Confinement from Instantons or Merons

Lattice 2004

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INTRODUCTION

- GOAL: INSIGHT INTO HOW QCD WORKS

- KEY FEATURES OF VACUUM AND HADRON STRUCTURE
  - CHIRAL SYMMETRY BREAKING
  - CONFINEMENT

- SEEK LOW ENERGY EFFECTIVE THEORY - ESSENTIAL DEGREES OF FREEDOM

  CLASSICAL SOLNS $\int d\phi\, e^{S(\phi)} \xrightarrow[\text{SPA}]{} \sum_i e^{S(\phi^i_{\text{CLASSICAL}})} + \cdots$

- INSTANTONS

  INSTANTONS & ZERO MODES DRIVE CHIRAL SYMM. BREAKING

  TOPOLOGICAL SUSCEPTIBILITY χ

  't HOOFT INTERACTION - BEHAVIOR OF $\langle \bar{\psi}\Gamma\psi(x) \quad \bar{\psi}\Gamma\psi(0) \rangle$

  SUCCESSFUL INSTANTON LIQUID PHENOMENOLOGY

  LATTICE EVIDENCE

    COOLING - DISTRIBUTION OF ρ

    SIMILARITY OF COOLED RESULTS & ALL GLUONS

    ZERO MODES

    ZERO MODE DOMINANCE

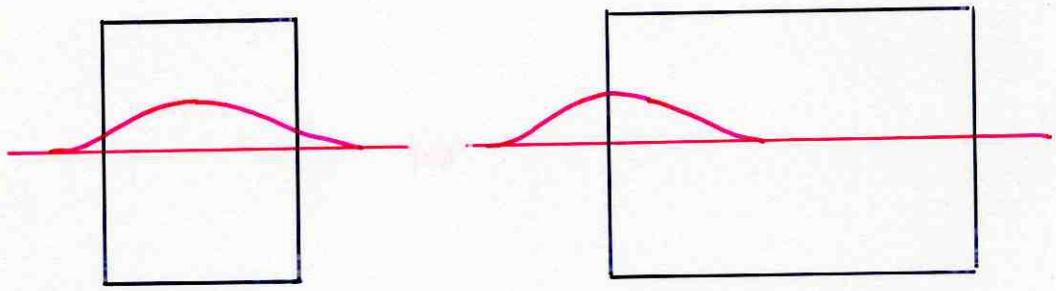
    χ

    η' MASS

CONFINEMENT

RANDOM ENSEMBLE OF SINGULAR GAUGE INSTANTONS DOES NOT CONFINE.

$$A^a_\mu = \frac{2\rho^2 \eta_{a\mu\nu} x_\nu}{x^2(x^2+\rho^2)} \sim \frac{1}{x^3} \Rightarrow \text{size} \sim \rho$$



$\Rightarrow$ CONVENTIONAL WISDOM: INSTANTONS DON'T CONFINE

QUESTIONS:

- WHAT ARE ALL THE RELEVANT CLASSICAL (PSEUDOPARTICLE) SOLUTIONS IN SU(2) YANG MILLS THEORY?
- WHAT ARE THE PROPERTIES OF EFFECTIVE THEORIES BASED ON THESE PSEUDOPARTICLE DEGREES OF FREEDOM?
- FOR EFFECTIVE THEORIES YIELDING STRING TENSION σ, HOW DO $\langle s \rangle$, χ, AND GLUEBALL MASSES COMPARE WITH LATTICE QCD?

EFFECTIVE THEORY WITH PSEUDOPARTICLE DEGREES OF FREEDOM

CONSIDER $A\{z_i, h_i\}$ COMPRISED OF MERONS OR INSTANTONS AT POSITIONS $\{z_i\}$ WITH COLOR ORIENTATIONS $\{h_i\}$

$$Z = \int dz_i \, dh_i \, e^{-\frac{1}{g^2} S[A\{z_i h_i\}]} \quad , \quad \langle O \rangle = \frac{1}{Z} \int dz_i \, dh_i \, e^{-\frac{1}{g^2} S[A\{z_i h_i\}]} \, O[A\{z_i h_i\}]$$

METROPOLIS SAMPLING OF $\{z_i h_i\}$

VIEW AS SIMPLIFIED PARAMETERIZATION OF LATTICE FIELD

SINGULAR GAUGE

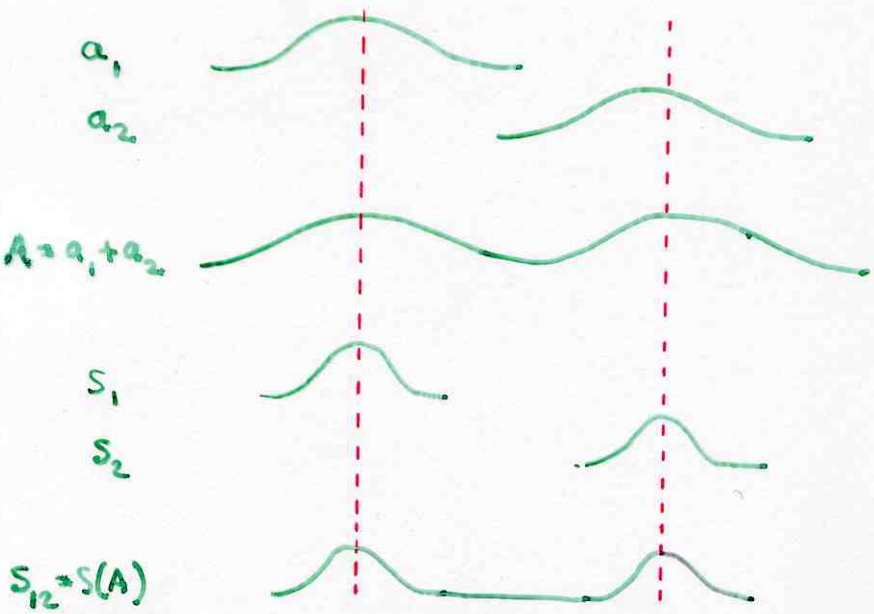
$$a_\mu^{SG} \sim \frac{2\rho^2}{x^2} \frac{\xi x_\nu}{x^2 - \rho^2}$$

$$A_\mu = \sum_i a_\mu^{SG}(z_i, h_i)$$

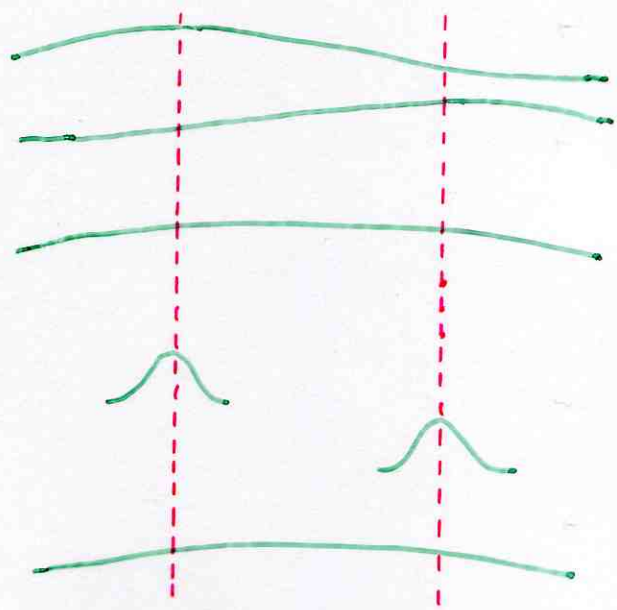
REGULAR GAUGE

$$a_\mu^{RG} \sim \frac{2 \xi x_\nu}{x^2 - \rho^2}$$

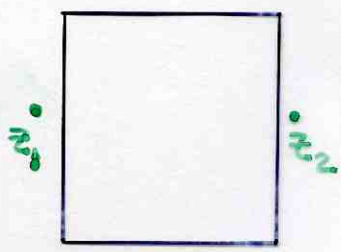
$$A_\mu = \sum_i a_\mu^{RG}(z_i h_i) = \sum_i g_i \, a^{SG}(z_i h_i) \, g_i^{-1}$$

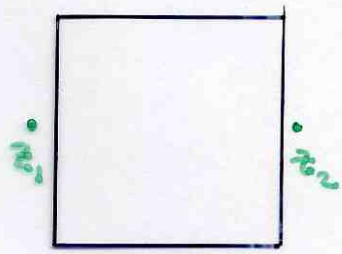


ENSEMBLE NEARLY RANDOM



ENSEMBLE HIGHLY CORRELATED





Instantons and Merons in SU(2) Yang-Mills Theory

Gauge fields A, action density

$$s(x) = \frac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}$$

and topological charge density

$$\tilde{s}(x) = \frac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

of instantons $\xi = 2$ and merons $\xi = 1$

$$A^a_\mu = \frac{\xi\,\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}$$

$$s(x) = \frac{3\xi^2}{2(x^2+\rho^2)^4}\Big[\big((2-\xi)x^2+2\rho^2\big)^2+4\rho^4\Big]$$

$$\tilde{s}(x) = \frac{6\xi^2\rho^2}{(x^2+\rho^2)^4}\Big[2\rho^2+(2-\xi)x^2\Big]$$

$$\nu = \frac{1}{8\pi^2}\int d^4x\,\tilde{s}(x) = \frac{\xi}{2}$$ topological charge

't Hooft tensor

$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu,\nu = 1,2,3 \\ \delta_{a\mu} & \nu = 0 \\ -\delta_{a\nu} & \mu = 0 \end{cases}$$

Instantons

Cancellation of abelian $\partial_\mu A_\nu - \partial_\nu A_\mu$ and non-abelian $ig[A_\mu, A_\nu]$ contributions to field strength

$$A \xrightarrow[|x|\to\infty]{} \frac{1}{x} \qquad F \xrightarrow[|x|\to\infty]{} \frac{1}{x^4}$$

Finite action

$$S = \int d^4x\, s(x) = 8\pi^2$$

x^{-4} decrease of field strength suggests transformation to "singular" gauge

$$A^a_\mu = \frac{2\rho^2}{x^2}\,\frac{\bar{\eta}_{a\mu\nu}x_\nu}{x^2+\rho^2}$$

in which for sufficiently large separations, gauge fields of instantons do not overlap – Instanton gas if $r_{ij} \gg \rho$.

Merons

No cancellation between abelian and non-abelian contributions to field strength

$$A \sim \frac{1}{x}, \quad F \sim \frac{1}{x^2}, \quad S \sim \int d^4x \frac{1}{x^4}.$$

Ultraviolet and infrared divergent action.

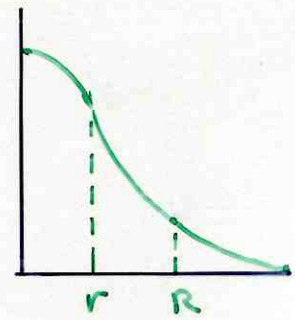
Unlike instantons in singular gauge ($A \sim x^{-3}$) merons always overlap.

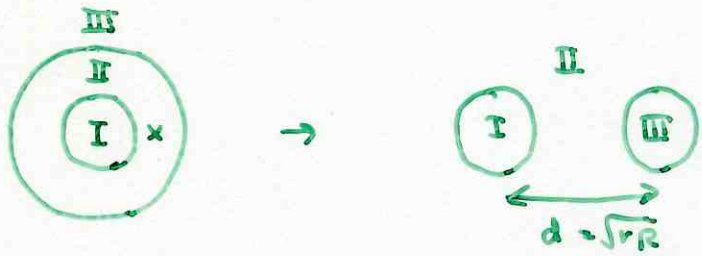
No dilute gas limit of meron ensemble.

KOSTERLITZ-THOULESS UNBINDING TRANSITION FOR MERONS

CALLAN DASHEN GROSS ANSATZ

$$A^a_\mu(x) = \eta_{a\mu\nu} x^\nu \begin{cases} \dfrac{2}{x^2+r^2} & x^2 < r^2 \quad \text{I} \\ \dfrac{1}{x^2} & r^2 < x^2 < R^2 \quad \text{II} \\ \dfrac{2}{x^2+R^2} & R^2 < x^2 \quad \text{III} \end{cases}$$





$$S \sim S_0 \ln d$$

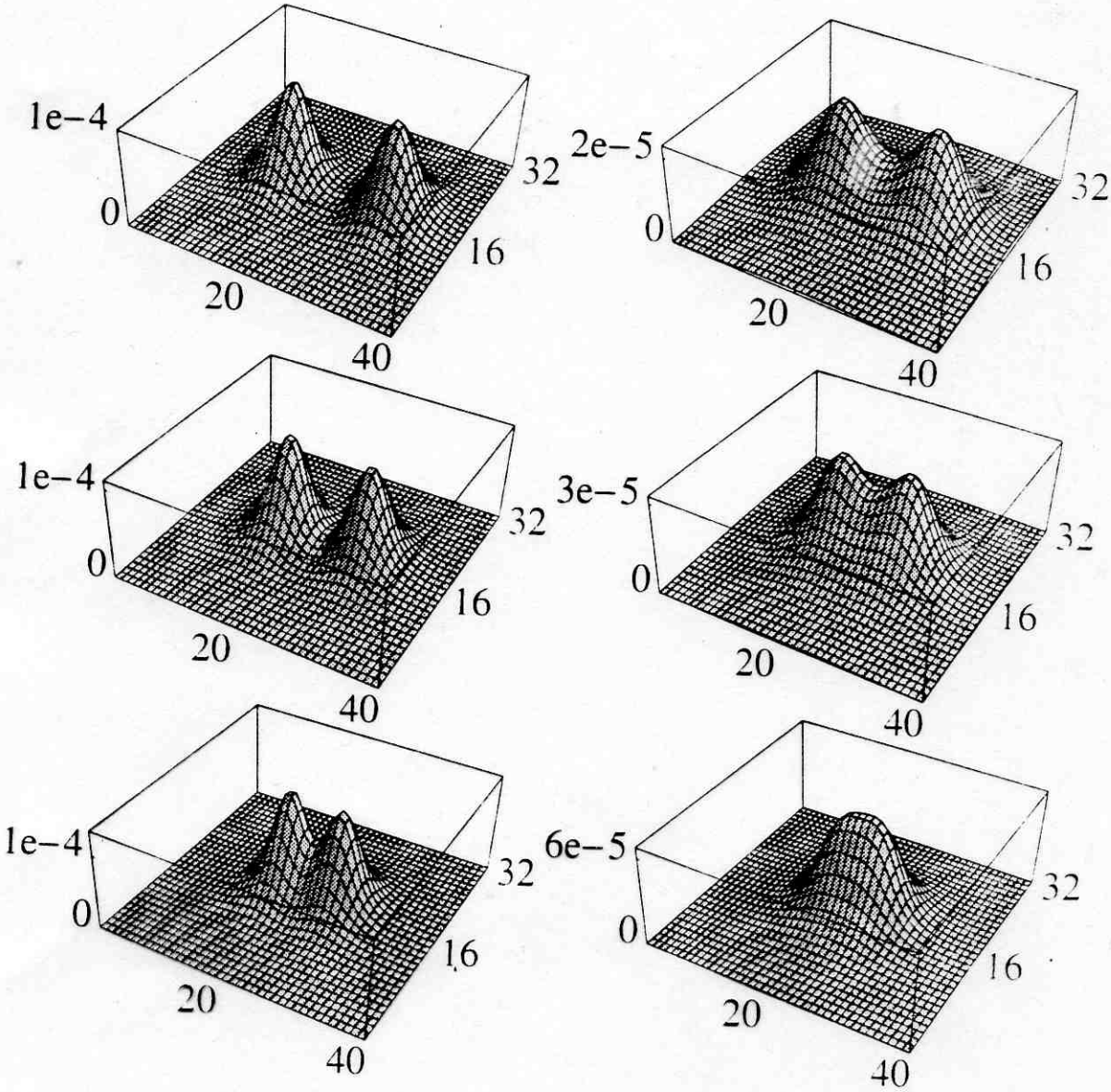
COOL CDG ANSATZ ON LATTICE TO FIND STATIONARY SOLUTIONS AND VERIFY LOGARITHMIC POTENTIAL

$$\int d^4 r_m \, e^{-\frac{1}{g^2} S(r_m)} = \int dr_m \, e^{3 \ln r_m - \frac{S_0}{g^2} \ln r_m}$$

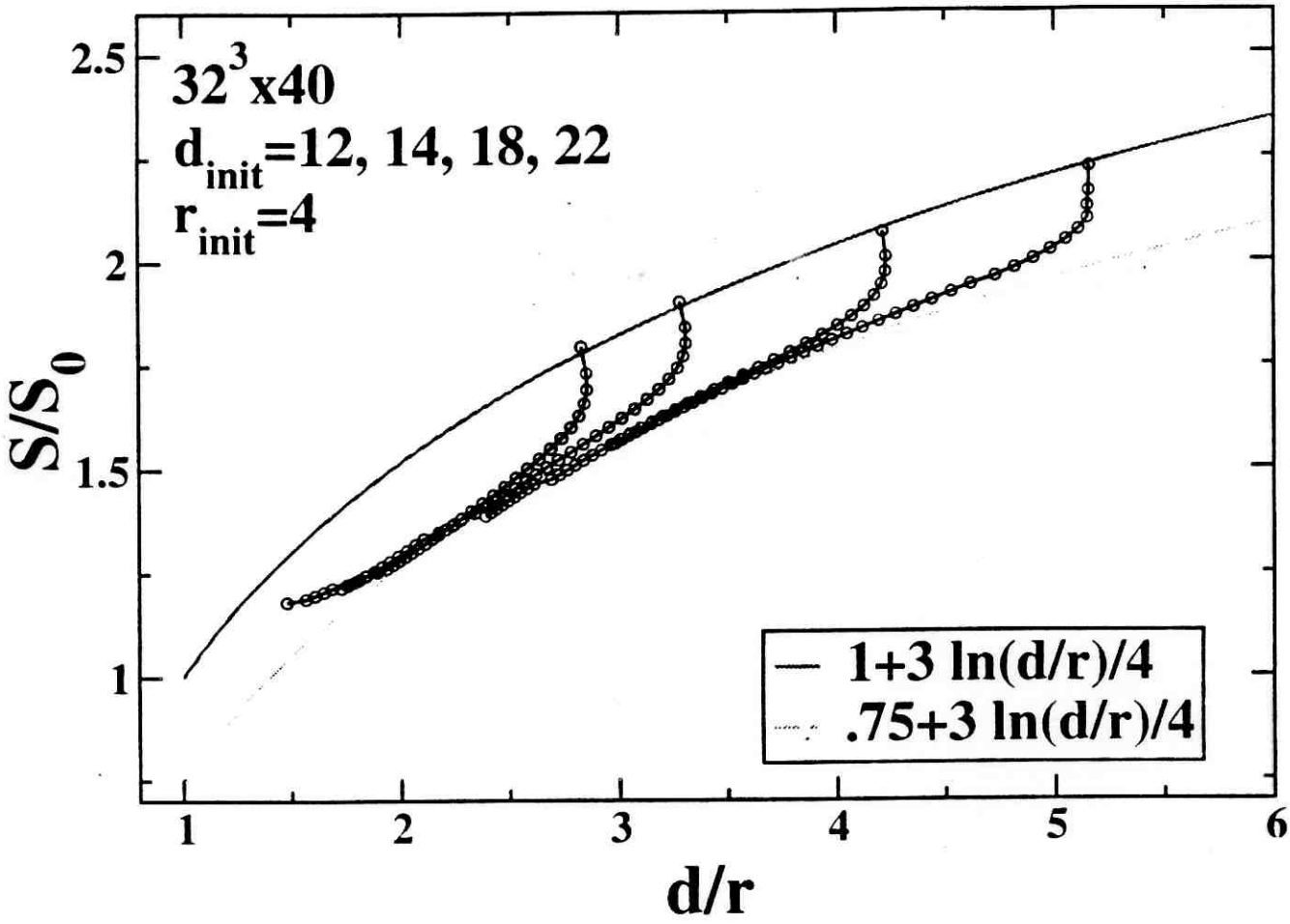
UNBINDING: $\quad 3 = \dfrac{S_0}{g^2(\rho)}$

$\rho\downarrow \quad g\downarrow$ BINDING WINS

$\rho\uparrow \quad g\uparrow$ ENTROPY WINS



J. Steele & JN PRL 85, 4207 (2000)


32^3x40
d_{init}=12, 14, 18, 22
r_{init}=4
1+3 ln(d/r)/4
.75+3 ln(d/r)/4
S/S$_0$
d/r
2.5
2
1.5
1
1
2
3
4
5
6

CONSIDER 3 PSEUDOPARTICLE OPTIONS

1) SINGULAR GAUGE INSTANTONS

$A \sim \frac{1}{x^3}$ → WELL SEPARATED

SUM OF RANDOM CONFIGURATION NEARLY SOLUTION ⇒

NO COLOR CORRELATIONS, NO CONFINEMENT

2) REGULAR GAUGE INSTANTONS

$A \sim \frac{1}{x}$ → STRONGLY OVERLAP

LARGE AVERAGE FIELD UNLESS CORRELATED

3) MERONS - SIMILAR TO (2)

USE METROPOLIS ON $\{z_i, h_i\}$ TO SAMPLE $e^{-\frac{1}{g^2} S[A\{z_i, h_i\}]}$

50 – 1000 PSEUDOPARTICLES IN BOX

EVALUATE S ON ~ 2,000 MESH POINTS

~ 200,000 CONFIGURATIONS

OBSERVABLES

WILSON LOOPS

$\langle S \rangle$

CORRELATION FUNCTIONS

χ

GLUEBALL MASSES

Meron Ensembles

Partition function

$$Z = \int dz_i \, dh_i \, e^{-\frac{1}{g^2} S[A(\boldsymbol{z}_i, h_i)]}$$

Superposition of Merons

$$A_\mu(x) = \sum_{i=1}^{N_M} h(i) a_\mu(x,i) h^{-1}(i)$$

with color orientations

$$h = h_0 + i\mathbf{h} \cdot \boldsymbol{\sigma} \qquad h_0^2 + \mathbf{h}^2 = 1$$

and locations of the center

$$a_\mu(x,i) = \frac{\sigma^a}{2g} \frac{\eta_{a\mu\nu}\Big(x_\nu - z_\nu(i)\Big)}{\Big(x - z(i)\Big)^2 + \rho^2}$$

with action density averaged over volume V

$$S[A] = \frac{1}{V} \int_V d^4x \, s(x).$$

Location of the merons restricted to a cube

$$-1 \leq z_\mu(i) \leq 1 \qquad V = 16$$

meron size

$$\rho$$

Fixed number of merons = number of antimerons

Meron size ρ regularizes UV divergence of action.

By appropriate choice of color orientations of merons total action can be made infrared finite

$$A_\mu^a(x) = \sum_{i=1}^{4} u^{ab}(i) a_\mu^b(x,i)$$

Example: with the choice

$$u(1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; u(2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$u(3) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \; u(4) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s(x) \xrightarrow[|x|\to\infty]{} \frac{1}{x^6}$$

MERON RESULTS

THERMODYNAMIC LIMIT

MAKE SURE LONG RANGE FIELDS TREATED CAREFULLY

DOUBLE L : 50 MERONS IN 2^4

800 MERONS IN 4^4

χ , $\langle s \rangle$ STABLE WITHIN 2%

WILSON LOOP →

SCALING

OBSERVE UNIVERSAL WILSON LOOP BEHAVIOR

LATTICE: a , g^2 PICK g^2

FIT $\sigma \Rightarrow a$

MERON EFFECTIVE THEORY

ρ, $n_M \equiv \frac{N_M}{V}$, g^2

PICK g^2

FIT $\sigma \Rightarrow$ 1 COMBINATION OF ρ, $n_M^{-1/4}$

INSENSITIVE TO OTHER COMBINATION

RESCALE $A \to \lambda A$

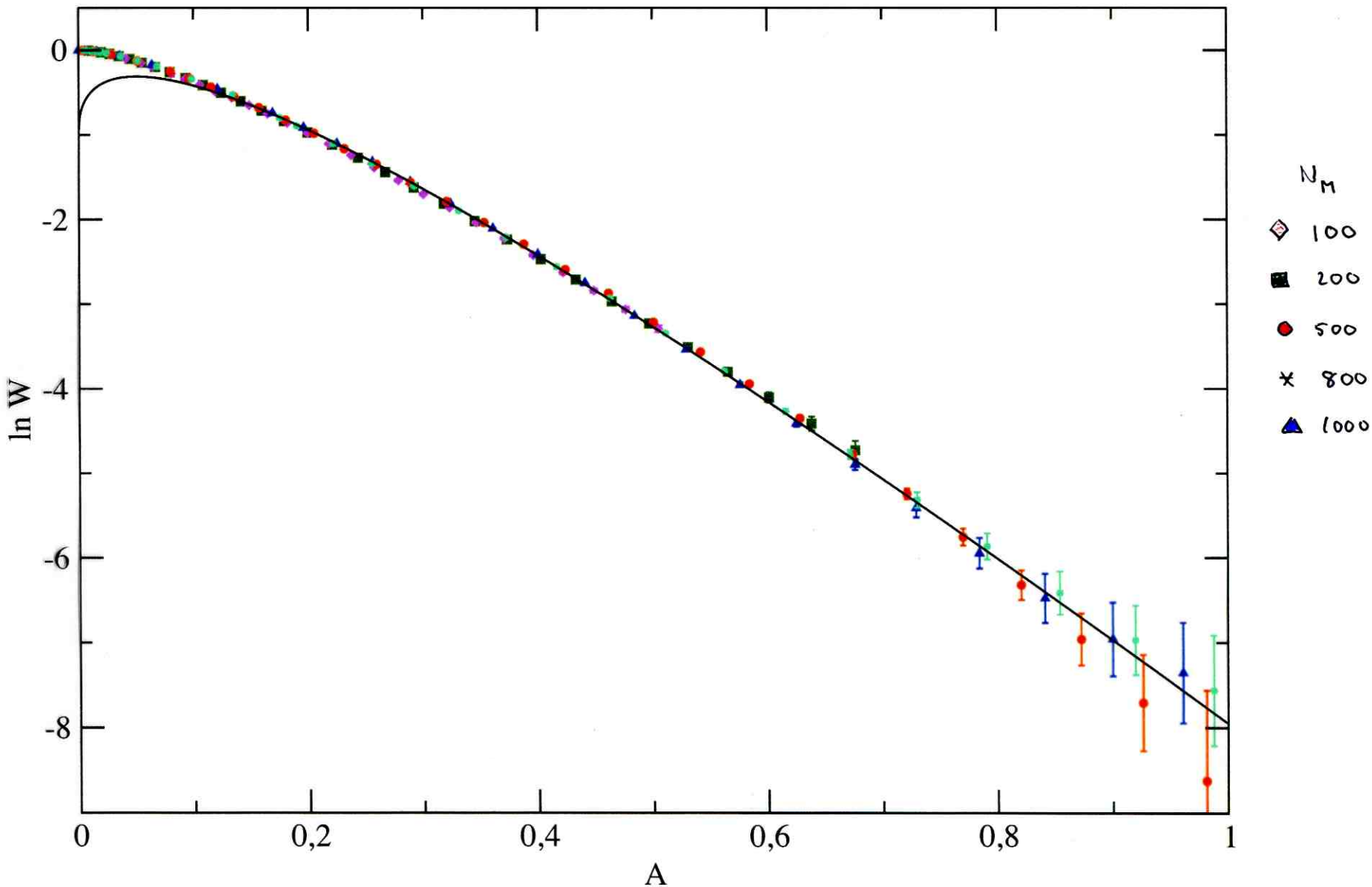
FIT $\ln \langle W \rangle = \omega + \tau \sqrt{\lambda} P - \sigma \lambda A$ →

CLEAR AREA LAW

RESULTS IN PHYSICAL UNITS (USE $\sqrt{\sigma} = 410$ MeV)

n_M (fm^{-4})	ρ (fm)	$\langle s \rangle$ (fm^{-4})	$\chi^{1/4}$ (MeV)
4.8	0.30	210	118
3.8	0.27	209	120
3.2	0.23	226	124
2.8	0.20	225	124
2.7	0.16	218	132
LATTICE		80-446	198 ± 3

SCALING OF WILSON LOOPS FOR MERON ENSEMBLES



RESULTS FOR REGULAR GAUGE INSTANTONS

SCALING

UNIVERSAL SCALING BEHAVIOR

CLEAR AREA LAW →

CORRELATION FUNCTIONS

$\langle \mathcal{O}(x)\ \mathcal{O}(0) \rangle$ point to point

$\langle \bar{\mathcal{O}}(t)\ \bar{\mathcal{O}}(0) \rangle \equiv \frac{1}{V} \int d^3\vec{x}\, d^3\vec{y}\, \langle \mathcal{O}(t,\vec{x})\, \mathcal{O}(0,\vec{y}) \rangle$ momentum projected

$\frac{1}{2} E \cdot B$: χ

$\frac{1}{4}(E^2 - B^2)$: 0^+

$T_i^+ = \epsilon^{abc} E_i^a E^b \cdot B^c$: 1^+

$T_i^- = \epsilon^{abc} B_i^a E^b \cdot B^c$ 1^-

$E_i E_j - \frac{1}{3} E^2 \delta_{ij} - B_i B_j + \frac{1}{3} B^2 \delta_{ij}$ 2^+ →

RESULTS IN PHYSICAL UNITS ($\sqrt{\sigma}$ = 410 MEV)

n_I (fm^{-4})	ρ (fm)	$\langle s \rangle$ (fm^{-4})	$\chi^{1/4}$ (MeV)
1.64	0.19	340	190
1.45	0.09	409	225
1.37	0.05	418	217
Lattice		80[a] – 496[b]	198 ± 3[c]

GLUEBALL MASSES (GEV) STATE	INSTANTONS	LATTICE[d]
1^-	1.7	4.2 (4)
1^+	2.0	3.8 (4)
2^+	1.2	2.3 (1)
0^+	1.0	1.5 (1)

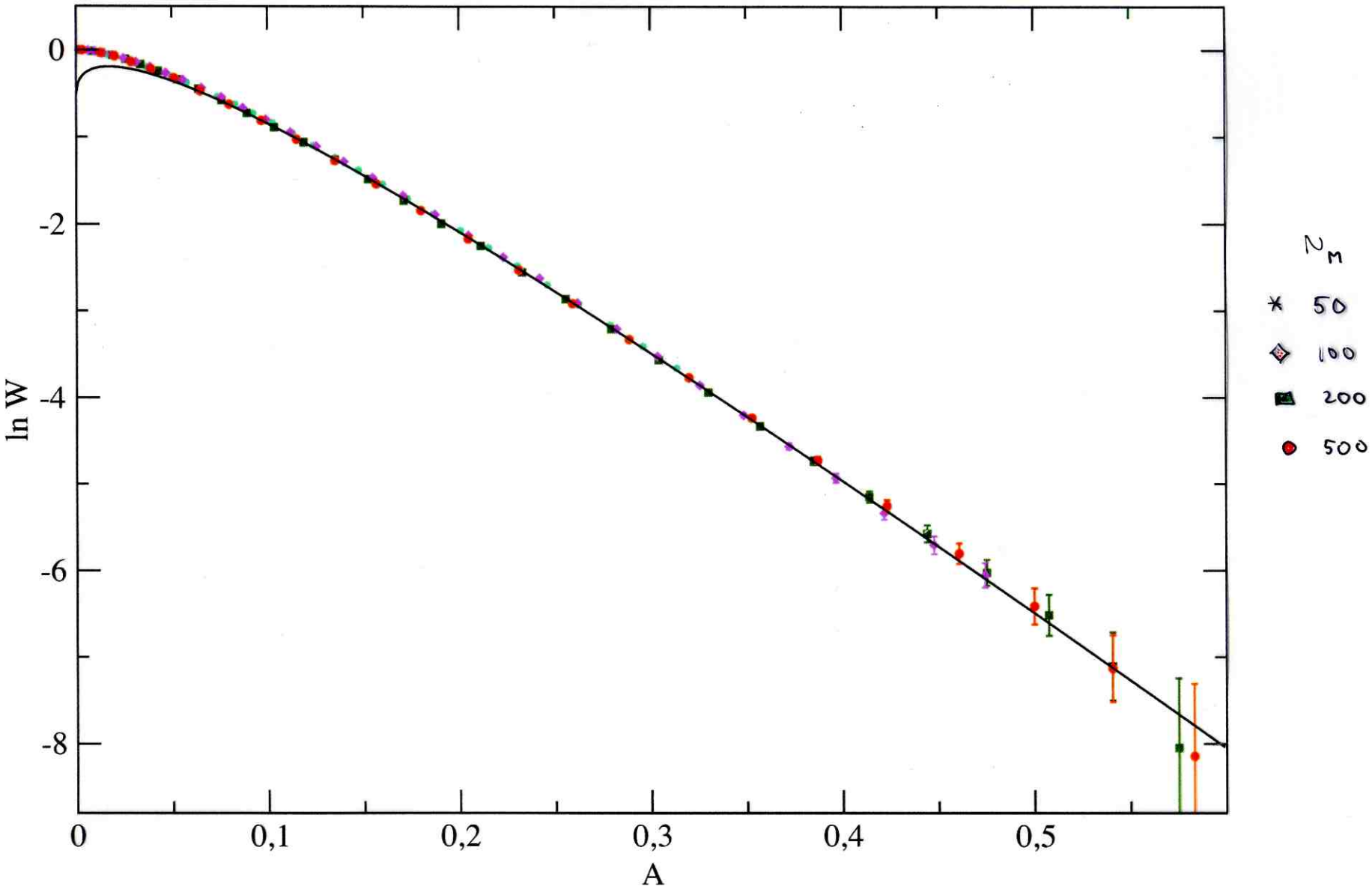
a) Campostrini et al Z. Phys. C 22 143 (1984)

b) Di Giacomo et al NP B 338 299 (1990)

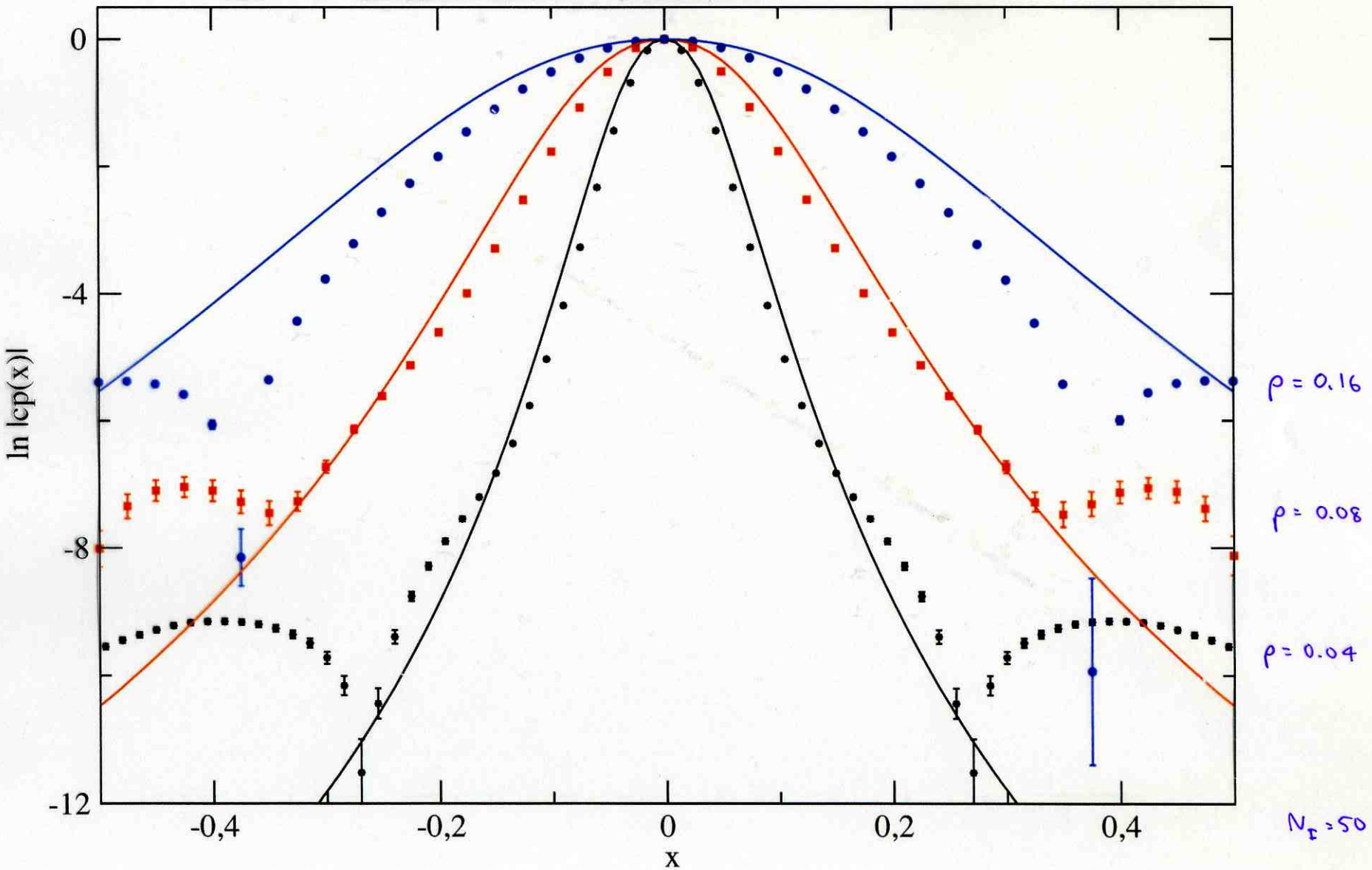
c) Lucini & Teper hep-lat/0103027

d) Teper hep-lat/9812187

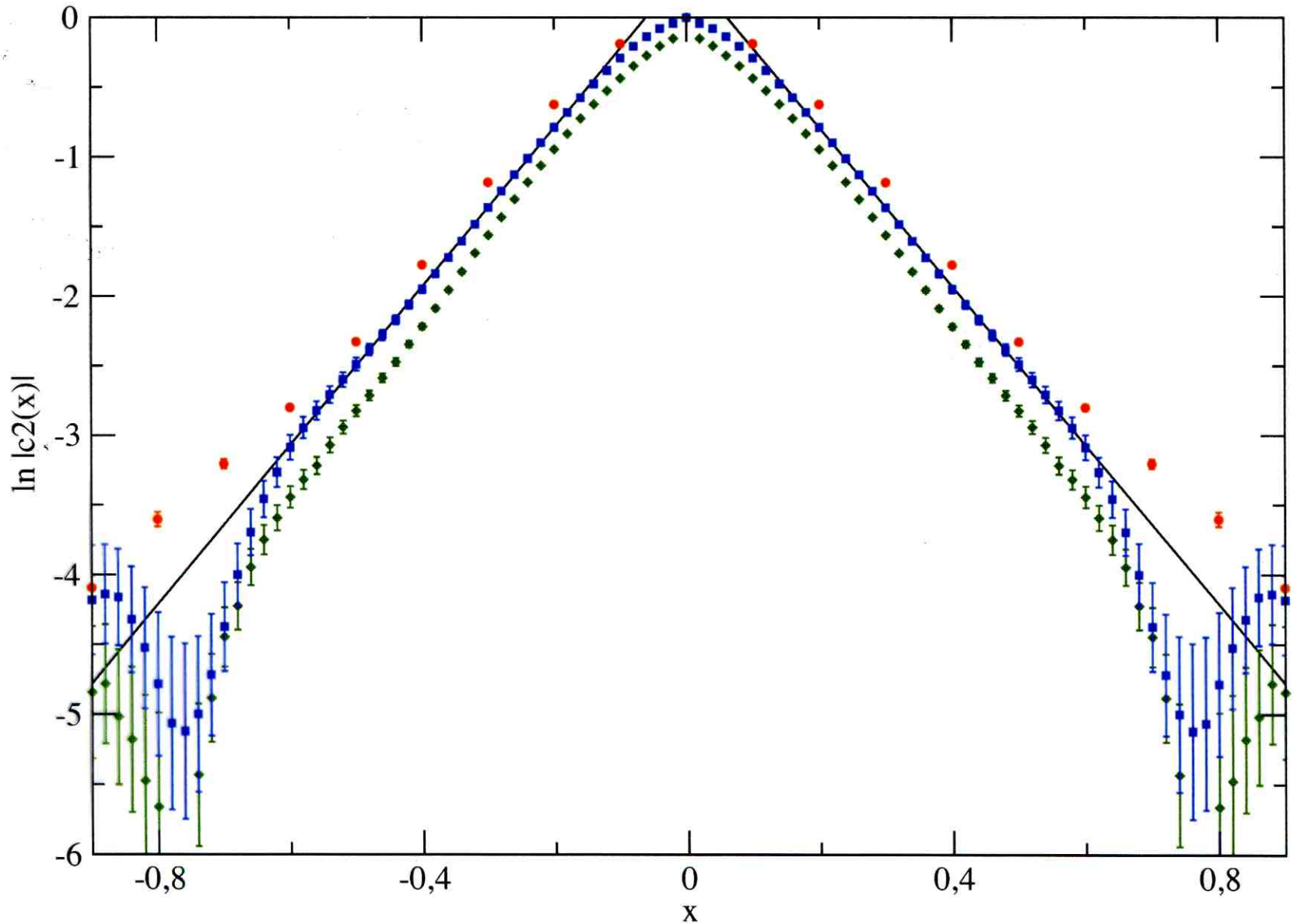
SCALING OF WILSON LOOPS FOR REGULAR GAUGE INSTANTON ENSEMBLES



$\langle \frac{1}{2} E \cdot B(x) \; \frac{1}{2} E \cdot B(0) \rangle$ χ



$\langle (E^2-B^2)\,(E^2-B^2) \rangle \qquad 0^+$



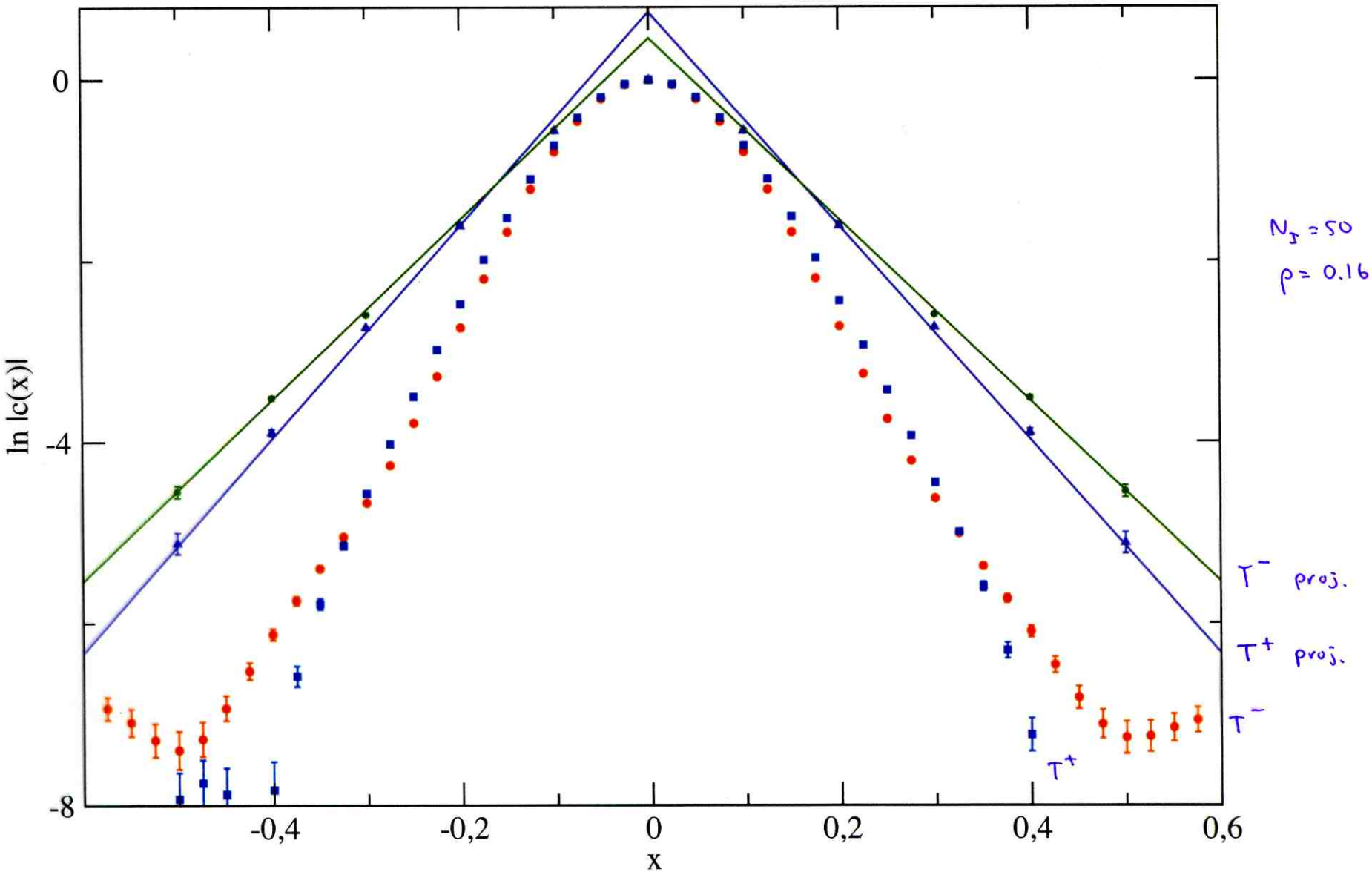
$N_I = 50$

$\rho = 0.16$

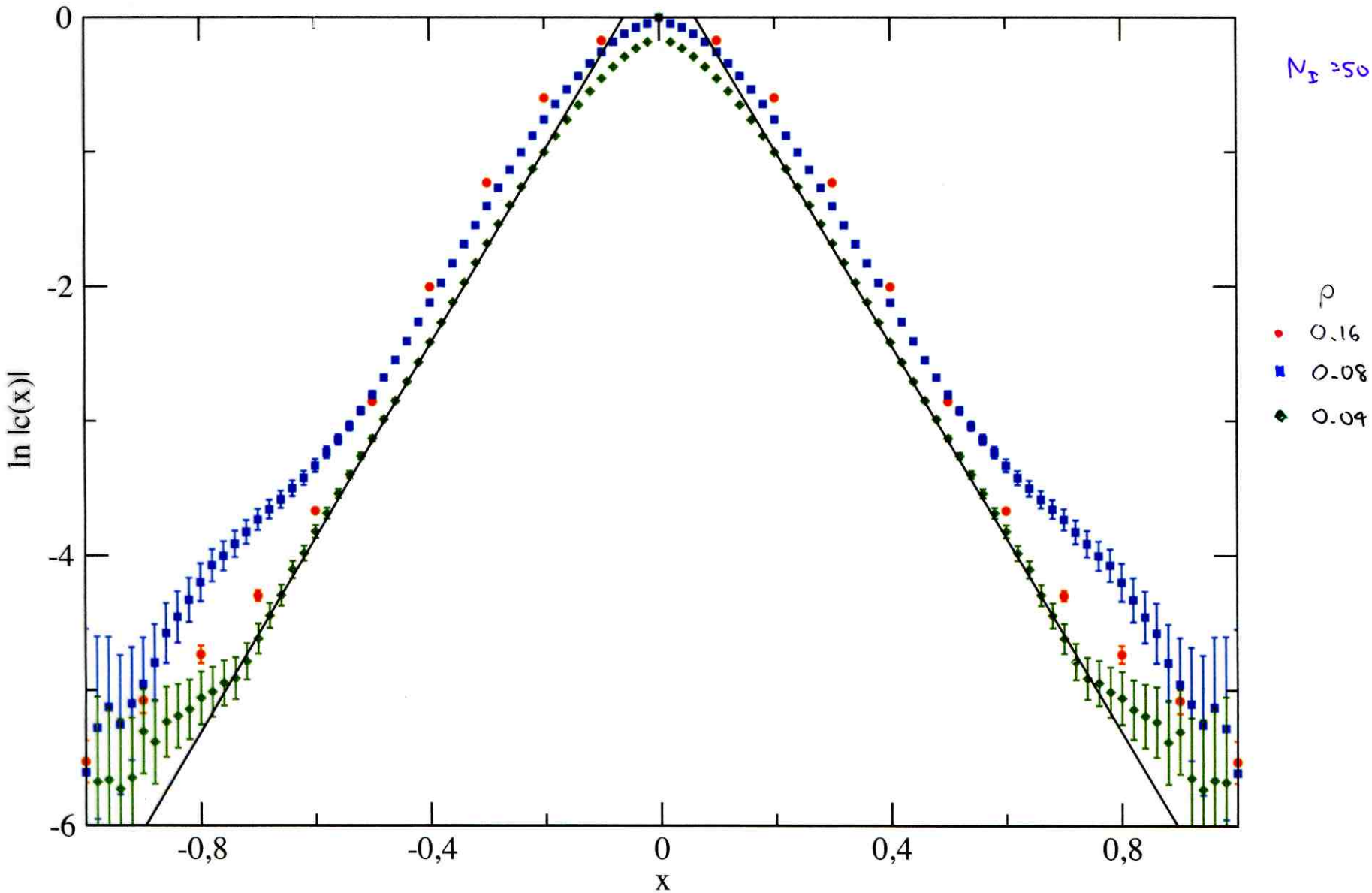
$\rho = 0.08$

$\rho = 0.04$

$\langle T_i \; T_i \rangle$ $\quad 1^+ , 1^-$



$\langle (E_i E_j - B_i B_j)(E_i E_j - B_i B_j) \rangle$ 2^4



SUMMARY

- IN CONTRAST TO SINGULAR GAUGE INSTANTONS, ENSEMBLES OF REGULAR GAUGE INSTANTONS OR MERONS PRODUCE CONFINEMENT

- WHEN SCALE IS SET BY σ, $\langle s \rangle$, χ, AND GLUEBALL MASSES ARE IN QUALITATIVE AGREEMENT WITH LATTICE QCD

- AT VERY LEAST, INSTANTON MODEL WITH CHIRAL SYMMETRY BREAKING + CONFINEMENT

- AT MOST, MOTIVATE MORE SERIOUS EXAMINATION OF ROLE OF INSTANTONS / MERONS IN CONFINEMENT