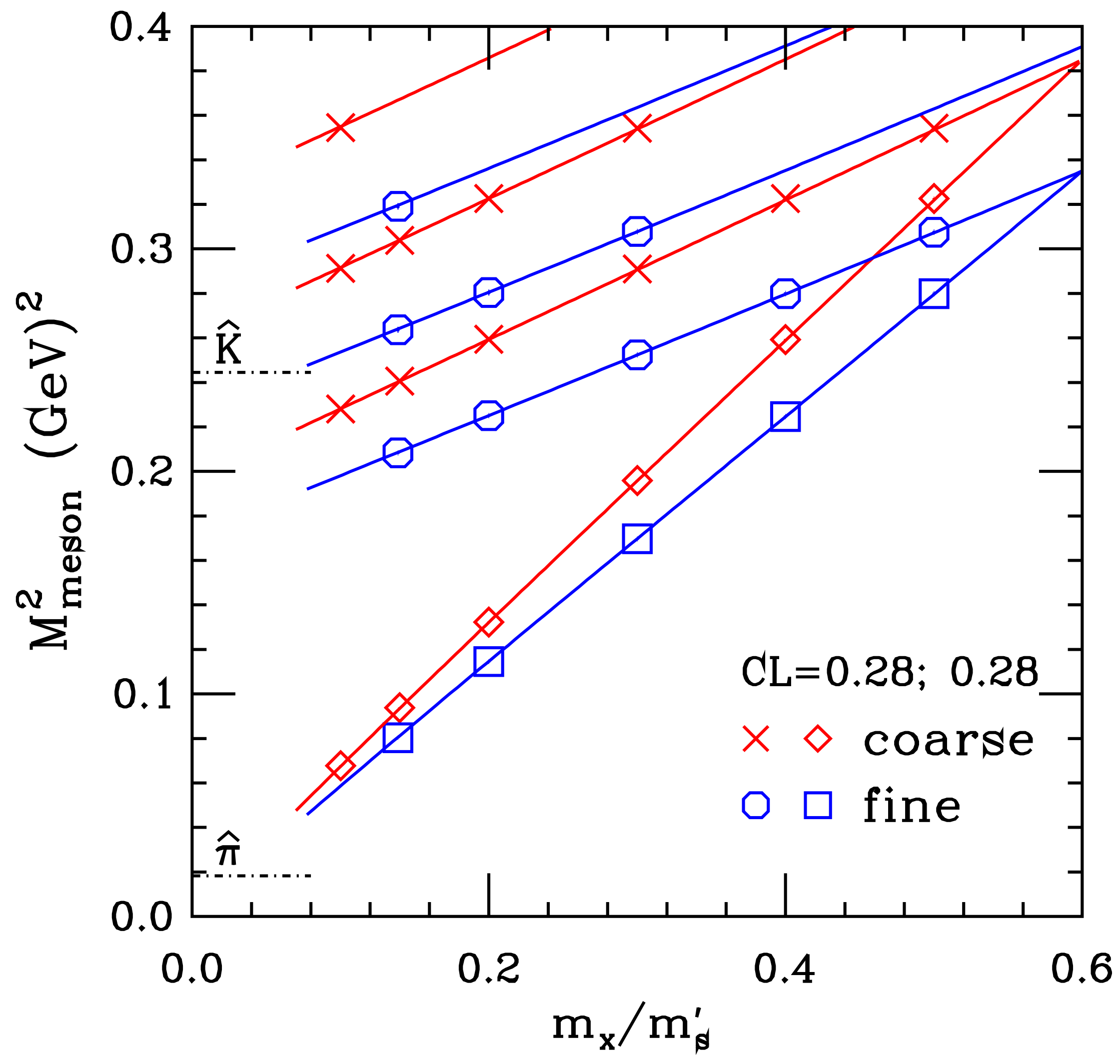
- Bottom line of standard continuum χPT with EM:

$$
\begin{aligned}
m_{\hat{\pi}}^2 &\approx (m_{\pi^0}^{\mathrm{QCD}})^2 \approx (m_{\pi^0}^{\mathrm{expt}})^2 \\
m_{\hat{K}}^2 &\approx \frac{(m_{K^0}^{\mathrm{QCD}})^2 + (m_{K^+}^{\mathrm{QCD}})^2}{2} \\
(m_{K^0}^{\mathrm{QCD}})^2 &\approx (m_{K^0}^{\mathrm{expt}})^2 \\
(m_{K^+}^{\mathrm{QCD}})^2 &\approx (m_{K^+}^{\mathrm{expt}})^2 - (1+\Delta_E)\left((m_{\pi^+}^{\mathrm{expt}})^2 - (m_{\pi^0}^{\mathrm{expt}})^2\right)
\end{aligned}
$$

- $\Delta_E = 0$ is “Dashen’s theorem.”
- Phenomenology: $\Delta_E \approx 1$.
- To be conservative, we take $0 \leq \Delta_E \leq 2$.
- More aggressively, we could use $\Delta_E = 0.84(25)$ from J. Bijnens and J. Prades, Nucl. Phys. B **490**, 239 (1997).

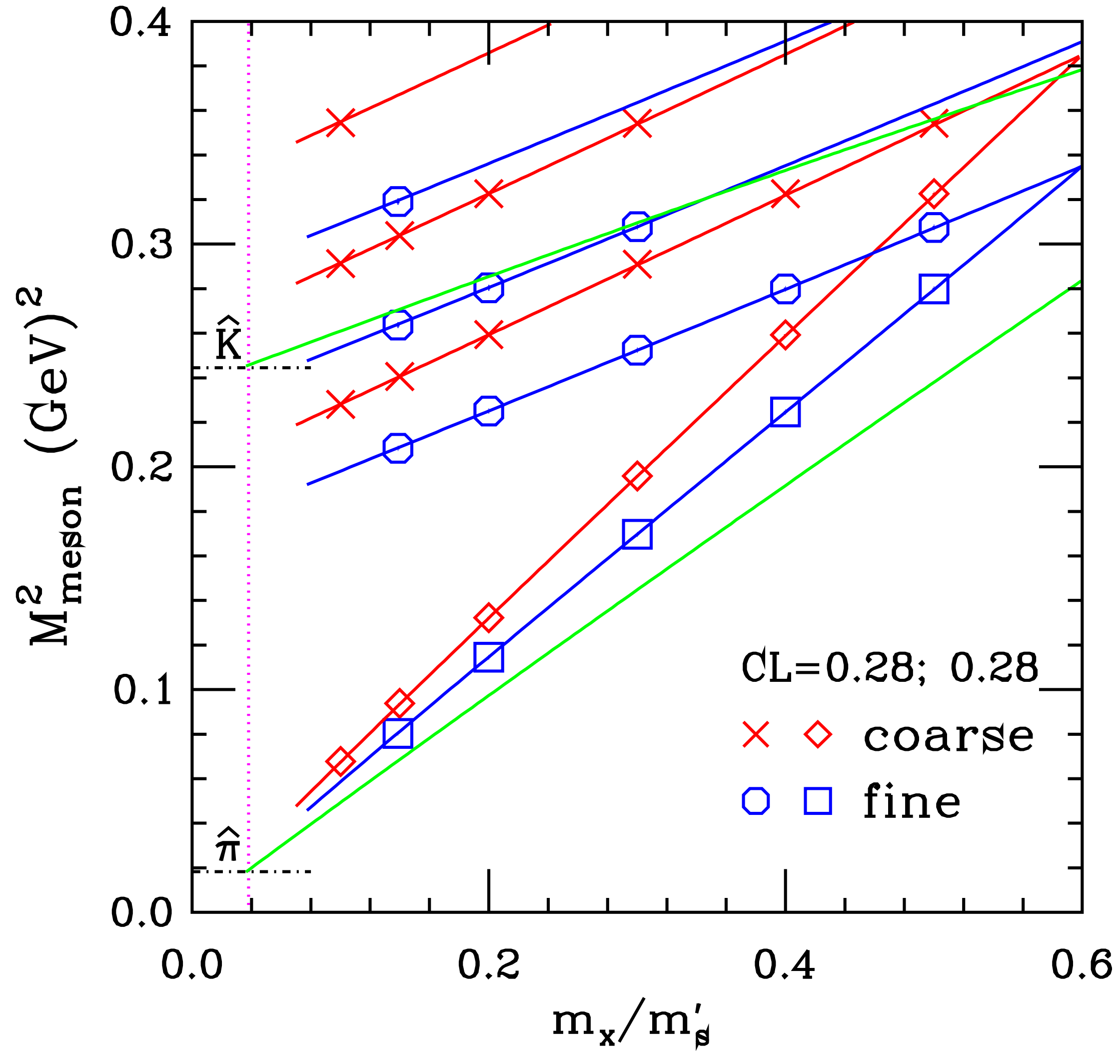
Finding quark masses

- Fit of partially quenched meson masses again, now shown without dividing by $m_x + m_y$.



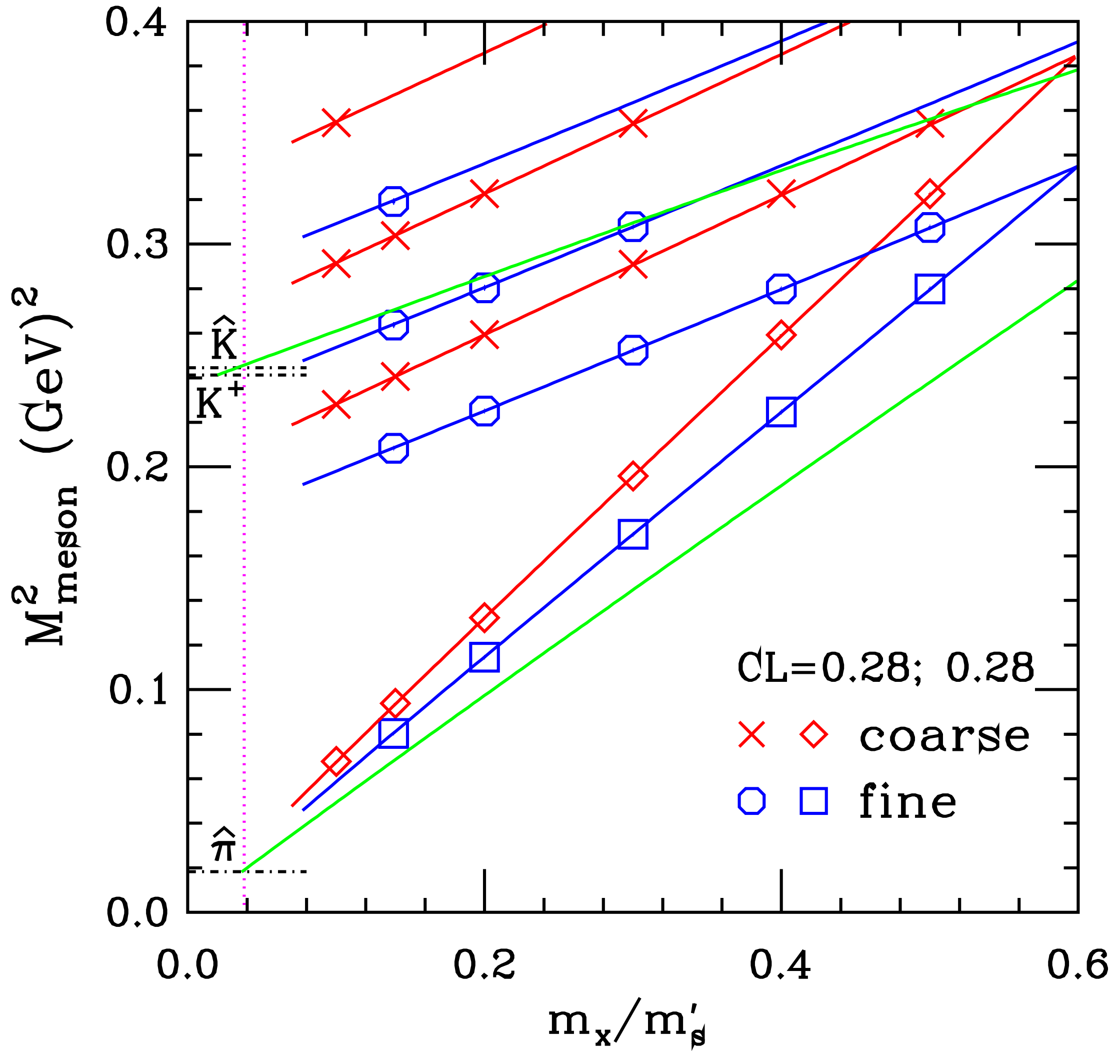
Finding quark masses

- Green lines are continuum extrapolated, full QCD.
- Have already adjusted m_s to make lines hits physical masses $m_{\hat{\pi}}^2$ and $m_{\hat{K}}^2$ at same value of light quark mass.
- Determines $\hat{m}$



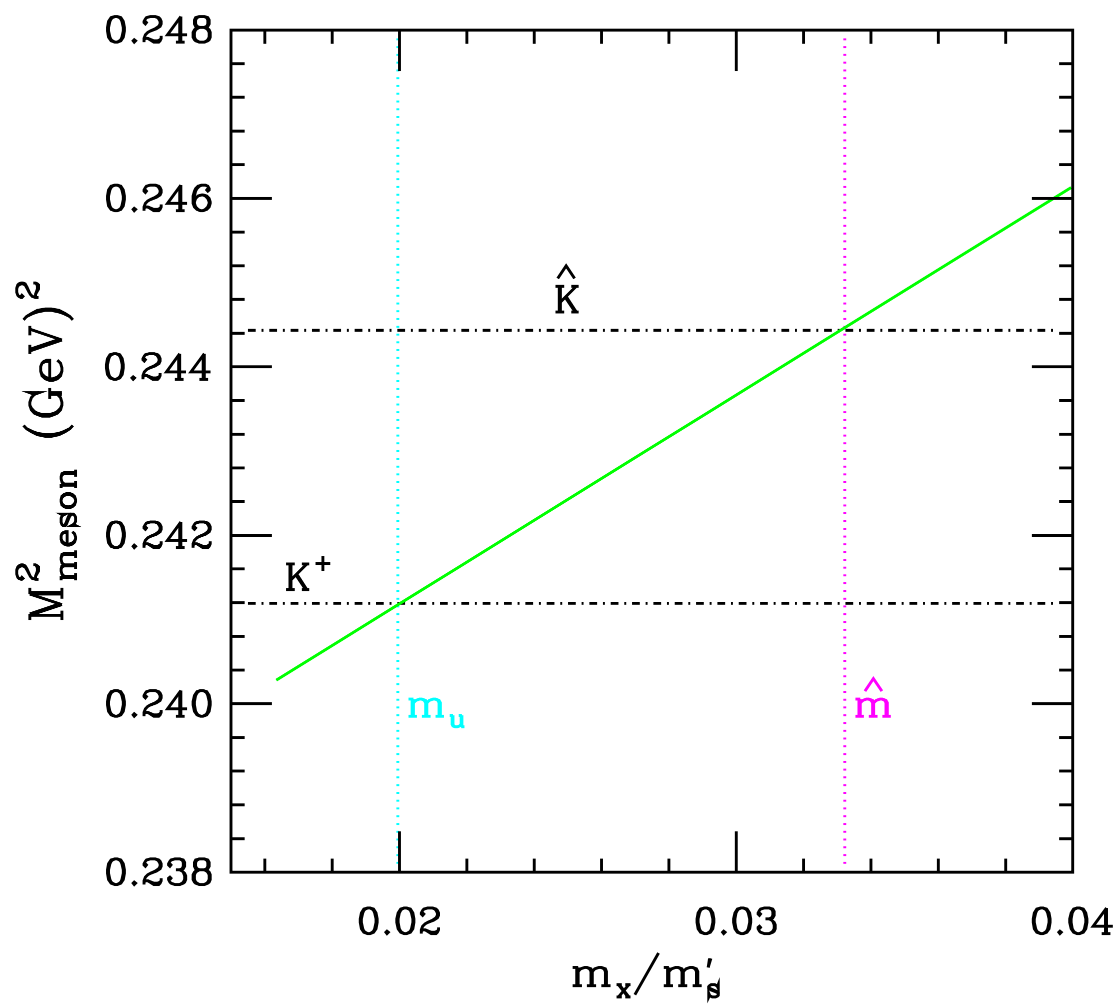
Finding quark masses

- Now fix light sea quark mass at $\hat{m}$, and continue extrapolation until line hits $(m_{K^+}^{\mathrm{QCD}})^2$



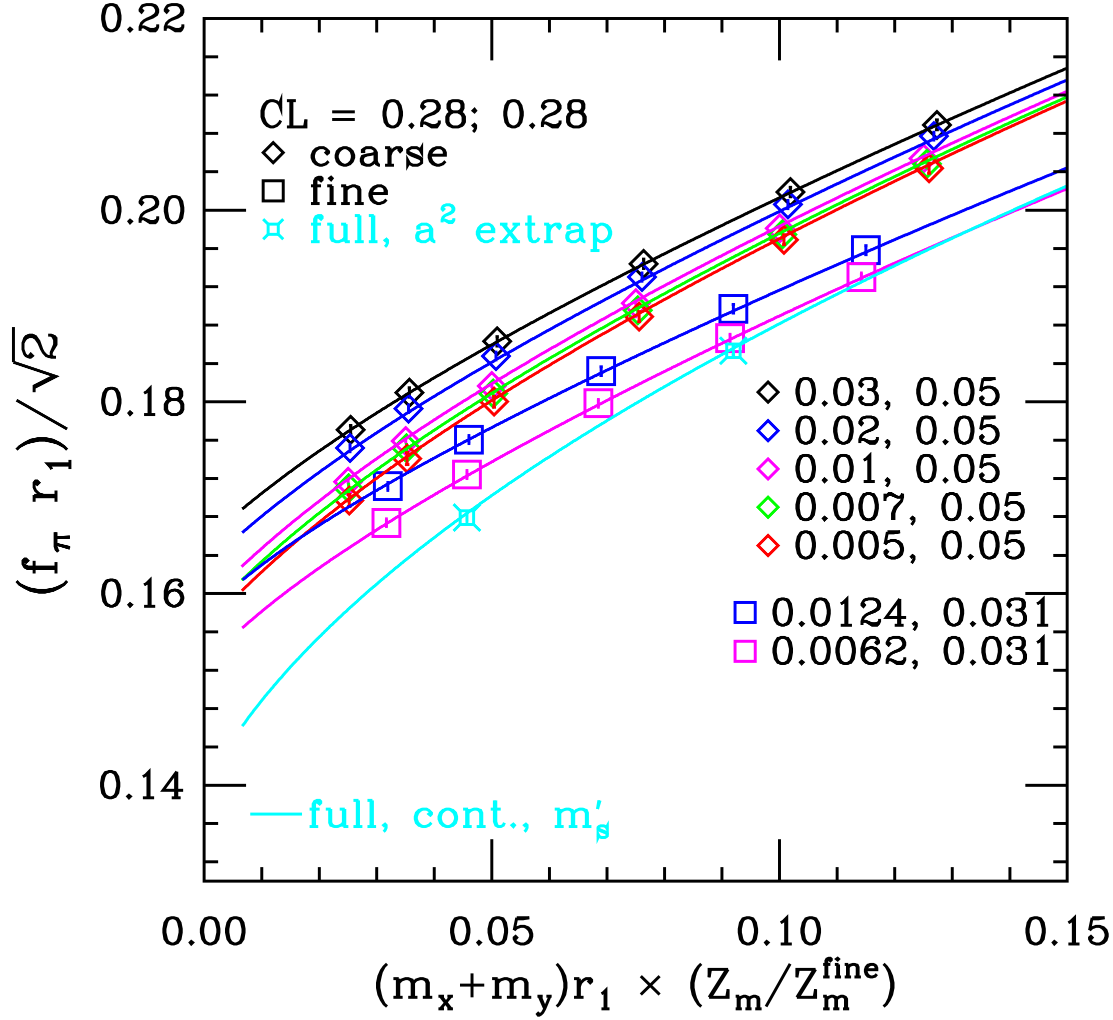
Finding quark masses

- Blow-up of region where full QCD line hits physical masses.
- m_u is determined up to small isospin-violating corrections (because sea quark masses still are $m_u = m_d = \hat{m}$).



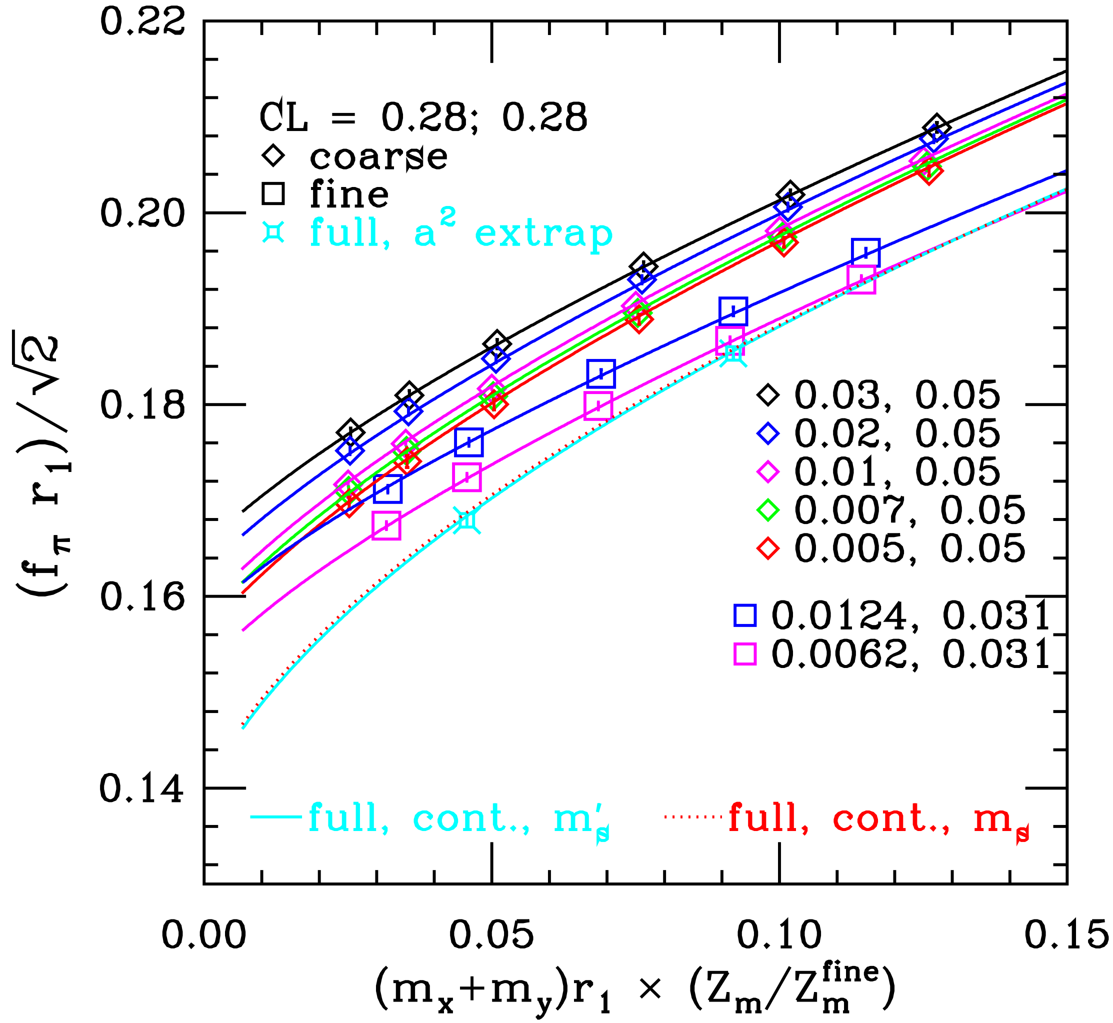
Extract f_π

- previous plot



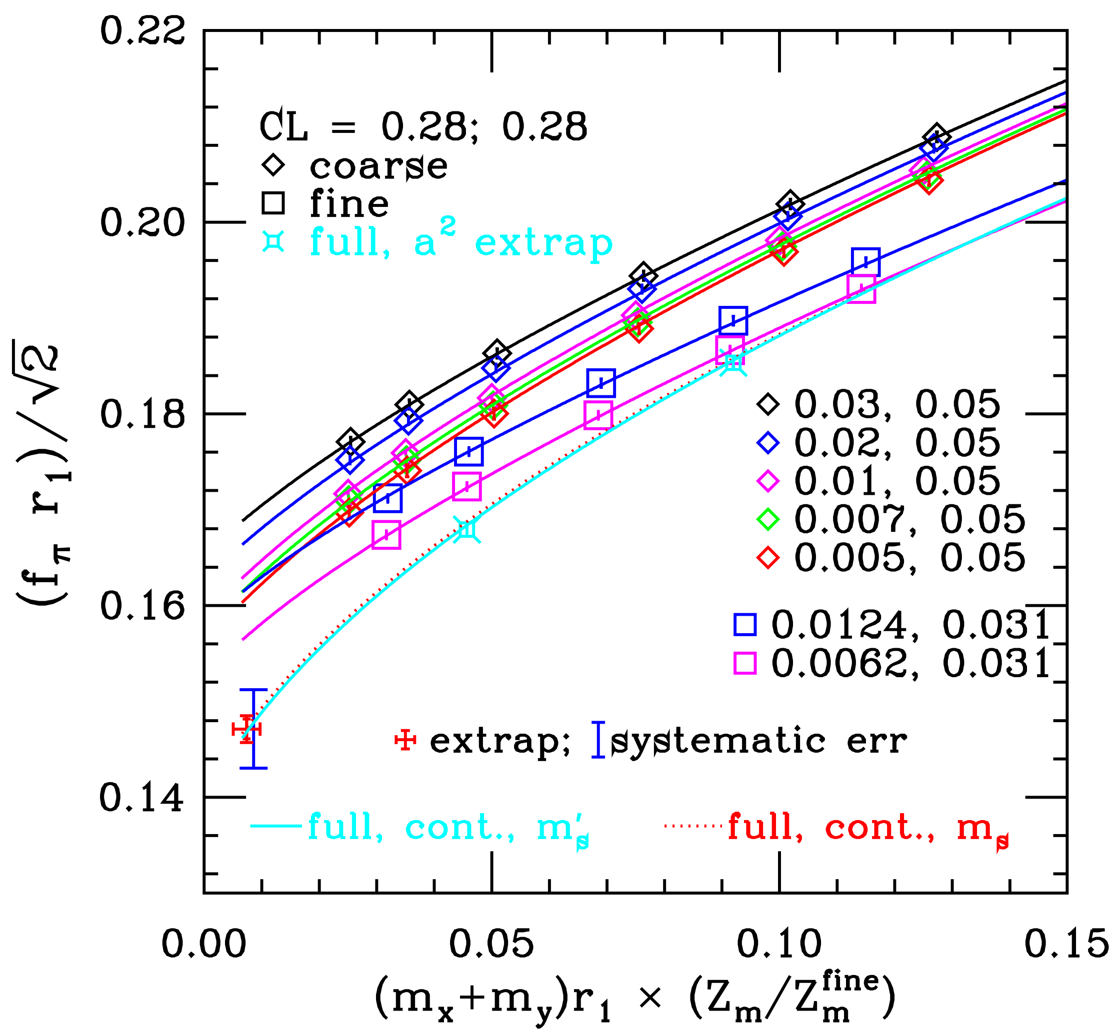
Extract f_π

- Adjust continuum-extrapolated, full QCD line to have physical m_s value.



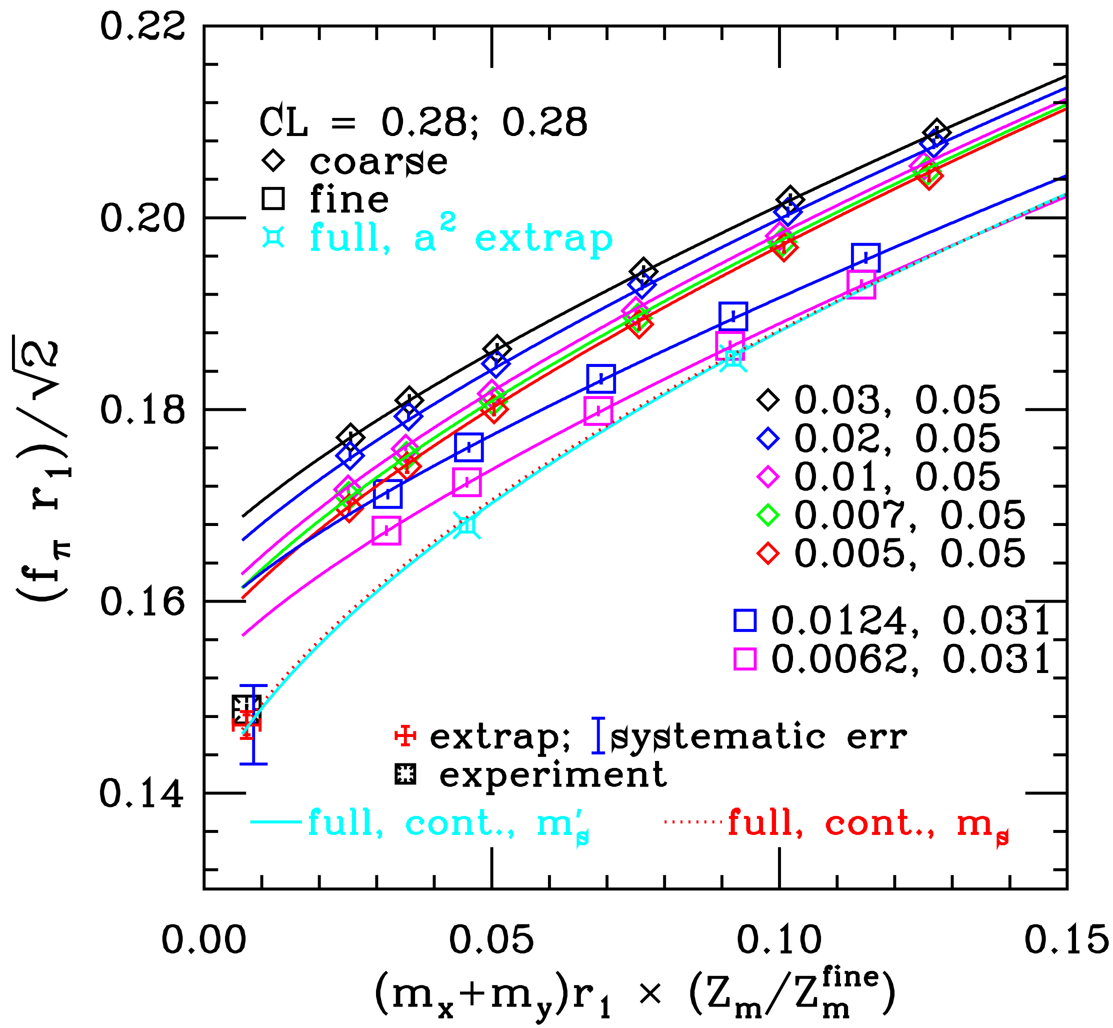
Extract f_π

- Extrapolate to physical $\hat{m}$ point.



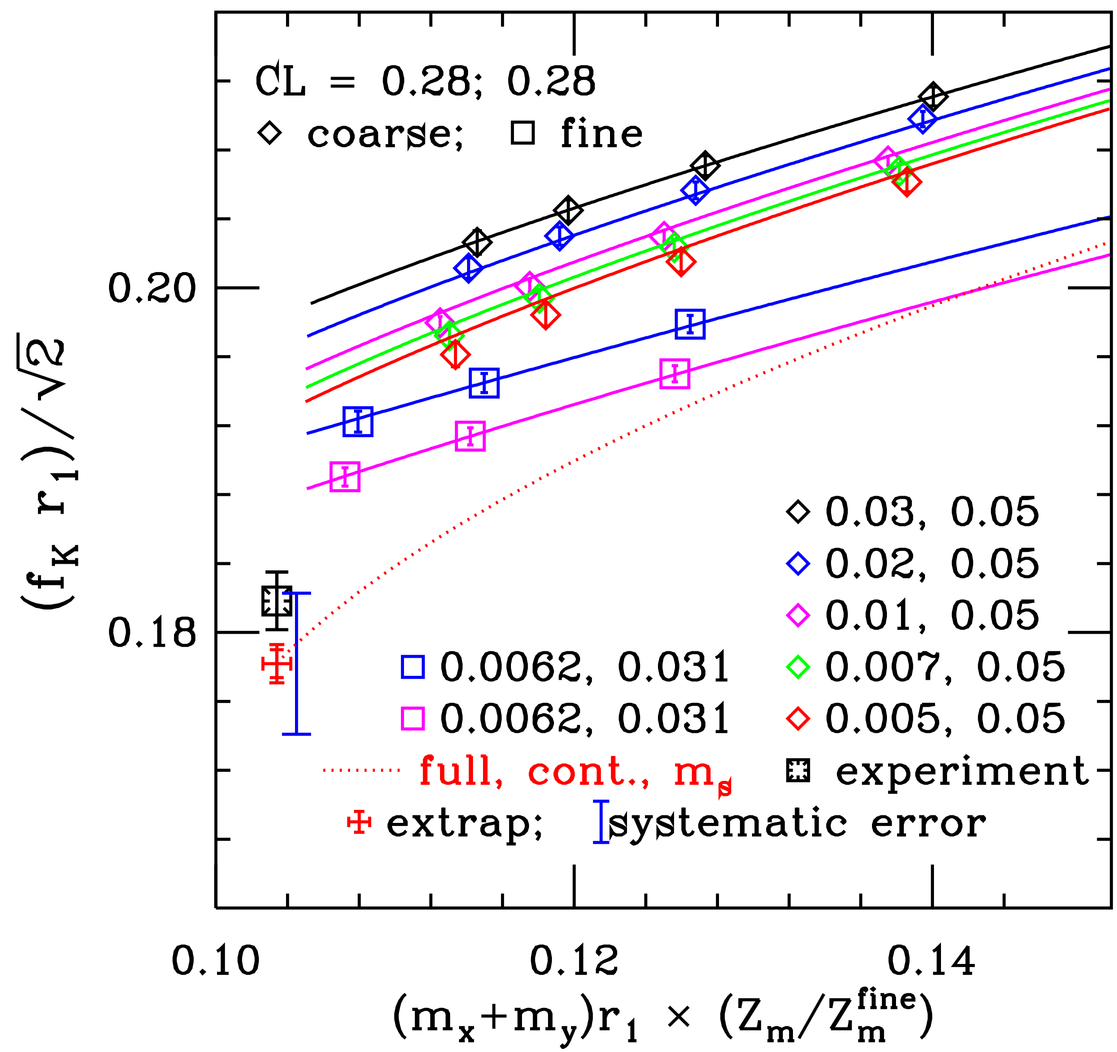
Extract f_π

- Comparison with experiment.



Extract f_K

- Similar procedure for f_K.
- But note that f_K is the decay constant of K^+.
- Here we need to extrapolate light valence quark to m_u, but light sea quark to $\hat{m}$.



Results: Decay Constants

$$\begin{aligned} f_\pi &= 129.5 \pm 0.9 \pm 3.6 \text{ MeV} \\ f_K &= 156.6 \pm 1.0 \pm 3.8 \text{ MeV} \\ f_K/f_\pi &= 1.210(4)(13) \end{aligned}$$

- First error is statistical; second is systematic.
- Chiral extrapolation errors and scale errors contribute almost equally to the systematic error on f_π and f_K. Scale errors are unimportant for the ratio.
- Results for f_π, f_K, and f_K/f_π consistent with experiment within their $\sim 3\%$, 2.5% and 1% errors, respectively.
- In fact, result for f_K/f_π can be turned around to compute $|V_{us}|$ (Marciano, hep-ph/0402299). Get: $|V_{us}| = 0.2219(26)$, compared to PDG value $0.2196(26)$.

Results: Masses

$$m_u/m_d = 0.43(0)(1)(8)$$

- Errors are from statistics, simulation systematics, and EM effects (conservative range), respectively.
- If instead we assume the result of Bijnens & Prades ($\Delta_E = 0.84 \pm 0.25$), we get $m_u/m_d = 0.44(0)(1)(2)$.

- Results from collaboration of HPQCD, UKQCD, & MILC [hep-lat/0405022]:

$$\begin{aligned} m_s^{\overline{\mathrm{MS}}} &= 76(0)(3)(7)(0)\ \mathrm{MeV}\ , \\ \hat{m}^{\overline{\mathrm{MS}}} &= 2.8(0)(1)(3)(0)\ \mathrm{MeV}\ , \\ m_s/\hat{m} &= 27.4(1)(4)(0)(1) \end{aligned}$$

- Errors are from statistics, simulation, perturbation theory, and electromagnetic effects. Scale for masses is $2\ \mathrm{GeV}$.

Results: Low Energy Constants

- Also get (in units of 10^{-3}, at chiral scale m_η):

$$
\begin{aligned}
2L_6 - L_4 &= 0.5(2)(4) \\
2L_8 - L_5 &= -0.2(1)(2) \\
L_4 &= 0.2(3)(2) \\
L_5 &= 1.9(3)(2)
\end{aligned}
$$

- Consistent with "conventional results" summarized in Cohen, Kaplan, & Nelson, JHEP 9911, 027 (1999): $L_5 = 2.2(5)$, $L_6 = 0.0(3)$, $L_4 = 0.0(5)$.
- Our result for $2L_8 - L_5$ is far from range $-3.4 \leq 2L_8 - L_5 \leq -1.8$ that would allow $m_u = 0$ (Kaplan & Manohar; Cohen, Kaplan & Nelson).
- Consistent with our direct determination of m_u.
- Need to look elsewhere for a solution of the strong CP problem.

Elephant in the room

In desperation I asked Fermi whether he was not impressed by the agreement between our calculated numbers and his measured numbers.

He replied, “How many arbitrary parameters did you use for your calculations?”

I thought for a moment about our cut-off procedures and said, “Four.”

He said, “I remember my friend Johnny von Neumann used to say, ‘With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.’"

With that, the conversation was over.

–Freeman Dyson

Elephant in the room

Could we fit a whole herd of elephants with our 40 (or 20 unconstrained) parameters?

If the physics isn't right, ~ 40 parameters WON'T allow you to fit the data:

- Comparable fits to continuum form (all taste-violating terms set to 0): 36 params, $\mathrm{CL} < 10^{-250}$.
- Comparable fits with all chiral logs and finite volume corrections omitted from fit function (*i.e.*, analytic function only) are poor $\Rightarrow$ Good evidence for chiral logarithms:
  - Remove finite volume effects from data first (*cf*. Becirevic & Villadoro): 38 params, $\mathrm{CL} < 10^{-38}$.
  - Don't remove finite volume effects from data: 38 params, $\mathrm{CL} < 10^{-186}$.
- Also tried separate linear fits of m_π^2 or f_π *vs.* quark mass:
  - m_π^2: 6 params, $\mathrm{CL} < 10^{-250}$.
  - f_π: 10 params, $\mathrm{CL} < 10^{-250}$.

Elephant in the room

Having wrestled for years with the problem of fitting an elephant, I can say with some certainty that at least 43 parameters. . . are required to give even a rough approximation to an elephant.

–Robert D. Phair