THE ROLE OF LATTICE QCD IN FLAVOR PHYSICS

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OUTLINE

1. Flavor physics and its motivations
2. First row unitarity and the Cabibbo angle
3. The unitarity triangle analysis
4. New Physics

see also talks by:
M. Wingate (LQCD)
I. Shipsey (Exp.)

Special thanks to P. Gambino, L. Giusti, G. Isidori, S. Sharpe, and all the members of the UTfit Collaboration.
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FLAVOR PHYSICS

AND ITS

MOTIVATIONS
CONCEPTUAL PROBLEMS  The most obvious:
- Gravity:  $M_{\text{Planck}} = (\hbar c/G_N)^{1/2} \approx 10^{19}$ GeV

PHENOMENOLOGICAL INDICATIONS
- Unification of couplings ($M_{\text{GUT}} \approx 10^{15}-10^{16}$ GeV)
- Dark matter ($\Omega_m \approx 0.35$)
- Neutrino masses
- Matter/Anti-matter asymmetry (not enough $CP$ in the SM)
- Cosmological vacuum energy

THE “NATURAL” CUT-OFF:
- $\delta m^2 = \frac{3G_F}{\sqrt{2} \pi} m_t^2 \Lambda^2 \approx (0.3 \Lambda)^2$
- $\Lambda = O(1 \text{ TeV})$
We do not understand flavor physics: Why 3 families? Why the hierarchy of masses?

We expect New Physics effects in the flavor sector:

**THE FLAVOR PROBLEM:** $\Lambda_{K^0-\bar{K}^0} \approx O(100 \text{ TeV})$

10 parameters in the quark sector ($6 m_q + 4 \text{CKM}$)

Is the $\text{CKM}$ mechanism and its explanation of $\text{CP}$ correct?
We need to control the theoretical input parameters at a comparable level of accuracy!!

\[ \varepsilon_K = (2.271 \pm 0.017) \times 10^{-3} \quad 0.7\% \]

\[ \Delta m_d = (0.503 \pm 0.006) \text{ ps}^{-1} \quad 1\% \]

\[ \sin(2\beta) = 0.734 \pm 0.054 \quad 7\% \]
FIRST ROW
UNITARITY AND THE
CABIBBO ANGLE
\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

**The most stringent unitarity test**

| Source     | \(|V_{ud}|\)       | Comment                                      |
|------------|--------------------|----------------------------------------------|
| SFT        | 0.9740 ± 0.0005    | Extremely precise, 9 expts                   |
| N β-dec    | 0.9731 ± 0.0015    | \(g_V/g_A\), will be improved at PERKEO, Heidelb. |
| \(\pi e_3\) | 0.9765 ± 0.0056    | Theor. clean, but BR=10^-8 PIBETA at PSI     |
| Average    | 0.9739 ± 0.0005    | G.Isidori et al., CKM 2002 Workshop          |

- **K→πev**: \(|V_{us}| = 0.2196 ± 0.0026\) PDG 2002 average
- **b→u**: \(|V_{ub}| = 0.0036 ± 0.0007\) \(|V_{ub}|^2 \approx 10^{-5}\)

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0042 ± 0.0019 \]

"Old" 2σ discrepancy
The NEW experimental results

BNL-E865
PRL 91, 261802
(and Moriond ’04)

KLOE
Moriond ’04

KTeV
hep-ex/0406001

$V_{us} f_+ K^0 \pi^- (0)$

KLOE
Moriond ’04

$V_{us} f_+ K^0 \pi^- (0)$

PDC
E865
KLOE (hep-ex/0308023)
KTeV

Unitarity and $V_{ud}$
Review (hep-ph/0307214)
Review (hep-ex/0402299)
Theory: 2 recent lattice contributions

1) LEPTONIC DECAYS:

\[
\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K \left(1 - \frac{m_{\mu}^2}{m_K^2}\right)^2}{|V_{ud}|^2 f_\pi^2 m_\pi \left(1 - \frac{m_{\mu}^2}{m_\pi^2}\right)^2} 0.9930(35) \]

Precise \(f_K/f_\pi\), MILC Latt.'03-04
Asqtad action, \(N_f=3\)

\[
\begin{align*}
  f_\pi &= 129.5 \pm 0.9_{\text{stat}} \pm 3.6_{\text{syst}} \text{ MeV} \\
  f_K &= 156.6 \pm 1.0_{\text{stat}} \pm 3.8_{\text{syst}} \text{ MeV} \\
  f_K/f_\pi &= 1.210 \pm 0.004_{\text{stat}} \pm 0.013_{\text{syst}}
\end{align*}
\]

C. Bernard, update of Marciano 2004: \(|V_{us}| = 0.2219(26)\)

Better agreement with unitarity!!
2) SEMILEPTONIC K13 DECAYS:
Precise (quenched) calculation of f(0), SPQcdR 2004

\[ \Gamma_{K^{0\pi}} = \frac{C^1 G_F^2 |V_{us}|^2 M_K^5}{192 \pi^3} S_{EW} (1+\delta^1_K) I^1_K f_+(0)^2 \]

The largest th. uncertainty from:
\[ f_+(0) = 1 - O(m_s - m_u)^2 \]
[Ademollo-Gatto theorem]

ChPT

\[ f_+(0) = 1 + f_2 + f_4 + O(p^8) \]

Vector Current Conservation

\[ f_2 = -0.023 \]
Independent of L_i (Ademollo-Gatto)

f_4 = -0.016 \pm 0.008

“Standard” estimate:
Leutwyler, Roos (1984)
(QUARK MODEL)
**ChPT: The complete \( O(p^6) \) calculation**


\[
f_4 = \Delta_{\text{loops}}(\mu) - \frac{8}{F_\pi^4} [C_{12}(\mu) + C_{34}(\mu)] \left( M_K^2 - M_\pi^2 \right)^2
\]

\( C_{12}(\mu) \) and \( C_{34}(\mu) \) can be determined from the slope and the curvature of the scalar form factor. Experimental data, however, are not accurate enough.

... and models

Jamin et al., \( f_4^{\text{LOC}} = -0.018 \pm 0.009 \) [Coupled channel dispersive analysis]

Cirigliano et al., \( f_4^{\text{LOC}} = -0.012 \) [Resonance saturation]

Cirigliano et al., \( f_4^{\text{LOC}} = -0.016 \pm 0.008 \) [QM, Leutwyler and Roos]

\( \mu = ??? \quad \Delta_{\text{loops}}(1\text{GeV}) = 0.004 \quad \Delta_{\text{loops}}(M_\rho) = 0.015 \quad \Delta_{\text{loops}}(M_\eta) = 0.031 \)

Cirigliano et al., \( f_4^{K^0\pi^-}(0) = 0.981 \pm 0.010 \)
1) Evaluation of $f_0(q_{\text{MAX}}^2)$

The basic ingredient is a double ratio of correlation functions:

$$R = \frac{\langle \pi | \bar{s} \gamma_0 u | K \rangle \langle K | \bar{u} \gamma_0 s | \pi \rangle}{\langle \pi | \bar{u} \gamma_0 u | \pi \rangle \langle K | \bar{s} \gamma_0 s | K \rangle}$$

$$= \frac{(M_K + M_\pi)^2}{4M_K M_\pi} f_0(q_{\text{max}}^2)^2$$

[FNAL for $B \rightarrow D^*$]
2) Extrapolation of $f_0(q^2_{\text{MAX}})$ to $f_0(0)$

Comparison of polar fits:

**LQCD:** \[ \lambda_+ = (25 \pm 2) \times 10^{-3} \]

**KTeV:** \[ \lambda_+ = (24.11 \pm 0.36) \times 10^{-3} \]

**LQCD:** \[ \lambda_0 = (12 \pm 2) \times 10^{-3} \]

**KTeV:** \[ \lambda_0 = (13.62 \pm 0.73) \times 10^{-3} \]
3) Chiral extrapolation

\[ R = \frac{f_+(0) - 1 - f_2^{\text{QUEN}}}{(M_K^2 - M_{\pi}^2)^2} \]

Computed in Quenched-ChPT

The dominant contributions to the systematic error come from the uncertainties on the \( q^2 \) and mass dependencies of the form factor.

\[ f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}} \]

[Quenching error is not included]  In agreement with LR!!
THE UNITARITY TRIANGLE ANALYSIS
### The Unitarity Triangle Analysis

The unitarity triangle analysis is a graphical method used to visualize the relationships between the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements. The key equation is:

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

This equation is an expression of the three fundamental constraints on the CKM matrix:

- **5 Constraints:**
  - \( \sin^2 \beta (\rho, \eta) \)
  - \( A(J/\psi K_S) \)
  - Hadronic Matrix Elements from LATTICE QCD

- **2 Parameters:**
  - \( \epsilon_K \)
  - \( \Delta m_d \)
  - \( \Delta m_d' / \Delta m_s \)

### Table: Constraints and Parameters

<table>
<thead>
<tr>
<th>(b→u)/(b→c)</th>
<th>( \bar{\rho}^2 + \bar{\eta}^2 )</th>
<th>( f_+, F(1), ... )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_K )</td>
<td>( \bar{\eta} [(1- \bar{\rho}) + P] )</td>
<td>( B_K )</td>
</tr>
<tr>
<td>( \Delta m_d )</td>
<td>( (1- \bar{\rho})^2 + \bar{\eta}^2 )</td>
<td>( f_{Bd}^2 B_{Bd} )</td>
</tr>
<tr>
<td>( \Delta m_d' / \Delta m_s )</td>
<td>( (1- \bar{\rho})^2 + \bar{\eta}^2 )</td>
<td>( \xi )</td>
</tr>
<tr>
<td>( A(J/\psi K_S) )</td>
<td>( \sin 2\beta (\bar{\rho}, \bar{\eta}) )</td>
<td></td>
</tr>
</tbody>
</table>
Bayesian and frequentist: 2 stat. approaches

and 2 dedicated workshop
The **Bayesian approach**

**The Bayes Theorem:** \( P(A/B) \sim P(B/A) \ P(A) \)

\[ f(\bar{\rho}, \bar{\eta}, \mathbf{x} | c_1, \ldots, c_m) \sim \prod_{j=1}^{m} f_j(c | \bar{\rho}, \bar{\eta}, \mathbf{x}) \prod_{i=1}^{N} f_i(x_i) \ f_o(\bar{\rho}, \bar{\eta}) \]

Integrate over \( \mathbf{x} \)

\[ f(\bar{\rho}, \bar{\eta} | \mathbf{c}) \sim \mathcal{L}(\mathbf{c} | \bar{\rho}, \bar{\eta}) \ f_o(\bar{\rho}, \bar{\eta}) \]

The p.d.f. \( f(x_i) \) represents our "degree of beliefs"

---

**The Frequentistic approach**

The theoretical likelihood do not contribute to the \( \chi^2 \) of the fit while the corresponding parameters take values within the "allowed" ranges. Instances where even only one of the parameters trespasses its range are not considered.
In the frequentistic approach the selected region does not have a precise statistical meaning ("at least 95"). Nevertheless, if same likelihood are used, the output results are very similar.

Estimates of the uncertainties for lattice determinations should be given by the lattice community.
Unitary Triangle Analysis:
LQCD INPUT PARAMETERS
**K – K mixing and \( B_K \)**

\[
\hat{B}_K = 0.86 \pm 0.06 \pm 0.14
\]

Error: 7% 16%

Projected: 7%

From the UT fit
\[
\hat{B}_K = 0.65 \pm 0.10
\]

15%

\[
\hat{B}_K = 0.87 \pm 0.06 \pm 0.13
\]

LATT03 average: D. Becirevic

Error: 7% 16%

Error from other sources \( \approx 10\% \) (mainly \( V_{cb} \))
B_{B_d/s} - \bar{B}_{B_d/s} mixing: f_{B_s} \sqrt{B_{B_s}} and \xi (I)

Stat & Syst

f_{B_s} \sqrt{B_{B_s}} = 276 \pm 38 \text{ MeV}

Error: 14%  

Projected: 5%

From the UT fit

f_{B_s} \sqrt{B_{B_s}} = 279 \pm 21 \text{ MeV}

8%
$B_{B_{d/s}} - \overline{B}_{B_{d/s}}$ mixing: $f_{B_s} \sqrt{B_{B_s}}$ and $\xi$ (II)

Stat.  Syst.

$\xi = 1.24 \pm 0.04 \pm 0.06$

Error:  3%  5%

Projected:  3%

From the UT fit

$\xi = 1.22 \pm 0.05$

4%

$\xi = 1.25 \pm 0.10$

LATT03 average: A. Kronfeld
$V_{cb}$ from exclusive semil. B-decays

**Exp. Theor.**

$V_{cb}^{\text{Excl.}} = (42.1 \pm 1.1 \pm 1.9) \cdot 10^{-3}$

Error: 2.6% 4.5%

Projected: ??

$F_{B \to D^*}(1) = 0.91 \pm 0.04$

$V_{cb}^{\text{Incl.}} = (41.4 \pm 0.7 \pm 0.6) \cdot 10^{-3}$

$V_{cb}^{\text{Aver.}} = (41.5 \pm 0.7) \cdot 10^{-3}$

Mainly from LQCD, FNAL
Compatible with QCDSR and HQET
+ Quark Model

$V_{cb} = A \lambda^2$

Dominant contribution to the average

Projected: ??
$V_{ub}$ from exclusive semi-leptonic $B$-decays

Exp. Theor.

$V_{ub}^{\text{Excl.}} = (32.4 \pm 2.4 \pm 4.6) \times 10^{-4}$

Error: 7% 14%

Projected: 7%

CLEO 2003

15-20% within quenching

LATTICE

All $q^2$

Ball '01

KRWYY

SPD

Average, all $q^2$

$q^2 > 16 \text{ GeV}^2$

Lattice QCD

FNAL '01

JLQCD

APE

UKQCD '00

Average, $(q^2 \geq 16 \text{ GeV}^2)$

$q^2 < 16 \text{ GeV}^2$

LCSR

Ball '01

KRWYY

Average, $q^2 < 16 \text{ GeV}^2$

Average of LOCD + LCSR

$\pi\ell\nu$: LOCD + LCSR

$\pi\ell\nu + p\ell\nu$: LOCD + LCSR
Unitary Triangle Analysis: RESULTS AND PERSPECTIVES

Roma, Genova, Torino, Orsay

www.utfit.org
FIT RESULTS

\[ \sin^2 \alpha = -0.14 \pm 0.25 \]

\[ \sin^2 \beta = 0.697 \pm 0.036 \]

\[ \gamma = (61.9 \pm 7.9)^\circ \]

\[ \bar{\rho} = 0.174 \pm 0.048 \]

\[ \bar{\eta} = 0.344 \pm 0.027 \]

\[ \sin 2 \alpha = -0.14 \pm 0.25 \]

\[ \sin 2 \beta = 0.697 \pm 0.036 \]
INDIRECT EVIDENCE OF CP VIOLATION

\[ \sin^2 \beta_{\text{UT Sides}} = 0.685 \pm 0.047 \]

\[ \sin^2 \beta_{J/\psi K_s} = 0.739 \pm 0.048 \]

Prediction (Ciuchini et al., 2000):

\[ \sin^2 \beta_{\text{UTA}} = 0.698 \pm 0.066 \]
Prediction for $\Delta m_s$

**$\Delta m_s$ NOT USED**

$$\Delta m_s = (20.5 \pm 3.2) \text{ ps}^{-1}$$

**WITH ALL CONSTRAINTS**

$$\Delta m_s = (18.0 \pm 1.6) \text{ ps}^{-1}$$

A measurement is expected at FERMILAB
IMPACT OF IMPROVED DETERMINATIONS

\[ B_K = 0.86 \pm 0.06 \pm 0.14 \]
\[ \xi = 1.24 \pm 0.04 \pm 0.06 \]
\[ V_{ub} = (32.4 \pm 2.4 \pm 4.6) \times 10^{-4} \text{ (exclusive only)} \]

\[ f_{Bs} \sqrt{B_{Bs}} = 276 \pm 38 \text{ MeV} \]

\[ \sin2\beta = 0.734 \pm 0.054 \]

\[ \Delta \rho = 28\% \rightarrow 17\% \text{ (-40\%)} \quad \Delta \eta = 7.8\% \rightarrow 5.2\% \text{ (-33\%)} \]
\[ \Delta m_s = (20.5 \pm 3.2) \text{ ps}^{-1} \]

\[ \Delta m_s = (20.7 \pm 1.9) \text{ ps}^{-1} \]
NEW PHYSICS
1) “To which extent improved experimental determinations will be able to detect New Physics?”

**Compatiblity between direct and indirect determinations as a function of the measured value and its experimental uncertainty**
2) “Given the present theoretical and experimental constraints, to which extent the UTA can still be affected by New Physics contributions?”

An interesting case:

New Physics in $B_d - \bar{B}_d$ mixing

The New Physics mixing amplitudes can be parameterized in a simple general form:

\[
M_d = C_d e^{2i\varphi_d} (M_d)^{SM}
\]

\[
\Delta m_d = C_d (\Delta m_d)^{SM}
\]

\[
A(J/\psi K_S) \sim \sin 2(\beta + \varphi_d)
\]
TWO SOLUTIONS:

Standard Model solution:
\[ C_d = 1 \quad \phi_d = 0 \]

\( \phi_d \) can be only determined up to a trivial twofold ambiguity:
\[ \beta + \phi_d \rightarrow \pi - \beta - \phi_d \]
HOW CAN WE DISCRIMINATE BETWEEN THE TWO SOLUTIONS?

\[ \Delta m_s, \eta [K_L \rightarrow \pi \nu \bar{\nu}], \gamma [B \rightarrow DK], |V_{td}| [B \rightarrow \rho \gamma], \ldots \]

\[ \gamma = 81^\circ \pm 19^\circ \pm 13^\circ \text{ (syst)} \pm 11^\circ \text{ (mod)} \]

Belle preliminary + LQCD(!)

Belle

Independent of NP
Coming back to the Standard Model:

15 YEARS OF ($\bar{\rho} - \bar{\eta}$) DETERMINATIONS
(The “commercial” plot)
CONCLUSIONS

LATTICE QCD CALCULATIONS HAD A CRUCIAL IMPACT ON TESTING AND CONSTRAINING THE FLAVOR SECTOR OF THE STANDARD MODEL

IN THE PRECISION ERA OF FLAVOR PHYSICS, LATTICE SYSTEMATIC UNCERTAINTIES MUST (AND CAN) BE FURTHER REDUCED

IMPORTANT, BUT MORE DIFFICULT PROBLEMS (NON LEPTONIC DECAYS, RARE DECAYS, ...) ARE ALSO BEING ADDRESSED