NLO anomalous dimension of $\Delta F = 2$ parity–odd four fermion operators in the Schrödinger Functional scheme

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in collaboration with

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ALPHA Collaboration

Phenomenology of four–fermion operators

A number of phenomenological applications (without penguins and power subtractions) involve four–fermion dimension–six operators:

- Study of $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ oscillations through $B_{K,B}$: $\mathcal{O}^{\Delta S=2}$, $\mathcal{O}^{\Delta B=2}$
- Study of the $\Delta I = 3/2$ sector of the decay $K \to \pi\pi$: $\mathcal{O}_9^{3/2}$, $\mathcal{O}_{10}^{3/2}$
- Study of the electropenguin contribution to $K \to \pi\pi$: $\mathcal{O}_7^{3/2}$, $\mathcal{O}_8^{3/2}$
- FCNC processes beyond the SM
  (SUSY, LR-symmetric models, multi–Higgs models)

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tmQCD for B_K

- B_K is given in QCD by

$$\langle \bar{K}^0 | \mathcal{O}^{\Delta S=2} | K^0 \rangle = \tfrac{8}{3} B_K F_K^2 m_K^2 \qquad \mathcal{O}^{\Delta S=2} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$$

- As parity does not change, only the parity–even contributions survive

$$\mathcal{O}^{\Delta S=2} \;\rightarrow\; \mathcal{O}_{VV+AA}$$

$$\text{Wilson reg.} \rightarrow\; \mathcal{O}^R_{VV+AA}(\mu) = Z_{VV+AA}(a\mu, g_0^2)[\mathcal{O}_{VV+AA}(a) + \textstyle\sum_{i=1}^4 Z_i(g_0^2)\mathcal{O}_i(a)]$$

- A twist rotation of the d–quark gives (Frezzotti et al., JHEP 0108:058,2001)

$$\mathcal{O}'_{VV+AA} = \cos(\alpha)\mathcal{O}_{VV+AA} - i\sin(\alpha)\mathcal{O}_{VA+AV} \quad \stackrel{\alpha=\pi/2}{\Longrightarrow} \quad -i\mathcal{O}_{VA+AV}$$

- $\mathcal{O}_{VA+AV}$ renormalizes multiplicatively!
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A basis of $\Delta F = 2$ $d = 6$ parity–odd 4–fermion operators

General structure of the operators

$$\mathcal{O}^{\pm}_{\Gamma^{(1)}\Gamma^{(2)}} = \overline{\psi}_1\Gamma^{(1)}\psi_2\overline{\psi}_3\Gamma^{(2)}\psi_4 \pm \overline{\psi}_1\Gamma^{(1)}\psi_4\overline{\psi}_3\Gamma^{(2)}\psi_2$$

$$\mathcal{O}^{\pm}_{\Gamma^{(1)}\Gamma^{(2)}\pm\Gamma^{(2)}\Gamma^{(1)}} = \mathcal{O}^{\pm}_{\Gamma^{(1)}\Gamma^{(2)}} \pm \mathcal{O}^{\pm}_{\Gamma^{(2)}\Gamma^{(1)}}$$

10 Operators

$$\mathcal{Q}_1^{\pm} = \mathcal{O}^{\pm}_{VA+AV}$$

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$$\mathcal{Q}_4^{\pm} = \mathcal{O}^{\pm}_{PS+SP}$$

$$\mathcal{Q}_5^{\pm} = \mathcal{O}^{\pm}_{T\tilde{T}}$$

Renormalization matrix

$$\begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}^{\pm}_{R} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix}^{\pm} \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}^{\pm}$$

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Non perturbative renormalization in the SF scheme

- Non perturbative renormalization of $\mathcal{Q}_1^{\pm} = \mathcal{O}_{VA+AV}^{\pm}$ in the SF scheme has been completed

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One–loop perturbative renormalization (aim of the present work!)

- checks the consistency of the scheme
- provides estimates of perturbative lattice artefacts
- allows a comparison: two–loop perturbative vs. non–perturbative running

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Schrödinger Functional (basics)

- A finite volume scheme. QCD is set up on a volume $T \times L^3$ ($T = L$), with periodic b.c. on spatial directions and Dirichlet b.c. on time
- The finite size L is used as the renormalization scale $\mu = \frac{1}{L}$, and the continuum limit is performed by letting $a \to 0$ at fixed L
- The main advantage is the possibility to compute the non-perturbative running in the continuum limit
- Correlation functions are defined in the SF in order to probe quantum operators. The operator is placed in the bulk at a physical distance from the boundaries ($x_0 = T/2$), and is probed by boundary sources
- Parity is conserved by the QCD lagrangian. In order to probe parity–odd operators, we need parity-odd boundary sources

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Renormalization of $\mathcal{O}^{\pm}_{VA+AV}$ in the SF scheme

- First we introduce bilinear boundary sources at times $x_0 = 0, T$

$$S_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{yz}} \bar{\zeta}_{f_1}(\mathbf{y})\Gamma\zeta_{f_2}(\mathbf{z}) \qquad S'_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y'z'}} \bar{\zeta}'_{f_1}(\mathbf{y'})\Gamma\zeta'_{f_2}(\mathbf{z'})$$

- The operator is probed by three bilinear sources

$$F^{\pm}(x_0) = \frac{a^3}{L^3} \sum_{\mathbf{x}} \langle S'[\Gamma_3]\mathcal{O}^{\pm}_{VA+AV}(x)S[\Gamma_2]S[\Gamma_1]\rangle$$

- $[\Gamma_1, \Gamma_2, \Gamma_3]$ must be parity-odd

- The operator correlation function $F^{\pm}$ must be properly normalized

$$f_1 = -\frac{1}{L^6}\langle S'[\gamma_5]S[\gamma_5]\rangle$$

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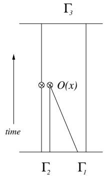

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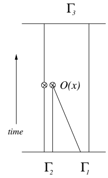

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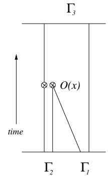
Renormalization of $\mathcal{O}^{\pm}_{VA+AV}$ in the SF scheme

- First we introduce bilinear boundary sources at times $x_0 = 0, T$

$$S_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{yz}} \bar{\zeta}_{f_1}(\mathbf{y})\Gamma\zeta_{f_2}(\mathbf{z}) \qquad S'_{f_1 f_2}[\Gamma] = a^6 \sum_{\mathbf{y'z'}} \bar{\zeta}'_{f_1}(\mathbf{y'})\Gamma\zeta'_{f_2}(\mathbf{z'})$$

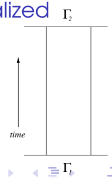
- The operator is probed by three bilinear sources



$$F^{\pm}(x_0) = \frac{a^3}{L^3} \sum_{\mathbf{x}} \langle S'[\Gamma_3]\mathcal{O}^{\pm}_{VA+AV}(x)S[\Gamma_2]S[\Gamma_1]\rangle$$

- $[\Gamma_1, \Gamma_2, \Gamma_3]$ must be parity-odd

- The operator correlation function $F^{\pm}$ must be properly normalized



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Renormalization of $\mathcal{O}^{\pm}_{VA+AV}$ in the SF scheme (2)

- Therefore we introduce a normalized c.f.

$$h^{\pm}(x_0) = \frac{F^{\pm}(x_0)}{f_1^{\alpha}\ k_1^{\beta}}\Bigg|_{\alpha+\beta=3/2}$$

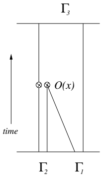
- and impose a renormalization condition on it:

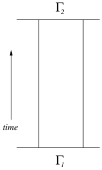
$$h^{\pm}(T/2)\Bigg|^{m_R=0}_{Renormalized} = h^{\pm}(T/2)\Bigg|^{m_0=0}_{Tree-Level}$$

- where

$$h^{\pm}(T/2)\Bigg|^{m_R=0}_{Renormalized} = Z_{\mathcal{O}}^{VA+AV,\pm}\ h^{\pm}(T/2)\Bigg|^{m_R=0}$$

- $Z_{\mathcal{O}}^{VA+AV,\pm}$ depends upon the choice of $[\Gamma_1, \Gamma_2, \Gamma_3]$ and (α, β)





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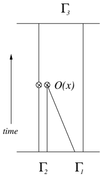

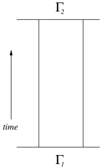

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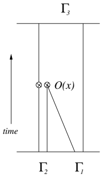

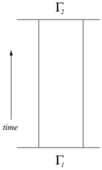

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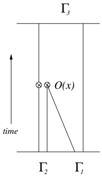

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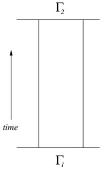

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Renormalization schemes

- We consider 9 different renormalization schemes

$[\Gamma_1,\Gamma_2,\Gamma_3]$	α	β
$[\gamma_5,\gamma_5,\gamma_5]$	3/2	0
$\frac{1}{3}\sum_{k=1}^{3}[\gamma_5,\gamma_k,\gamma_k]$	3/2	0
$\frac{1}{3}\sum_{k=1}^{3}[\gamma_k,\gamma_5,\gamma_k]$	3/2	0
$\frac{1}{3}\sum_{k=1}^{3}[\gamma_k,\gamma_k,\gamma_5]$	3/2	0
$\frac{1}{6}\epsilon_{ijk}[\gamma_i,\gamma_j,\gamma_k]$	3/2	0

$[\Gamma_1,\Gamma_2,\Gamma_3]$	α	β
$\frac{1}{3}\sum_{k=1}^{3}[\gamma_5,\gamma_k,\gamma_k]$	1/2	1
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$\frac{1}{6}\epsilon_{ijk}[\gamma_i,\gamma_j,\gamma_k]$	0	3/2

Perturbative expansion

- Correlation functions are expanded to one loop order...

$$F^{\pm}(x_0) = F^{(0)\pm}(x_0) + \bar{g}^2\left[F^{(1)\pm}(x_0) + m_c^{(1)}\frac{\partial F^{(0)\pm}}{\partial m_0}(x_0)\right] + O(g^4)$$

$$f_1 = f_1^{(0)} + \bar{g}^2\left[f_1^{(1)} + m_c^{(1)}\frac{\partial f_1^{(0)}}{\partial m_0}\right] + O(\bar{g}^4)$$

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- ...as well as the renormalization constant

$$Z_{\mathcal{O}}^{VA+AV,\pm} = 1 + \bar{g}^2 Z^{(1)} + O(\bar{g}^4)$$

$$Z^{(1)} = -\left\{\frac{h^{(1)}}{h^{(0)}} + \frac{m_c^{(1)}}{h^{(0)}}\frac{\partial h^{(0)}}{\partial m_0}\right\} + \alpha\left\{\frac{f_1^{(1)}}{f_1^{(0)}} + \frac{m_c^{(1)}}{f_1^{(0)}}\frac{\partial f_1^{(0)}}{\partial m_0}\right\} + \beta\left\{\frac{k_1^{(1)}}{k_1^{(0)}} + \frac{m_c^{(1)}}{k_1^{(0)}}\frac{\partial k_1^{(0)}}{\partial m_0}\right\}$$

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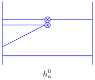
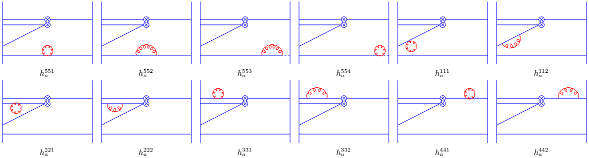
One loop Feynman Diagrams for $F^{\pm}(x_0)$

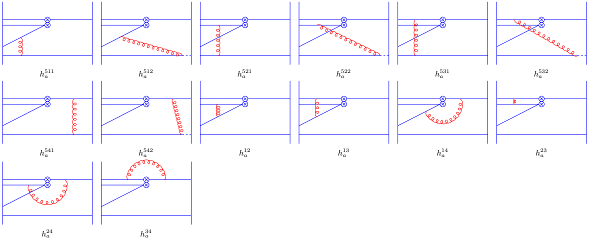
Tree Level



Self Energies



Leg – Leg



NLO-AD in the SF scheme from matching with DRED

- Once the NLO-AD is known in a reference scheme, it can be computed in any other scheme by a matching procedure

Matching Formula: DRED → SF

$$\gamma_{SF}^{(1)\pm} = \gamma_{DRED}^{(1)\pm} + 2b_0 \left[B_{SF}^{\pm} - \frac{Z^{\pm}}{16\pi^2} \right] + \gamma^{(0)} \frac{c_{1,0} + c_{1,1} N_f}{4\pi}$$

(Martinelli,Phys.Lett.B141:395,1984), (Frezzotti et al.,Nucl.Phys.B373:781-794,1992)

- Results for the 9 SF schemes

$[\Gamma_1, \Gamma_2, \Gamma_3]$	α	β	$\gamma_{SF}^{(1)+}/\gamma_{SF}^{(0)+}$	$\gamma_{SF}^{(1)-}/\gamma_{SF}^{(1)-}$
$[\gamma_5, \gamma_5, \gamma_5]$	3/2	0	$0.0204(1) + 0.008023(7)N_f$	$-0.46703(6) + 0.038908(3)N_f$
$[\gamma_5, \gamma_k, \gamma_k]$	3/2	0	$-0.3145(2) + 0.02832(1)N_f$	$-0.10915(8) + 0.017218(5)N_f$
$[\gamma_k, \gamma_5, \gamma_k]$	3/2	0	$0.0667(1) + 0.005216(7)N_f$	$-0.49019(6) + 0.040312(3)N_f$
$[\gamma_k, \gamma_k, \gamma_5]$	3/2	0	$-0.2878(2) + 0.02670(1)N_f$	$-0.15903(8) + 0.020242(5)N_f$
$\epsilon_{ijk}[\gamma_i, \gamma_j, \gamma_k]$	3/2	0	$-0.2397(2) + 0.02379(1)N_f$	$-0.18397(9) + 0.021753(5)N_f$
$[\gamma_5, \gamma_k, \gamma_k]$	1/2	1	$-0.3608(3) + 0.03113(2)N_f$	$-0.08596(8) + 0.015813(5)N_f$
$[\gamma_k, \gamma_5, \gamma_k]$	1/2	1	$0.0205(2) + 0.00802(1)N_f$	$-0.46703(6) + 0.038908(3)N_f$
$[\gamma_k, \gamma_k, \gamma_5]$	1/2	1	$-0.3341(3) + 0.02951(2)N_f$	$-0.13585(8) + 0.018837(5)N_f$
$\epsilon_{ijk}[\gamma_i, \gamma_j, \gamma_k]$	0	3/2	$-0.3091(3) + 0.02799(2)N_f$	$-0.14921(9) + 0.019647(5)N_f$

Perturbative Running

- The step scaling function measures the running of the operator with the renormalization scale $\mu = 1/L$

$$\sigma^{\pm}[\bar{g}^2(L)] = \frac{Z_{\mathcal{O}}^{VA+AV,\pm}(2L)}{Z_{\mathcal{O}}^{VA+AV,\pm}(L)}$$

- It can be expanded in perturbation theory

$$\sigma^{\pm}(u) = 1 + \sigma_1^{\pm} u + \sigma_2^{\pm} u^2 + O(u^3)\big|_{u=\bar{g}^2(L)}$$

$$\sigma_1^{\pm} = \gamma^{(0)\pm} \ln 2$$

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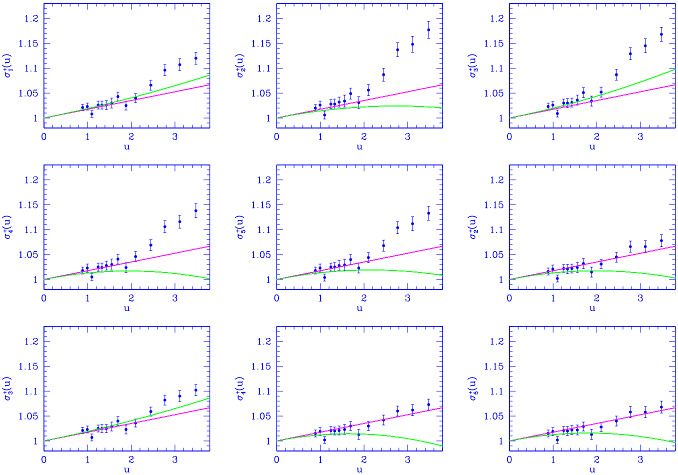
$\mathcal{O}^{+}_{VA+AV}$: perturbative vs. non-perturbative running



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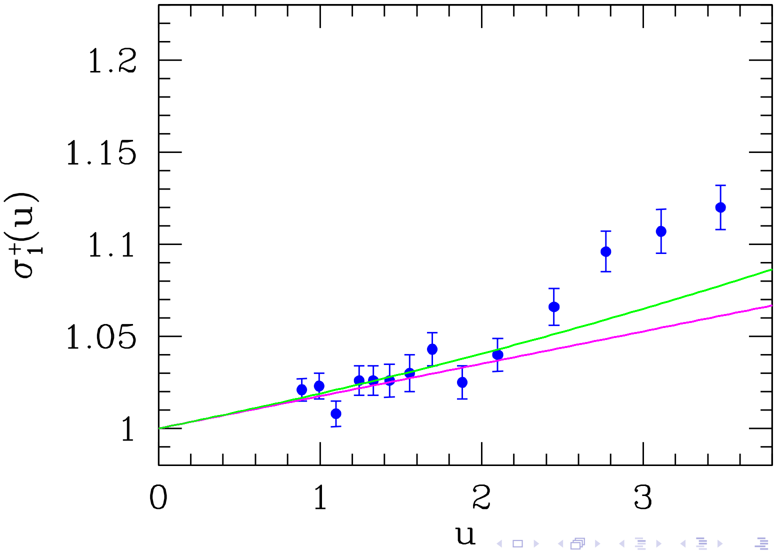

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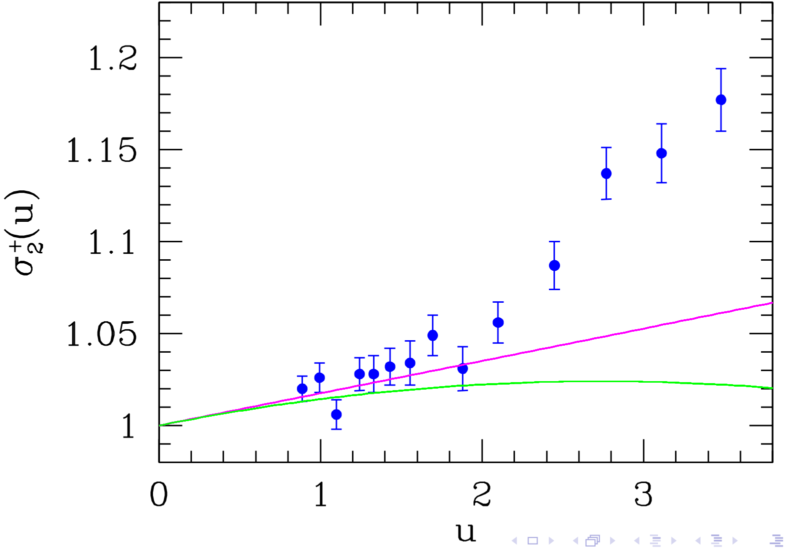

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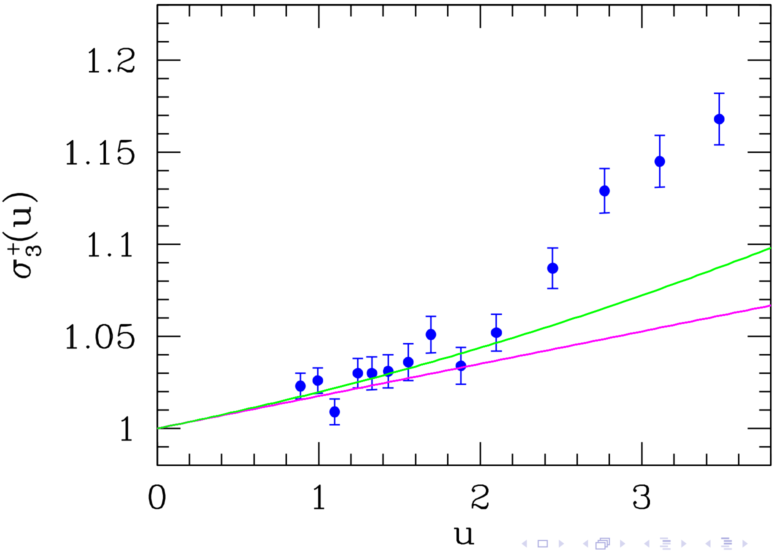

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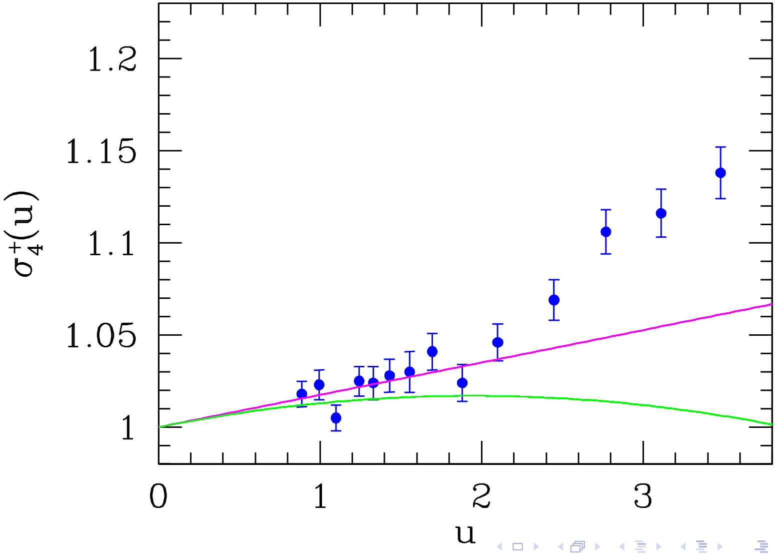

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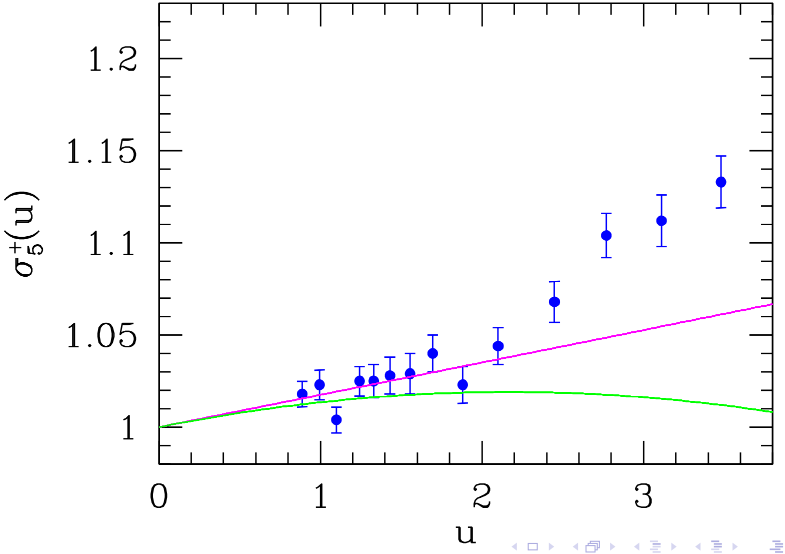

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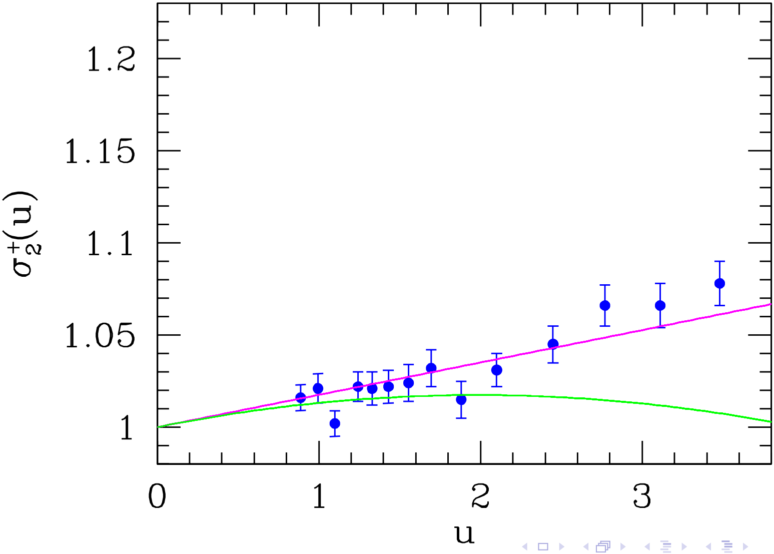

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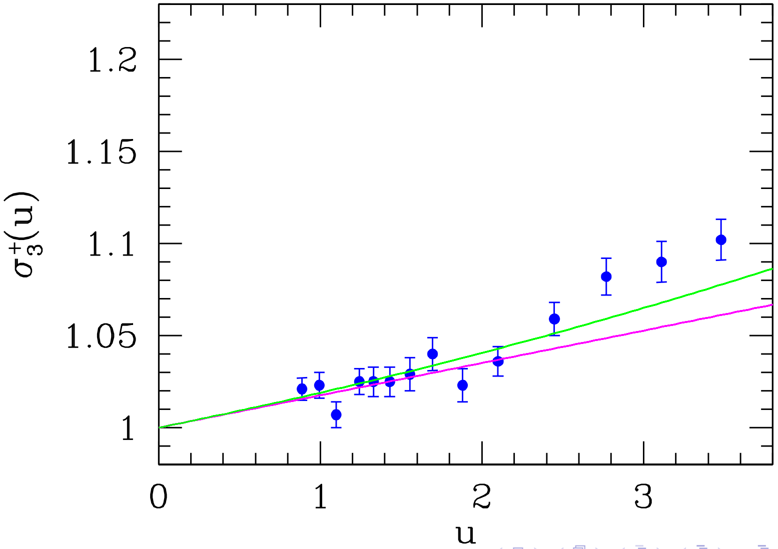

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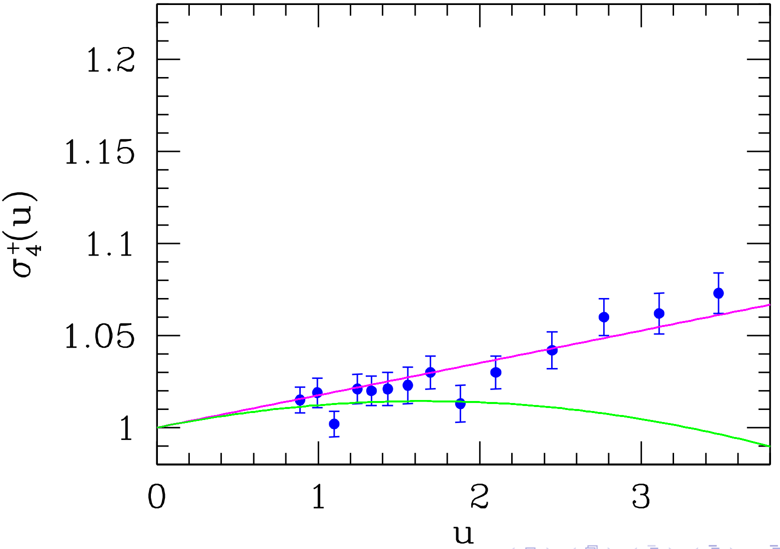

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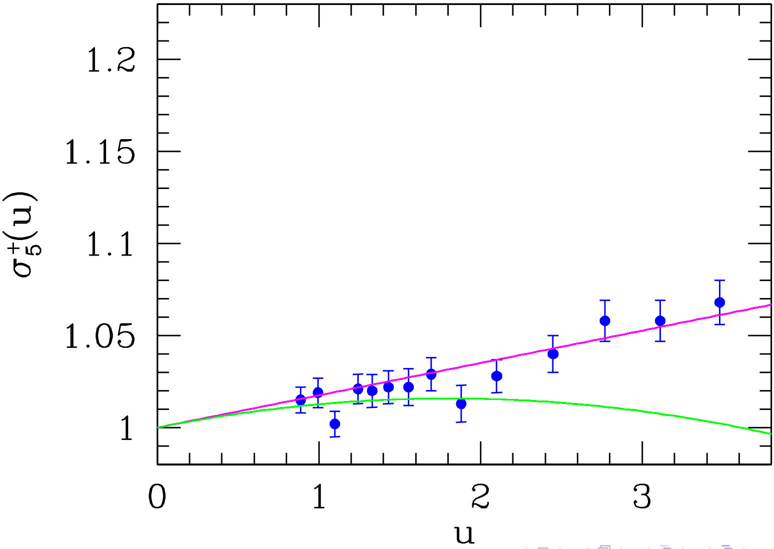

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- NLO-AD has been computed from matching with the DRED scheme
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