Recent results on moments of parton distribution functions

Ines Wetzorke

NIC / DESY Zeuthen

for the

ZeRo Collaboration

χ_{LF} Collaboration

M. Guagnelli

F. Palombi

R. Petronzio

K. Jansen

A. Shindler

I. W.

S. Capitani

C. Urbach

M. Papinutto

Selected Topics

- $N_f = 2$ non-pert. Renormalization
- Finite Size Effects
- $\langle x \rangle$ with Wilson tm fermions

Moments of PDF's on the Lattice

- Relation to matrix elements of leading twist operators between hadron states

$$\langle h(p)|\mathcal{O}_{\mu_1\cdots\mu_n}|h(p)\rangle \;=\; \langle x^{n-1}\rangle_q \, p_{\mu_1}\ldots p_{\mu_n} + \text{terms } \delta_{\mu_i\mu_j}$$

- non-singlet operators belong to two irreducible representation of $H(4)$:

$$\mathcal{O}_{44}(x) \;=\; \bar\psi(x)[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k\,]\frac{\tau^3}{2}\psi(x)$$

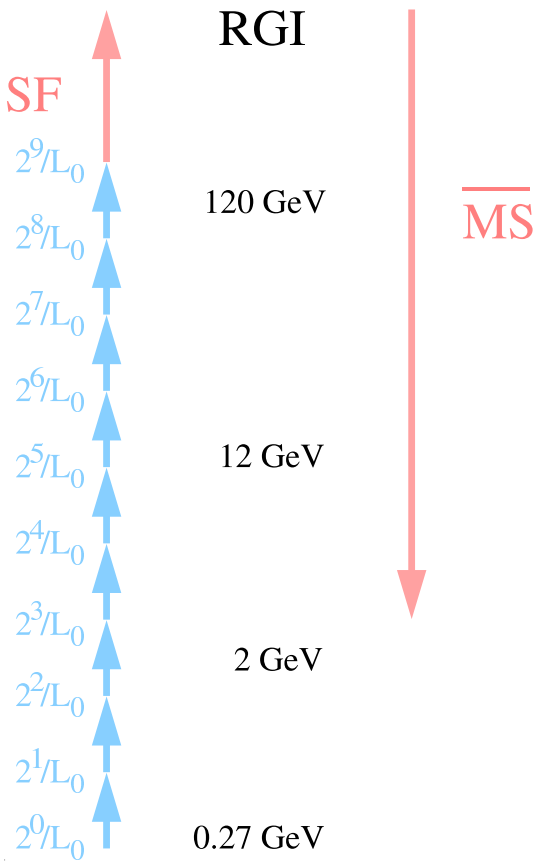
$$\mathcal{O}_{12}(x) \;=\; \bar\psi(x)[\gamma_1 \overleftrightarrow{D}_2 + \gamma_2 \overleftrightarrow{D}_1\,]\frac{\tau^3}{2}\psi(x)$$

- calculate renormalized matrix element at low energy scale μ_0 ...

$$\langle x\rangle(\mu_0) = \lim_{a\to 0}\lim_{m\to 0}\frac{\langle h(p)|\mathcal{O}_{NS}^{n=2}|h(p)\rangle}{Z(1/\mu_0)}$$

- ... and relate different scales μ_0 and μ by the step scaling function σ_Z

$$\langle x\rangle(\mu) = \langle x\rangle(\mu_0)\,\sigma_Z(\mu/\mu_0) = \langle x\rangle(\mu_0)\,\frac{Z(1/\mu_0)}{Z(1/\mu)}$$



- Renormalization group invariant matrix element:

$$\langle x\rangle^{RGI} = \frac{\langle h(p)|\mathcal{O}_{NS}^{n=2}|h(p)\rangle}{Z(1/\mu_0)}\,\sigma(\mu/\mu_0,\bar{g}(\mu))\,\mathcal{F}_{SF}(\bar{g}^2(\mu))$$

$$\mathcal{F}_{SF}(\bar{g}^2(\mu)) = \bar{g}^2(\mu)^{-\gamma_0/2b_0}\,\exp\{-\int_0^{\bar{g}(\mu)} dg[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{2b_0}]\}$$

➡ simple conversion of $\langle x\rangle^{RGI}$ to any desired scheme (e.g. $\overline{MS}$):

$$\langle x\rangle_{\overline{MS}}(\mu) = \langle x\rangle^{RGI}/\mathcal{F}_{\overline{MS}}(\bar{g}^2(\mu))$$

SF Finite Volume Renormalization Scheme

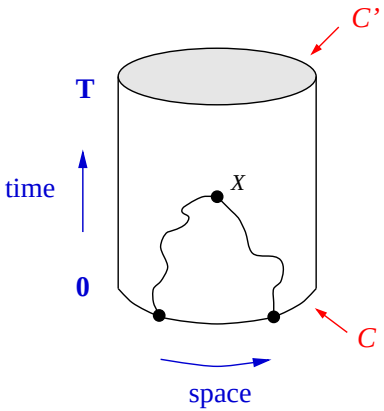
- scale $\mu^{-1} = L$ changed keeping pL, x_0/L and $m_q = 0$ fixed
- computation of $\sigma(\mu/\mu_0, \bar{g}(\mu))$ requires series of L^4 and $(2L)^4$ lattices

quenched	unquenched
9 couplings	6 couplings
$\bar{g}^2$=0.89 to 3.48	$\bar{g}^2$=0.98 to 3.33
4 lattice spacings	3 lattice spacings
$L = 6, 8, 12, 16$	$L = 6, 8, 12$
$2L = 12, 16, 24, 32$	$2L = 12, 16, 24$
2 action method	2 operator method
unimproved Wilson + non-pert. $O(a)$ impr. Clover	unimproved operator + improved operator

ALPHA Collaboration

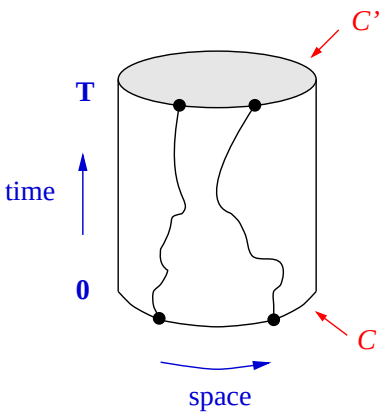
M. Della Morte, F. Knechtli, J. Rolf, R. Sommer, IW, U. Wolff

- correlation functions involve boundary fields:



$$f_{\mathcal{O}}(x_0/L) = -\frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y},\mathbf{z}} \langle \mathcal{O}(\mathbf{x})\, \bar{\zeta}(\mathbf{y})\, \Gamma\, \frac{\tau^3}{2}\, \zeta(\mathbf{z}) \rangle$$

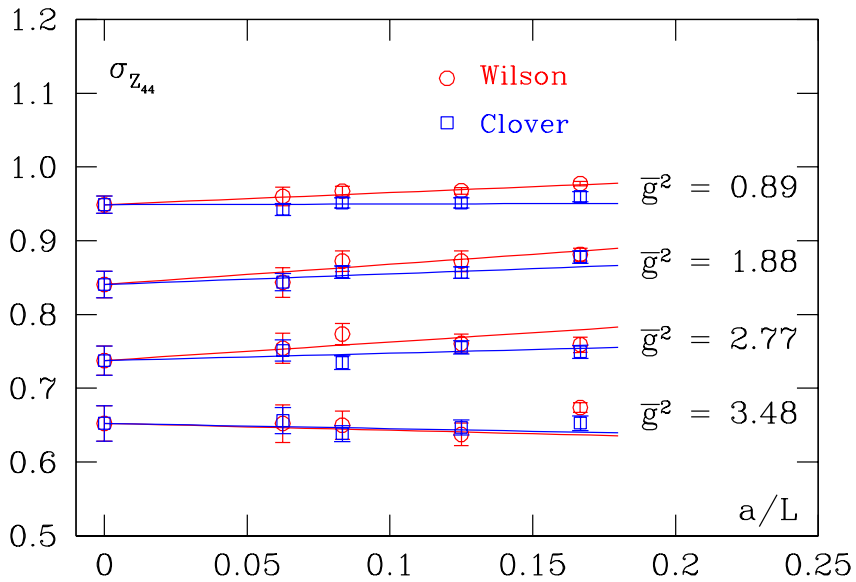
- boundary-to-boundary correlator:

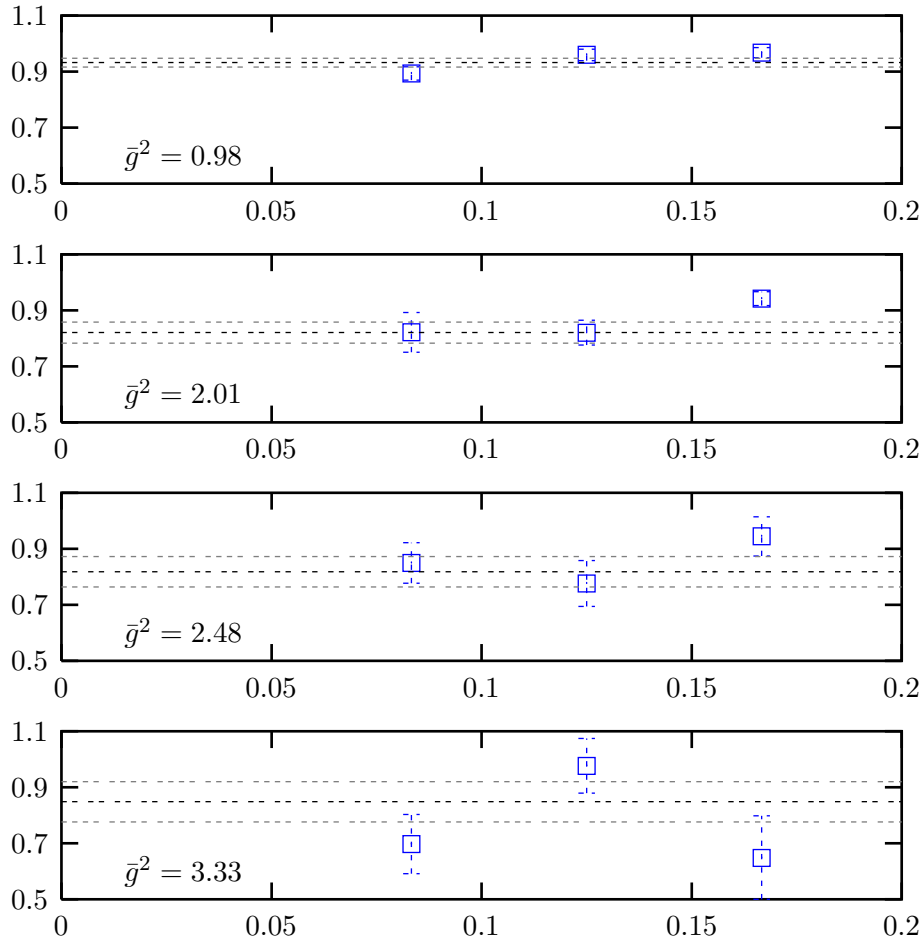


$$f_1 = -\frac{a^{12}}{L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}'(\mathbf{u})\, \gamma_5\, \frac{\tau^3}{2}\, \zeta'(\mathbf{v})\, \bar{\zeta}(\mathbf{y})\, \gamma_5\, \frac{\tau^3}{2}\, \zeta(\mathbf{z}) \rangle$$

$$\Rightarrow \quad Z(a/L, x_0/L) = \frac{f_{\mathcal{O}}(x_0/L)}{\sqrt{f_1}} \left(\frac{f_{\mathcal{O}}^{(0)}(x_0/L)}{\sqrt{f_1^{(0)}}} \right)^{-1}$$

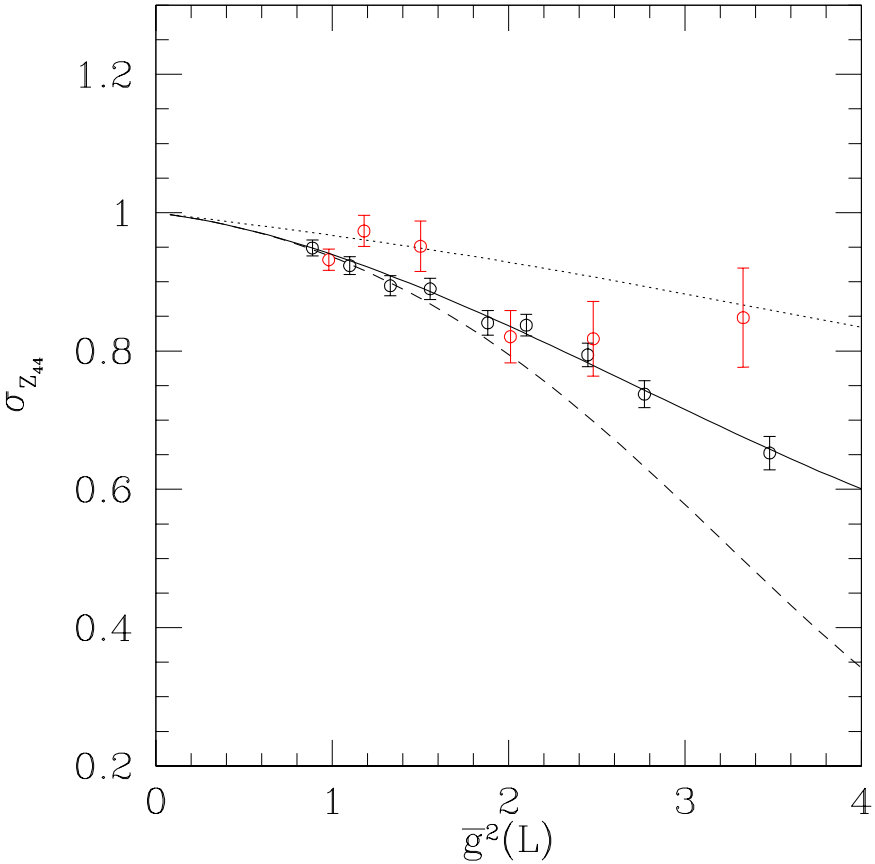
Continuum Extrapolation $\sigma_{Z_{44}}$

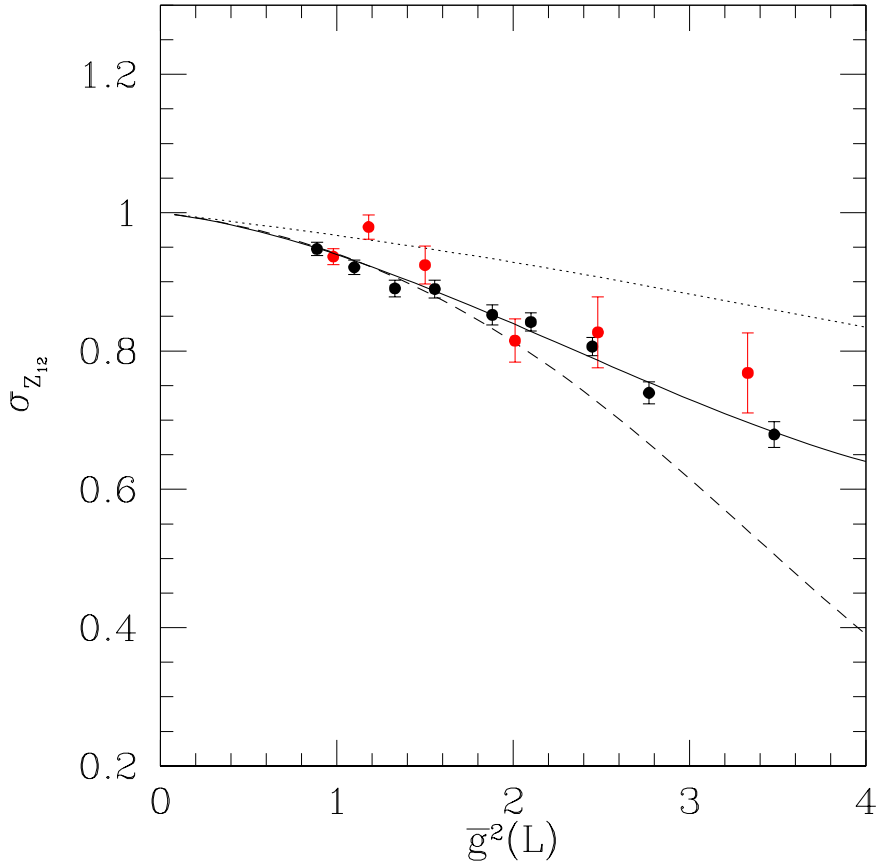




quenched versus
unquenched

Continuum step scaling functions

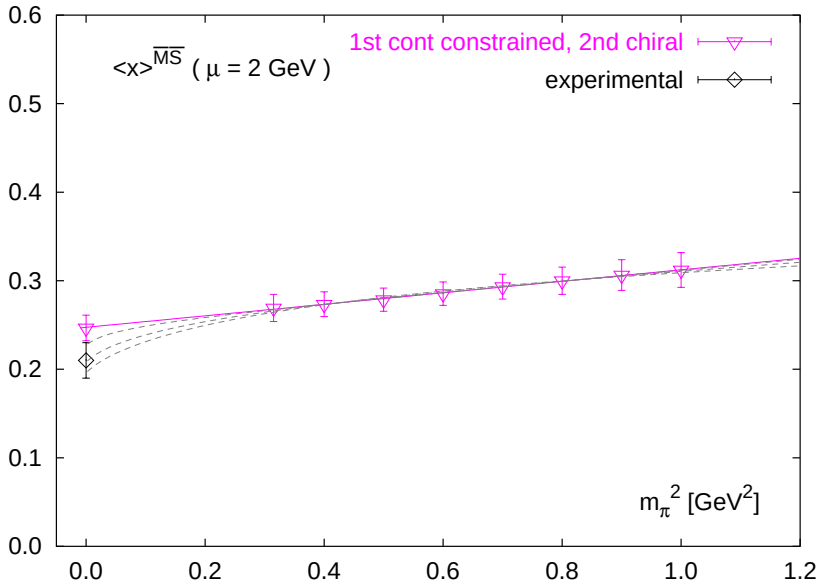


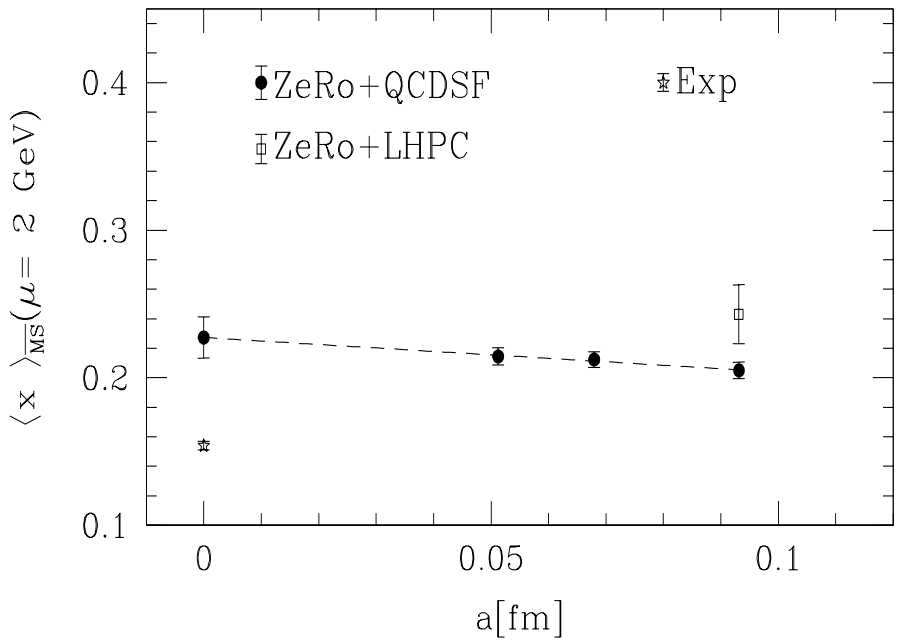


➡ 2- and 3-parameter fits: $\sigma(\bar{g}^2(L)) = 1 - \gamma_0 \ln(2)\, \bar{g}^2 + c_4 \bar{g}^4 + c_6 \bar{g}^6 + c_8 \bar{g}^8$

$\langle x \rangle^{\overline{MS}}(\mu = 2\ GeV)$ for Pion & Proton

ZeRo Collaboration, hep-lat/0405027





$$\langle x \rangle_q^{NS} = a \left[1 - \frac{1}{(4\pi f_\pi)^2}\, m_\pi^2\, \ln\left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right) \right] + b\, m_\pi^2 \quad \text{with} \quad \begin{array}{l} \Lambda = 0.4,\ 0.7,\ 1.0\ \text{GeV} \\ f_\pi = 93\ \text{MeV} \end{array}$$

Detmold, Melnitchouk, Thomas, PRD 68 (2003) 034025

Systematic Errors

- non-perturbative renormalization ✓
- continuum extrapolation ✓
- finite size effects ?
- chiral extrapolation ?
- quenching effects ?

Finite Size Effect Study

$\beta = 6.1$

L/a	T/a	N
12	36	3200
14	36	1600
16	42	800
24	42	800
32	56	185

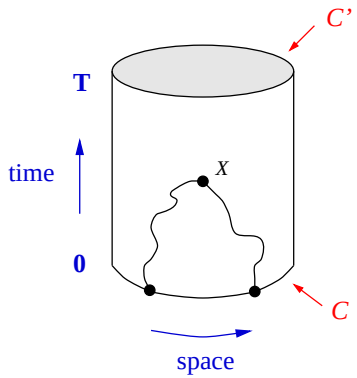
ZeRo Collaboration
hep-lat/0403009

- Schrödinger functional scheme
- non-pert. impr. Clover action
- $L = 1.0 - 2.9$ fm, $T = 2.8 - 4.4$ fm
- $a = 0.079$, 0.093 fm
- 3 κ's (560 MeV $< m_\pi <$ 850 MeV)
- observables: $m_\pi, \langle x \rangle, m_N, \langle x \rangle_N$

$\beta = 6.0$

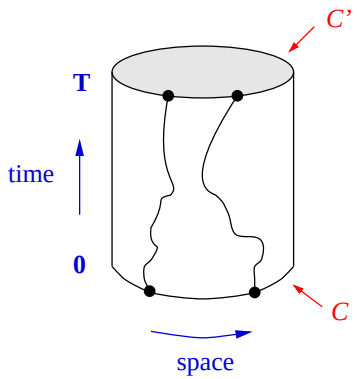
L/a	T/a	N
12	32	8160
16	32	2200+2144
24	32	1100+2481
32	32	1090+610

Schrödinger Functional Scheme

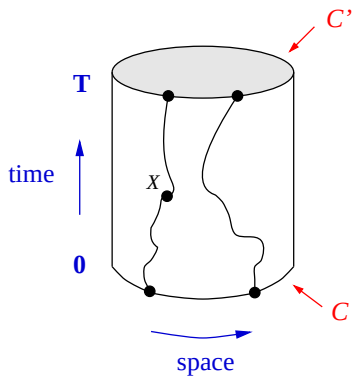


$$f_{A^I}(x_0) = -\frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y},\mathbf{z}} \langle A^I(\mathbf{x})\, \bar{\zeta}(\mathbf{y})\, \gamma_5\, \frac{\tau^3}{2}\, \zeta(\mathbf{z})\rangle = -\langle A^I(\mathbf{x})\, \mathcal{O}_0\rangle$$

$$A^I_\mu(x) = A_\mu(x) + a\, c_A \tfrac{1}{2}(\partial^*_\mu + \partial_\mu)\, P(x)$$



$$f_1 = -\langle \mathcal{O}_0 \mathcal{O}_T\rangle \sim e^{-m_\pi T}$$

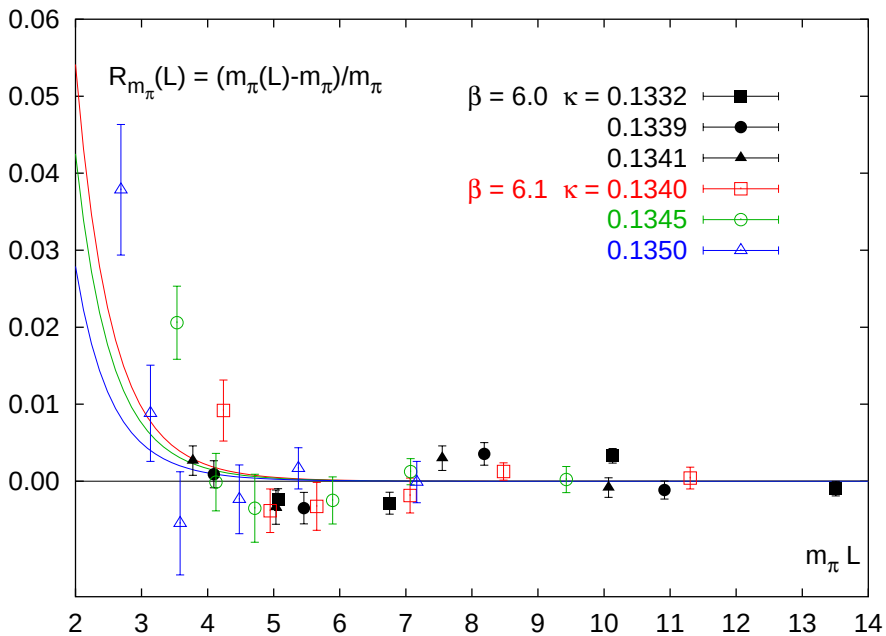


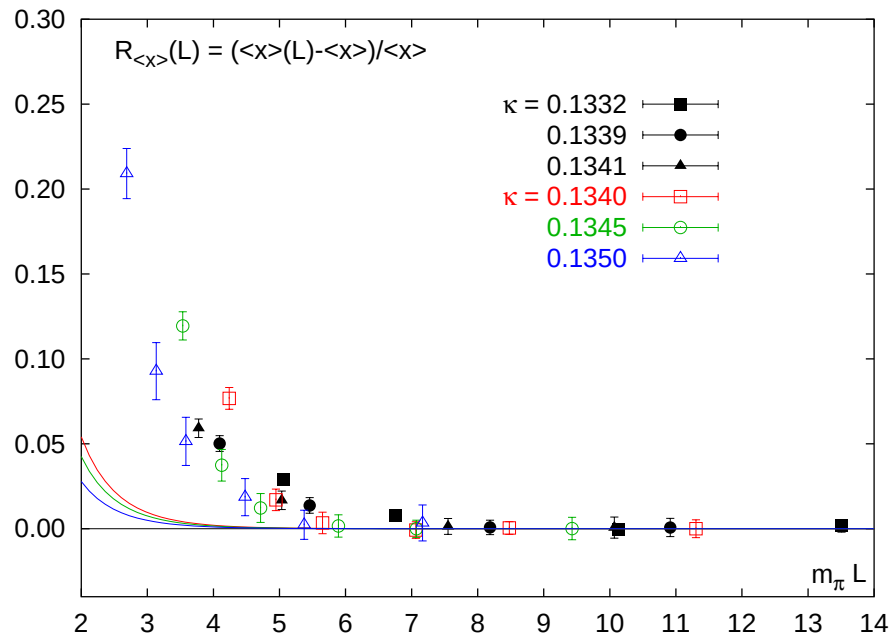
$$f_M(x_0) = a^3 \sum_{\mathrm{x}} \langle \mathcal{O}_0\, \mathcal{O}_{44}(\mathbf{x})\, \mathcal{O}_T\rangle \sim e^{-m_\pi T} \langle \pi|\mathcal{O}_{44}|\pi\rangle \{1 + \ldots\}$$

$$\Rightarrow \langle x\rangle^{bare}_{SF} \equiv \frac{2\kappa}{m_\pi}\langle \pi|\mathcal{O}_{44}|\pi\rangle = \frac{2\kappa}{m_\pi} f_M(x_0)/f_1$$

- Relative systematic deviation for the pion mass & matrix element:

$$R_{m_\pi} = \frac{m_\pi(L) - m_\pi(L=\infty)}{m_\pi(L=\infty)} \simeq \frac{3}{4(2\pi)^{3/2}} \left(\frac{m_\pi}{F_\pi}\right)^2 \left[\frac{e^{-m_\pi L}}{(m_\pi L)^{3/2}} + \frac{2\, e^{-\sqrt{2}\, m_\pi L}}{(\sqrt{2}\, m_\pi L)^{3/2}}\right]$$





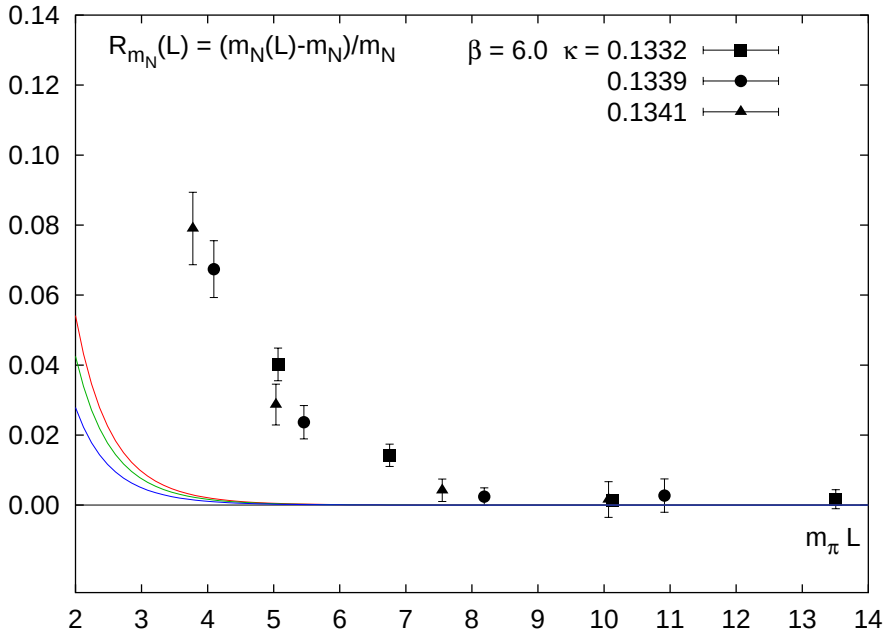
➡ $m_\pi a$: no FSE for $m_\pi L \geq 4$

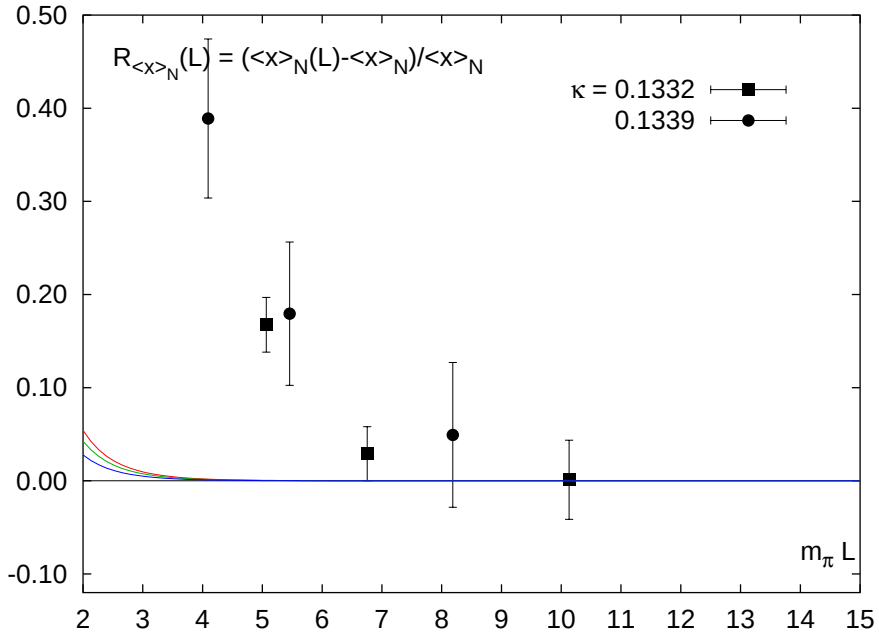
$\langle x \rangle$: strong effects up to $m_\pi L \approx 5$,

e.g. deviation of 5 % at $m_\pi L = 4$

- Relative systematic deviation for the nucleon mass & matrix element:

$$R_X = \frac{X(L) - X(L=\infty)}{X(L=\infty)} \qquad X = m_N, \langle x \rangle_N$$





➡ $m_N a$: FSE of 3-4% at $m_\pi L \approx 5$

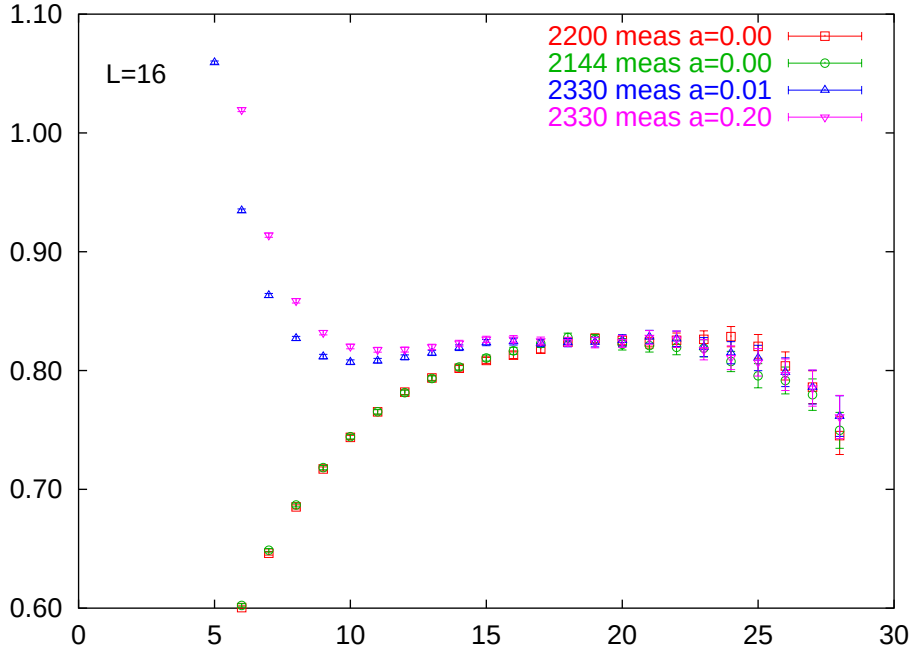
$\langle x \rangle_N$: strong effects up to $m_\pi L \approx 7$,

e.g. deviation of 20 % at $m_\pi L = 5$

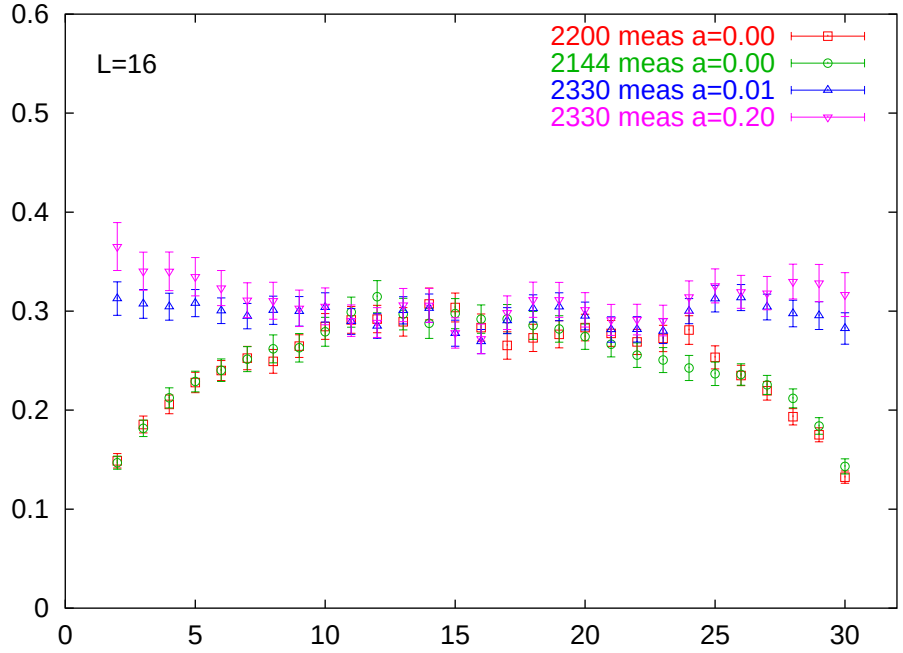
- Improvement with smeared nucleon source:

$$\sum_{xyz} e^{-ax^2} e^{-ay^2} e^{-az^2} \epsilon_{abc}\, \zeta_a(x)\zeta_b(y)\zeta_c(z)$$

nucleon mass



nucleon matrix element



Systematic Errors

- non-perturbative renormalization ✓
- continuum extrapolation ✓
- finite size effects !
- chiral extrapolation ?
- quenching effects ?

$\langle x \rangle$ with Wilson tm fermions

R. Frezzotti, G.C. Rossi, hep-lat/0306014

Wilson tmQCD lattice action in the twisted base (χ)

$$S[\chi, \bar{\chi}, U] = \bar{\chi}(D[U] + m_0 + \mu i \gamma_5 \tau_3)\chi$$

rescale with $\chi \to \frac{\sqrt{2\kappa}}{a^{3/2}}\chi\,, \quad \overline{\chi} \to \frac{\sqrt{2\kappa}}{a^{3/2}}\overline{\chi}\,, \quad \kappa = (2am_0 + 8r)^{-1}\,, \quad \tilde{\mu} = 2\kappa\mu\,.$
for the standard hopping parameter representation

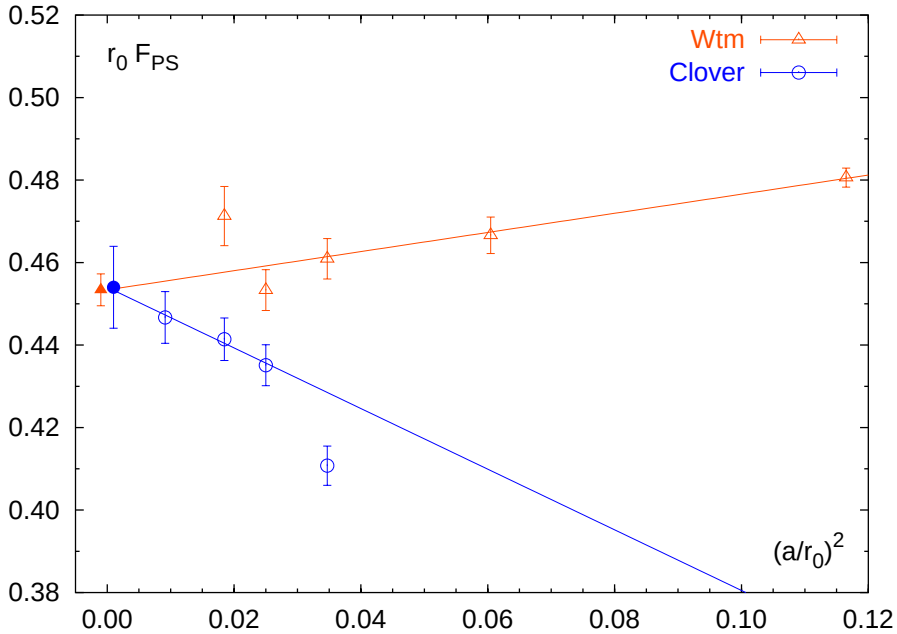
Advantages

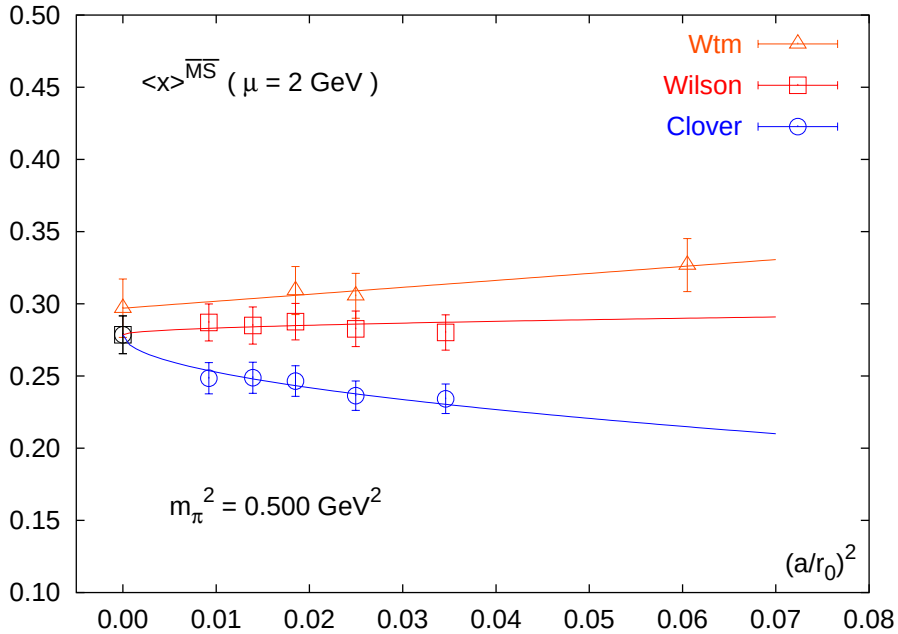
- twisted mass μ provides natural IR-cutoff
  - ➡ no accidental small eigenvalues, exceptional configurations
- full $O(a)$ improvement for parity odd correlators at $\omega = \pi/2$
- approach seems feasable down to $m_\pi \approx 270$ MeV
  - ➡ talks of A. Shindler, M. Papinutto, C. Urbach, F. Farchioni

- Scaling behavior for pion decay constant and pion matrix element:

$$\langle x \rangle^{\overline{MS}}(2\ GeV) = \frac{\langle x \rangle^{bare}}{Z_{RGI}} \cdot \frac{1}{\mathcal{F}_{\overline{MS}}(\bar{g}^2(\mu))} \qquad Z_{RGI}(g_0) = \frac{Z^{SF}(g_0, \mu_0)}{\sigma^{UV}_{INV}(\mu_0)}$$

ZeRo Collaboration, hep-lat/0405027





χ_{LF} Collaboration, PLB 586 (2004) 432

➡ talk of A. Shindler

promising scaling behavior of $\langle x \rangle$

Conclusions & Outlook

- $N_f = 2$ non-perturbative renomalization factors for $\mathcal{O}_{44}$ and $\mathcal{O}_{12}$ will be available soon
- strong FSE for nucleon mass (4%) and nucleon matrix element (20%) at $m_\pi L \approx 5$
- $\langle x \rangle$ with Wilson tm fermions shows promising scaling behavior

  ➡ approach seems feasable down to $m_\pi \approx 270$ MeV