The gluon and ghost propagator and the influence of Gribov copies

— Progress report on first results —

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Fermi National Accelerator Laboratory — Lattice 2004 — June 21-26, 2004

Contents

Introduction and motivation

- Non-perturbative study of gluon and ghost propagators is of interest for understanding the confinement phenomenon.

- In the infrared ($q \to 0$): Singular behavior of the ghost propagator

$$G^{ab} = \delta^{ab} \frac{Z_{gh}(q^2)}{q^2} \qquad \text{(Landau gauge)}$$

is related to

  – Kugo-Ojima confinement criterion, [Ojima, Kugo, (1978,1979)]

  – absence of colored states in physical spectrum.

- In the infrared ($q \to 0$): Suppression of the gluon propagator:

$$D^{ab}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z_{gl}(q^2)}{q^2} \qquad \text{(Landau gauge)}$$

is connected to the confinement of gluons.

Behavior of both are intimately connected !?

Results from Dyson-Schwinger-Equations

- needs gauge fixing (here Landau gauge)
- the infinite set of equations has to be truncated

Alkofer et.al [1997-2004]: scaling behavior as $q \to 0$:

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z_{gl}(q^2)}{q^2} \qquad Z_{gl}(q^2) \propto (q^2)^{2\kappa}$$

$$G^{ab} = \delta^{ab}\frac{Z_{gh}(q^2)}{q^2} \qquad Z_{gh}(q^2) \propto (q^2)^{-\kappa}$$

critical exponent: $\kappa \approx 0.595$

Running coupling in MOM scheme

$$\alpha_s(q) \sim [Z_{gh}(q^2)]^2 \cdot Z_{gl}(q^2) \cdot \alpha_s(\mu)$$

Is there an IR fixed point $\alpha_s(q) \to \text{const.}$ *as* $(q \to 0)$ *?*

Gribov, Zwanziger:

Infrared behavior related to the restriction of gauge fields $A_\mu(x)$ to the Gribov region:

$$\Omega = \left\{ A_\mu(x) : \partial_\mu A_\mu = 0, -\partial_\mu D_\mu^{ab} \geq 0 \right\}$$

Expect gauge fields belonging to the Gribov horizon to dominate

Zwanziger [2004]:

exact non-pertubative quantization of continuum theory is provided by Faddeev-Popov formula

$$\delta_\Omega(\partial_\mu A_\mu)\,\mathrm{det}(-\partial_\mu D_\mu^{ab}) e^{-S_{YM}[A]}$$

The Gribov copies in this region are claimed to not affect observables even on the lattice.

Landau gauge $\partial_\mu A_\mu(x) = 0$ does not determine $A_\mu(x)$ non-ambiguously. $\leftarrow$Gribov copies

. . . on the Lattice

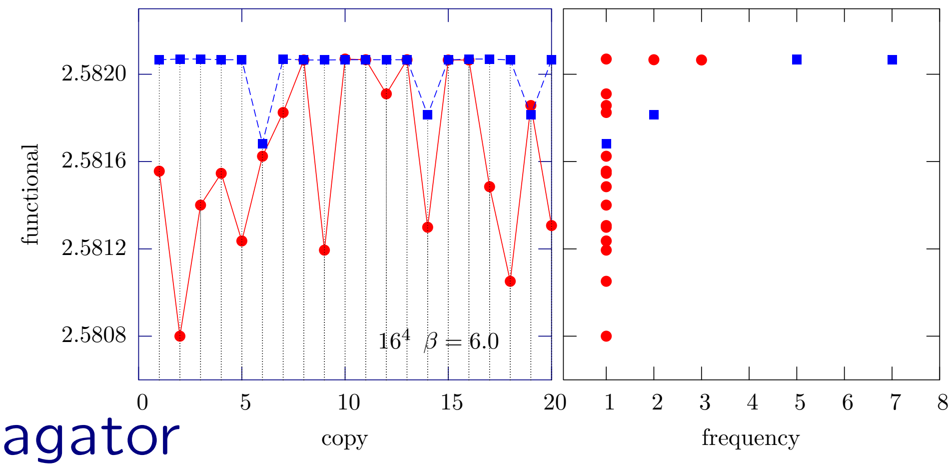
To compare DSE results gauge fixing is neccessary on the lattice

- vector potential: $A_\mu(x) \equiv \frac{1}{2i}(U_{x\mu} - U^\dagger_{x\mu})\big|_{\text{traceless}}$
- Landau gauge: $\max_x \mathrm{Tr}\left(\partial_\mu A_\mu(x) \cdot [\partial_\mu A_\mu(x)]^\dagger\right) < \epsilon$
- fundamental modular region: $\Lambda \equiv \{U : F_U(1) \geq F_U(g) \forall g\}$

$$\boxed{\begin{array}{l}\text{interest on the maximum of the gauge functional} \\ F_U(g) = \frac{1}{4V}\sum_{x,\mu} \mathfrak{Re}\,\mathrm{Tr}\, U^g_{x\mu} \text{ with } U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\mu}\end{array}}$$

well known: the maximum is not unique (Gribov copies)

Example: 1 gauge field configuration $\{U_{x\mu}\}$,
30 $\times$ gauge-fixed



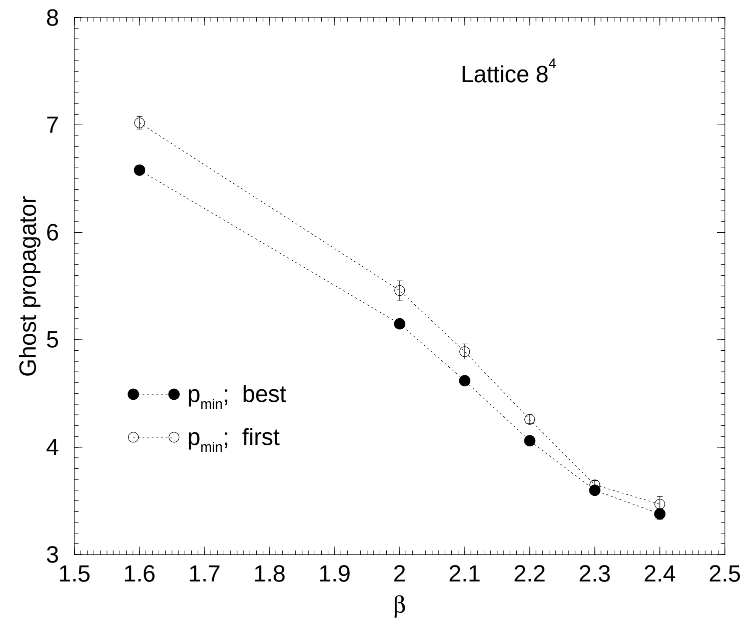
Are Observables affected by a Gribov noise ?

in particular: Ghost- and Gluonpropagator

Effect of Gribov copies (other studies)

SU(2) Ghostpropagator: Gribov noise observable for $0 \leq \beta < 2.6$

Cucchieri [NPB 508, 353 (1997)], Bakeev et.al [PRD 69,074507 (2004)]



SU(3) Gluonpropagator:

Silva, Oliveira [NPB 690, 177 (2004)]

$(\beta = 5.8, 12^4)$

1. the Gribov noise is small $< 10\%$
2. Gribov copies change lowest momenta components
3. effect of Gribov copies can be resolved by double (or more) statistical errors

Our study: Gluon- and Ghostpropagator

- on the same gauge-fixed configurations ($16^4, 24^4; \beta = 5.8, 6.0, 6.2$)

Applied numerical techniques

- Parallelized (MPI) SU(3) code running on IBM 690p (HLRN)
- Update: standard Wilson action and hybrid overrelaxation
- Gauge fixing: start with random gauge copies and apply standard overrelaxation (vs. simulated annealing)
- Ghost propagator:

$$G(k) = \frac{1}{3V} \sum_{xy} e^{-2\pi i\, k\cdot(x-y)} \left\langle \left(M^{-1} \right)^{aa}_{xy} [U] \right\rangle$$

  using conjugate gradient algorithm with source:

  $\psi^{ac}(y) = \delta^{ac}\, e^{2\pi i\, k\cdot y}$ $\qquad k \neq (0,0,0,0)$ (Cucchieri [1997])

- Gluon propagator: fast Fourier transform [FFTW package]

Remarks on gauge fixing algorithms

Goal: Find those gauge transformations which maximize (globally)

(Over-)relaxation:

$(g_x)^\omega$ where g_x is

local maximum

Simulated annealing

ground state of spin

sytem

$\exp\left\{ -F_U(g)/T \right\}$

$$F_U(g) = \frac{1}{4V} \sum \mathfrak{Re}\,\mathrm{Tr}\, U^g_{x\mu}$$

......

Fourier acceleration

Smeared gauge fixing

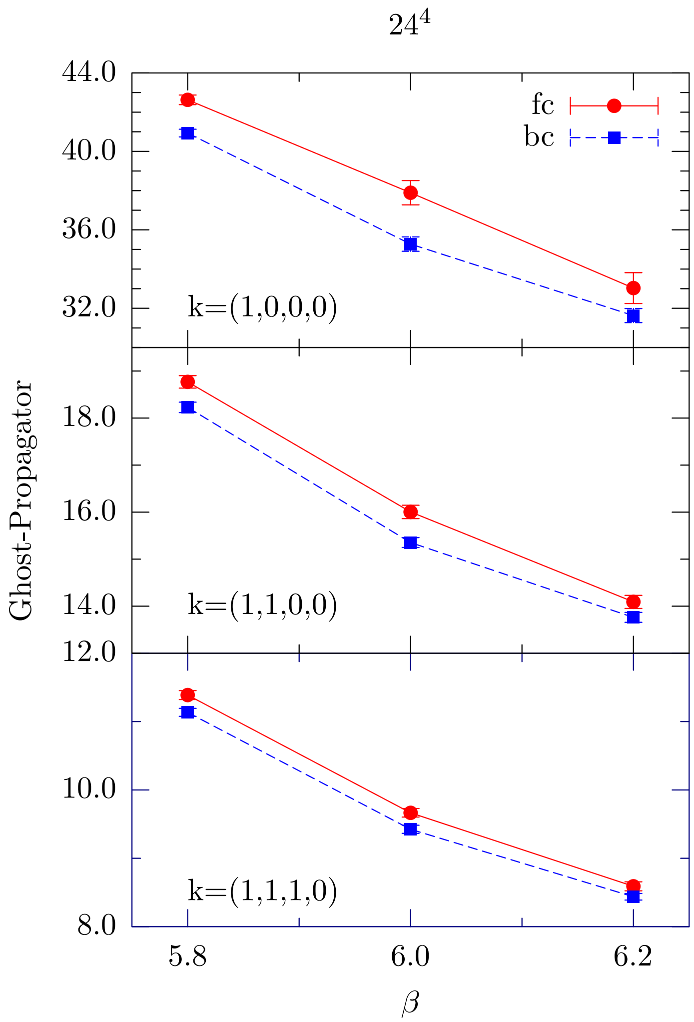
start with g_x which

gauge fix the smeared

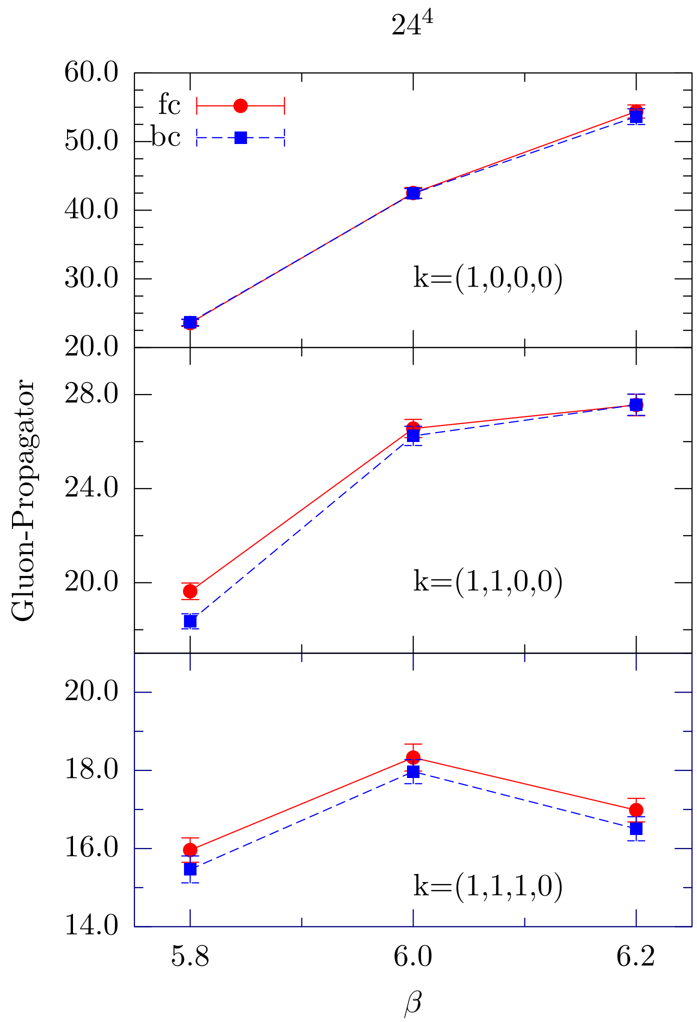
configuration $\tilde{U}_{x,\mu}$

Gribov noise at lowest momenta

Ghostpropagator



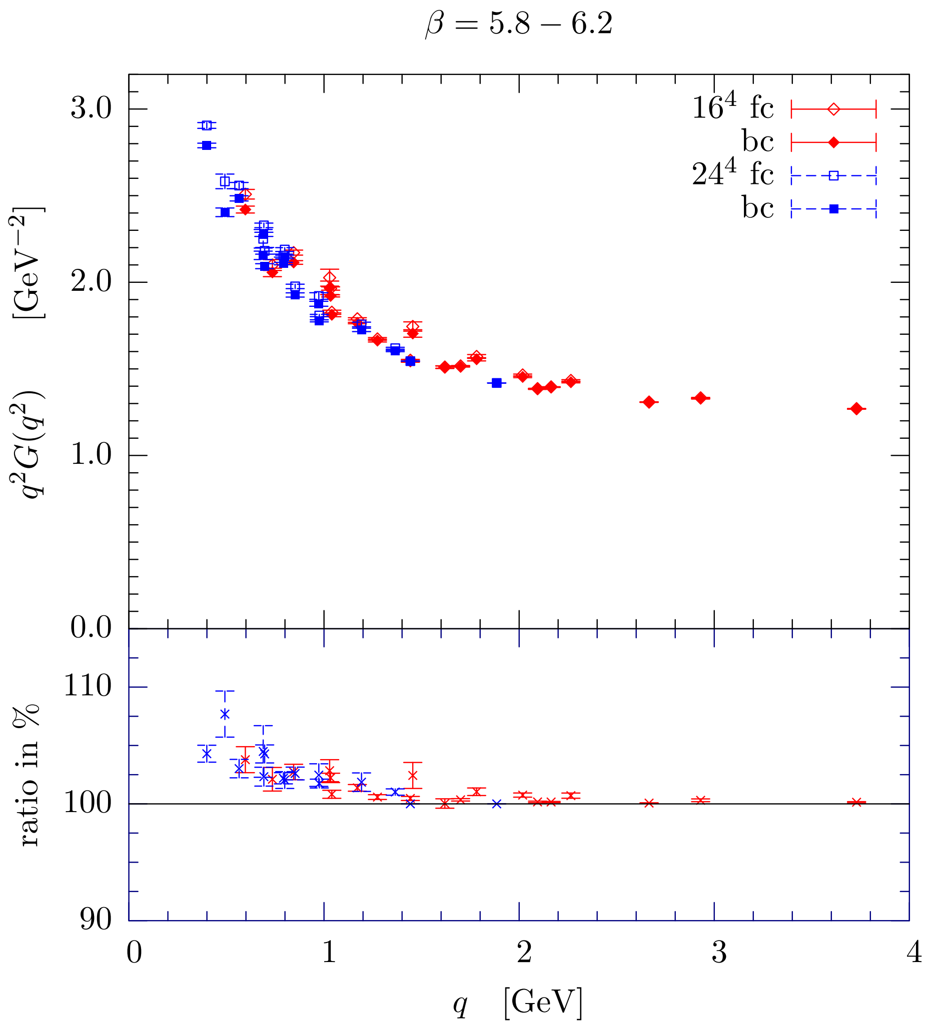
Gluonpropagator



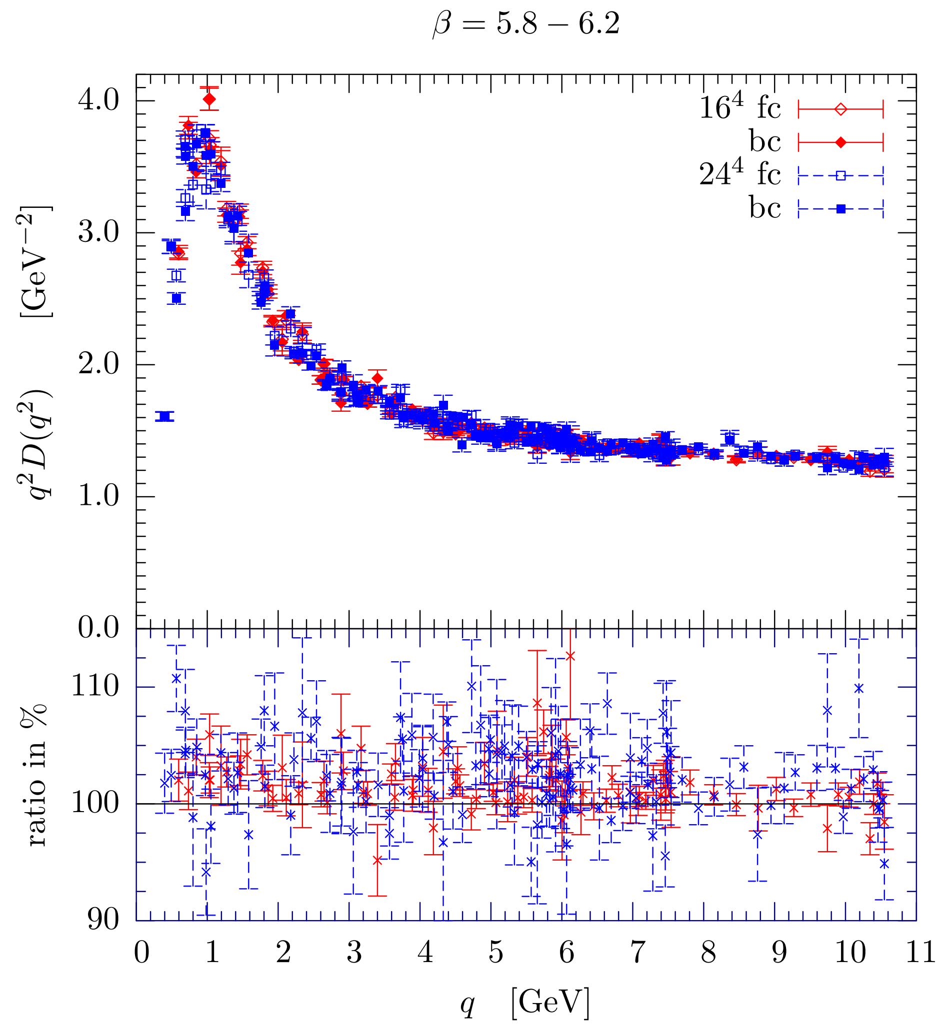
- Gribov noise most visible at lowest momenta
- The lower β the larger the noise

The dressing functions in the infrared

for the Ghostpropagator



for the Gluonpropagator



- Ghost: the Gribov noise increase as the momenta decrease
- Gluon: there is a small ($\sim$ 2%) Gribov noise for all momenta

The influence on the fit parameters

very preliminary !

- expected behaviour of the dressing functions in the infrared:

$$Z_{gh}(q^2) \propto (q^2)^{-\kappa} \qquad Z_{gl}(q^2) \propto (q^2)^{2\kappa} \qquad q \to 0$$

- a global fit (common κ) has been performed to both data, simultaneously, but the lattice sizes considered are to small to fit properly

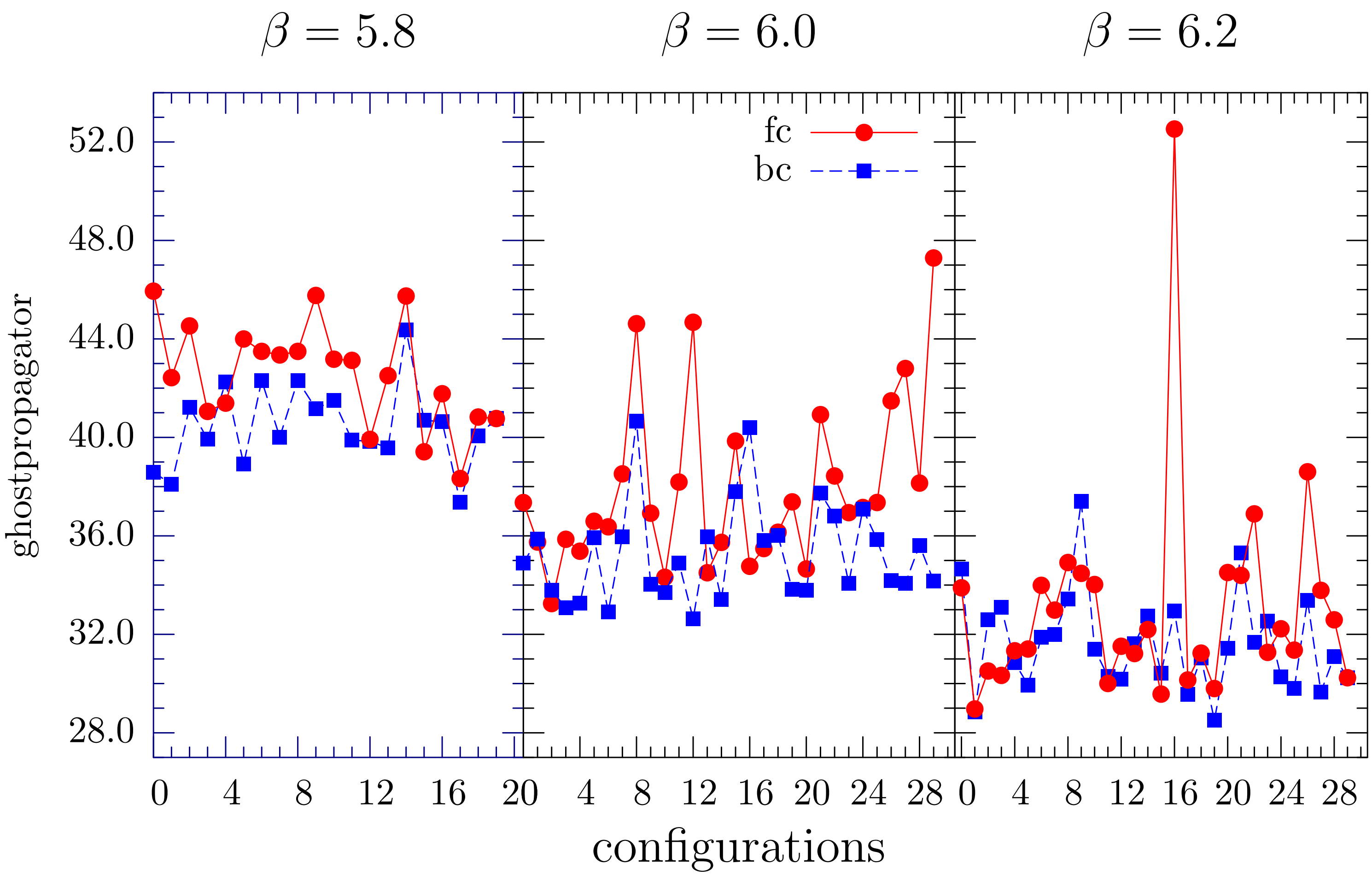
cut	copy	κ	χ^2/ndf
$q < 0.3$	fc	0.55(3)	60
	bc	0.52(2)	68
$q < 0.4$	fc	0.24(1)	43
	bc	0.22(1)	66
$q < 0.5$	fc	0.25(1)	29
	bc	0.24(1)	45

- exp. behaviour of the gluon propagator (to be checked yet)
[Leinweber et.al PRD 60,094507 (1999)]

$$Z_{gl}(q^2) \quad = \quad C\left[\frac{AM^{2\alpha}}{(q^2+M^2)^{1+\alpha}} + \frac{L(q^2,M^2)}{q^2+M^2}\right]$$

Appearance of 'exceptional' configurations

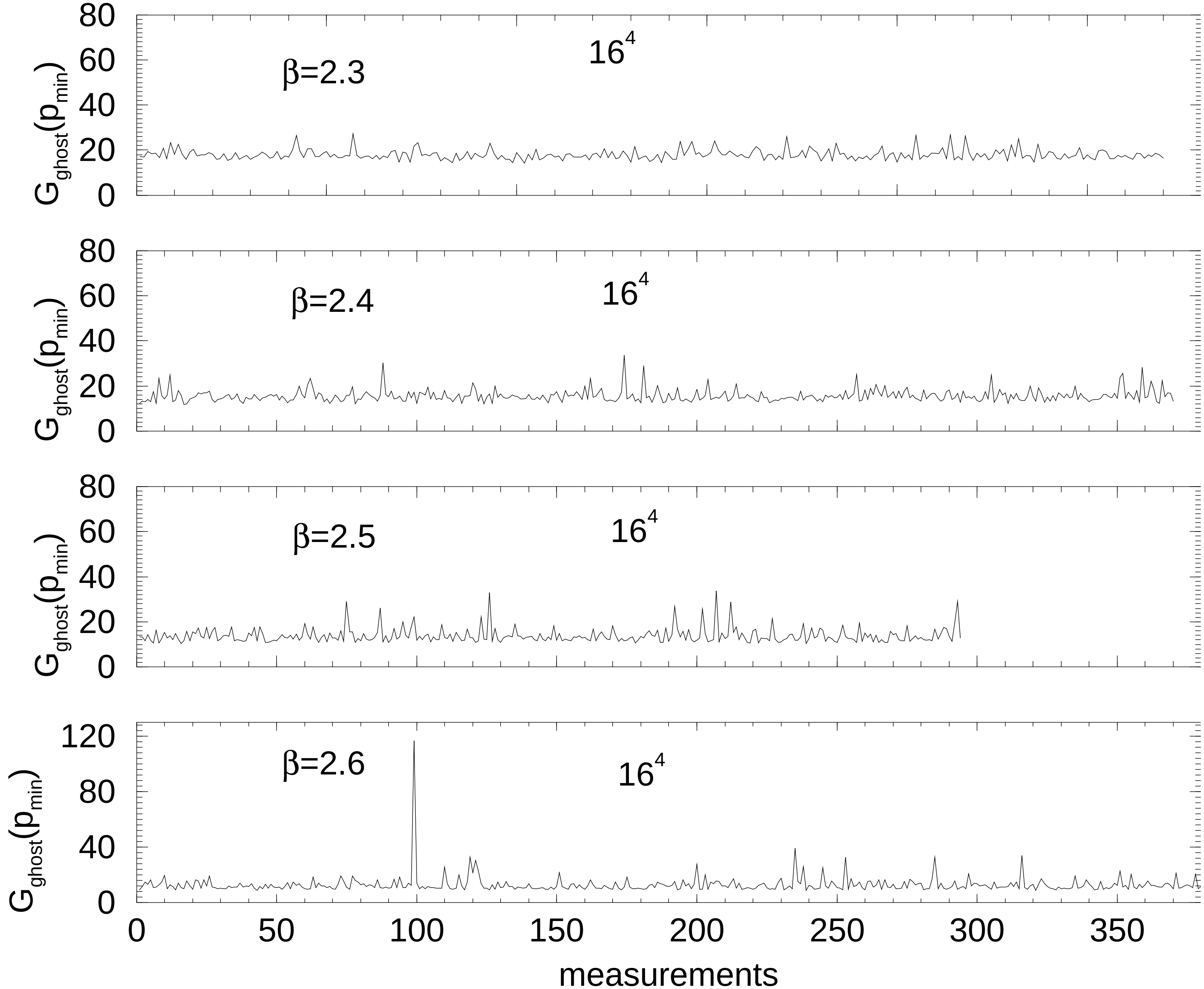
SU(3): For increasing β there are spikes in the history of the Ghostpropagator



here the lowest momentum (1,0,0,0) on a 24^4 lattice is shown

Appearance of 'exceptional' configurations

SU(2): [Bakeev et.al [PRD 69,074507 (2004)]



Conclusions and future plans

1. Gribov effects have to be carefully taken into account in particular at small β or larger lattice sizes
   - better algorithms for finding the global maxima are desired.
2. Infrared behavior: Larger lattices than 24^4 are needed to explore the low momentum limit much better.
   - to investigate the behavior of the dressing functions $Z_{gh}(q^2)$ and $Z_{gl}(q^2)$ coming from DSE
   - the (probable) finiteness of α_s in the infrared can be investigated
3. There are exceptional configurations as β increases.
   - Can it spoil the ghost propagator estimate ?
   - We will investigate the spectrum of the Faddeev-Popov operator.