Chiral Perturbation Theory at non-zero lattice spacing

Oliver Baer
Tsukuba University

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Introduction: Chiral extrapolation

Some numbers I heard the last four days:

Collaboration	Fermion type	Nf	smallest m_π/m_ρ	Talk/poster by
UKQCD	Wilson	2	~0.44	
SPQcdR	Wilson	2	~0.66	Tarantino
qq+q	Wilson	2	~0.47	Scholz
CP-PACS	Wilson	2	~0.35	Namekawa
CP-PACS	Wilson	2+1	~0.62	Ishikawa
MILC	Staggered	2+1	~0.3	Bernard
RBC	Domain Wall	2	~0.53	Izubuchi

physical value: 0.18

Still needed: rather long extrapolation in the light quark mass

Chiral Perturbation Theory (ChPT)

Analytic guidance is required for the extrapolation

$\Longrightarrow$ provided by ChPT

ChPT: Effective theory for low-energy QCD

Weinberg, Gasser, Leutwyler

Gives the quark mass dependence of observables

Example:

$$m_\pi^2 = M_0^2 \left[1 + \frac{M_0^2}{16\pi^2 N_f f^2} \ln M_0^2 - \frac{8 L M_0^2}{f^2} \right]$$

$$M_0^2 = (m_u + m_d) B$$

f, B, L: low-energy constants

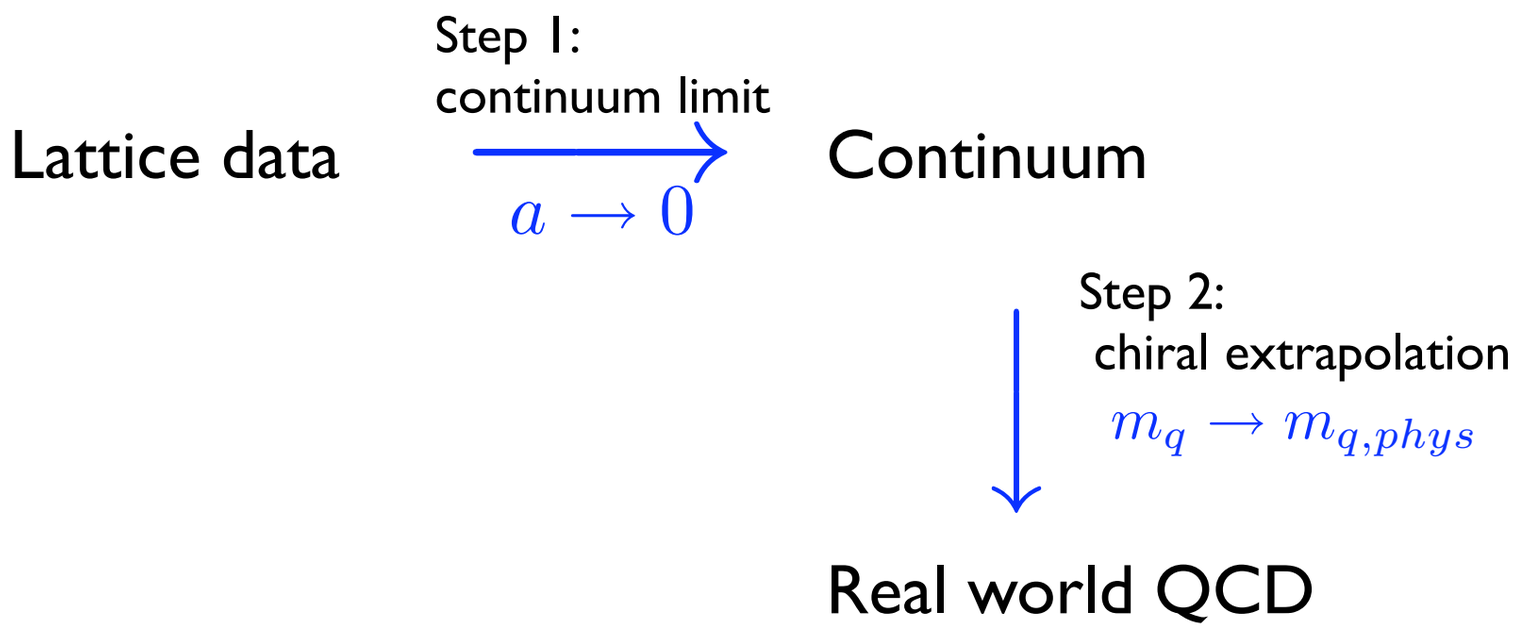
Does ChPT describe the lattice data ?

Smoking gun: curvature due to chiral logs

Panel discussion
Lattice 2002, Boston:

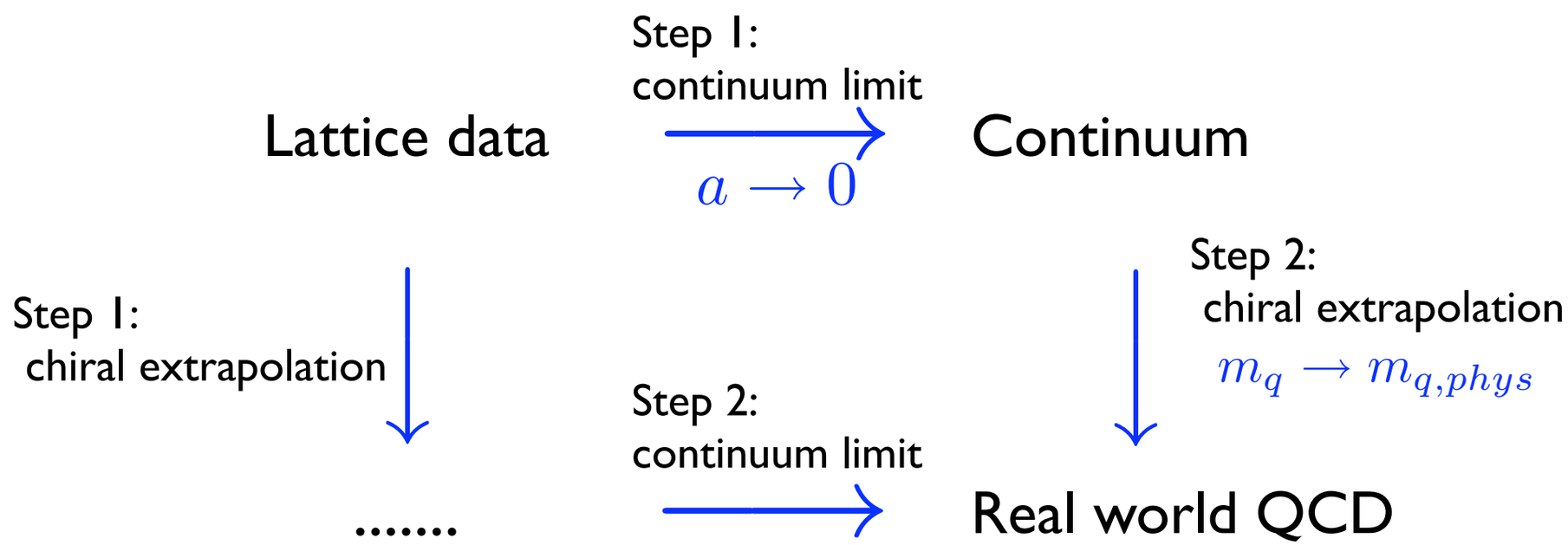
A potential problem

ChPT formulae derived for $a = 0$ only !



A potential problem

ChPT formulae derived for $a = 0$ only !



What about the opposite order ?

Outline of the talk

- Introduction (done)
- Wilson Chiral Perturbation Theory (WChPT)
  - Theory
  - Applying to numerical data
  - tmQCD, Mixed fermion theories, Baryon ChPT
- Staggered Chiral Perturbation Theory (SChPT)
  - Theory
  - Applying to numerical data
- Conclusion

Strategy

Two-step matching to effective theories:

Lee, Sharpe '98
Sharpe, Singleton '98

1. Lattice theory $\longrightarrow$ Symanzik's effective theory

   continuum theory making the a-dependence explicit

2. Symanzik's effective theory $\longrightarrow$ ChPT

   including the a-dependence

$\Longrightarrow$ Chiral expressions for m_π, f_π ... with explicit a-dependence

Symanzik's action for Lattice QCD with Wilson fermions

Locality and symmetries of the lattice theory

$$\Rightarrow \qquad S_{eff} = S_{QCD} + a\,c \int \overline{\psi}\, i\sigma_{\mu\nu} G_{\mu\nu} \psi + \mathcal{O}(a^2)$$

- At $O(a)$ only one additional operator (making use of EOM)
- c : unknown coefficient ("low-energy constant")
- $O(a^2)$: dim-6 operators: - fermion bilinears
  - 4-fermion operators

Sheikholeslami, Wohlert

Reminder: Chiral Lagrangian

Fields: $\Sigma(x) = \exp\left(\frac{2\,i}{F}\,\pi^a(x)T^a\right)$ T^a : Group generators

Lagrangian: $\mathcal{L}_{eff}\left[\Sigma, M\right] = \mathcal{L}_{eff}\left[\Sigma', M'\right]$ M: Quark mass matrix

$\Sigma' = L\Sigma R^\dagger \qquad M' = LMR^\dagger$ L, R : Left, Right transformations

Expand in powers of derivatives and masses: $\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$

$$\mathcal{L}_2 = \frac{f^2}{4}\mathrm{tr}\left[\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\right] - \frac{f^2 B}{2}\,\mathrm{tr}\left[\Sigma^\dagger M + M^\dagger\Sigma\right]$$

f, B : undetermined low-energy constants

Chiral Lagrangian including a

$$S_{eff} = S_{QCD} + a\,c \int \overline{\psi}\, i\sigma_{\mu\nu} G_{\mu\nu} \psi + \mathcal{O}(a^2)$$

Pauli term breaks the chiral symmetry exactly like the mass term in S_{QCD}

$\Longrightarrow$ a enters chiral Lagrangian exactly like the mass term

$$\Longrightarrow \qquad \mathcal{L}_2 = \frac{f^2}{4} \mathrm{tr}\left[\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right] - \frac{f^2 B}{2} \mathrm{tr}\left[\Sigma^\dagger M + M^\dagger \Sigma\right]$$

$$- \frac{f^2 W_0}{2}\, a\, \mathrm{tr}\left[\Sigma + \Sigma^\dagger\right]$$

Sharpe, Singleton '98
Rupak, Shoresh '02

W_0 : new undetermined low-energy constant

includes $c = c(g_0^2)$ not really a constant (weak a dependence)

$\mathcal{L}_4$ -Lagrangian:

$$\mathcal{L}_4 = \mathcal{L}_4(p^4, p^2 m, m^2) \;+\; \mathcal{L}_4(p^2 a, ma) \;+\; \mathcal{L}_4(a^2)$$

Gasser, Leutwyler '85 — Rupak, Shoresh '02 — Rupak, Shoresh, OB '03; Aoki '03

- No O(4) symmetry breaking terms in $\mathcal{L}_4(a^2)$ (start at $\mathcal{O}(a^2 p^4)$)
- Total number of low-energy constants: 10 L_i + (5 + 3) W_i = 18 !

L_i are physical ChPT parameters $\longrightarrow$ values are interesting

(independent of a)

Role of W_i : parameterize a dependence $\longrightarrow$ values are less interesting

Note: W_i are not universal (depend on the lattice action)

Power counting

The Power counting is non-trivial because of

1. the additive mass renormalization $\propto \frac{1}{a}$

2. two symmetry breaking parameters $a\,,\ m_{\mathrm{quark}}$

   $\Longrightarrow$ their relative size matters

Leading order pion mass (degenerate case)

$$M_0^2 = 2mB + 2aW_0 \qquad m = m_u = m_d$$

Leading a-effect: Shift in the pion mass

But: This shift might already be absorbed in m_c

e.g. $m_\pi^2 = 0$ for $m' = Z_m(m_0 - m_{\rm cr}) = 0$

$\Longrightarrow$ Express the observables m_π, f_π in terms of m'

Note: additional shifts enter at a^2, ...

Two expansion parameters: $\frac{2Bm'}{(4\pi f)^2}$ $\frac{2W_0 a}{(4\pi f)^2}$

Both need to be small

Relative size determines which terms are LO, NLO etc.

Example:

$\mathcal{L}_2(p^2, m')$ $\mathcal{L}_4(a^2)$

$\downarrow$ $\downarrow$

	$f^2 B\, m' \mathrm{tr}\left[\Sigma + \Sigma^\dagger\right]$	$W\, a^2 \left(\mathrm{tr}\left[\Sigma + \Sigma^\dagger\right]\right)^2$
$m' \gg a^2$	LO	NLO
$m' \approx a^2$	LO	LO

The proper power counting depends on the relative size of m' and a

Different power counting have been discussed:

If $m' \gg a^2 \rightarrow$ continuum like ChPT + small $\mathcal{O}(a^n)$ corrections

Rupak, Shoresh, OB '03

If $m' \approx a^2 \rightarrow$ qualitatively different !

Non-trivial phase diagram — Sharpe, Singleton '98

Modification of chiral logs — Aoki '03

Spontaneous flavor and parity breaking

Potential energy: Sharpe, Singleton '98

$(\mathrm{N_f} = 2)$

$$V = -c_1 m' \mathrm{tr}\left[\Sigma + \Sigma^\dagger\right] + c_2 a^2 \left(\mathrm{tr}\left[\Sigma + \Sigma^\dagger\right]\right)^2$$

$c_1(f, B)$

$c_2(f, B, W_i)$

A: $\mathrm{sign}\, c_2 = +1 \quad \Longrightarrow \quad \Sigma_{\mathrm{vacuum}} \neq \pm 1$ Aoki phase Aoki '85

flavor and parity are broken

massless pions at $a \neq 0$

B: $\mathrm{sign}\, c_2 = -1 \quad \Longrightarrow \quad \Sigma_{\mathrm{vacuum}} = \pm 1$ no flavor/parity breaking

no massless pions

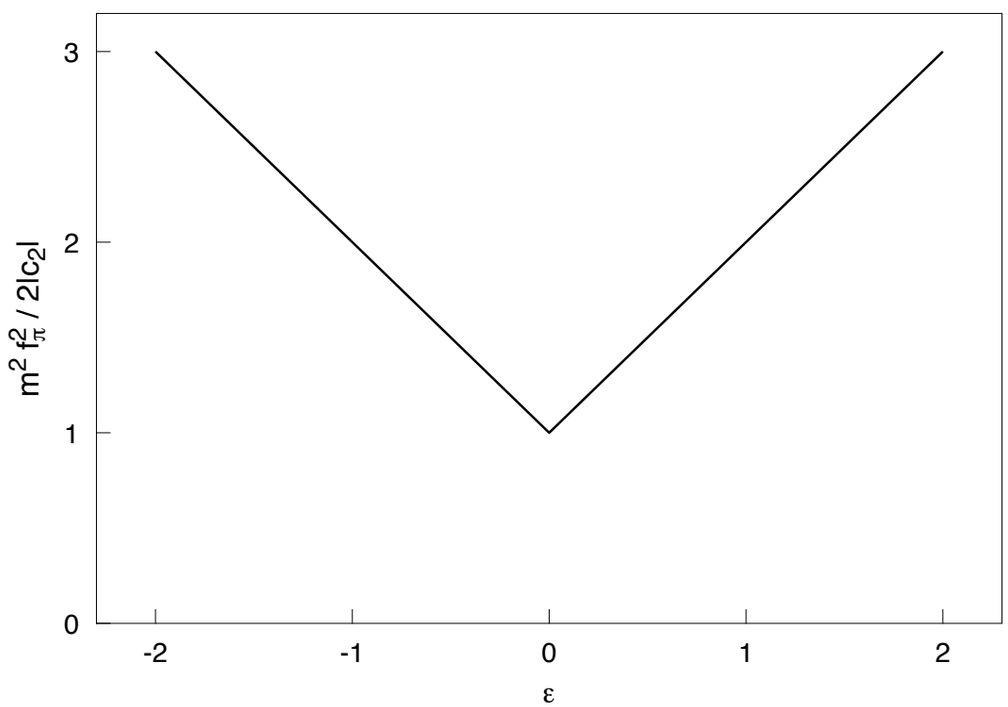
Quenched case: Same two possibilities

Golterman, Sharpe, Singleton

this conference

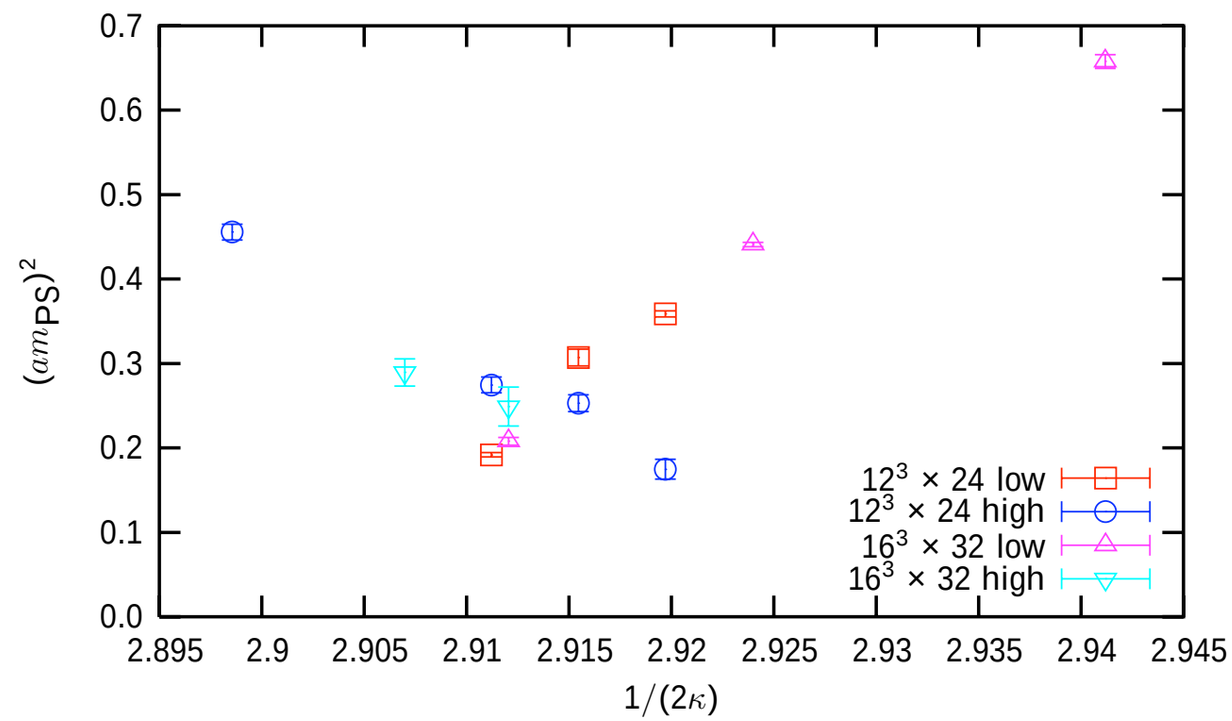
Scenario B

Theory:



Sharpe, Singleton '98

Data: Plaquette action + unimproved Wilson
$\beta = 5.2$



Urbach
this conference

Pseudo-scalar mass to 1-loop (2 deg flavor)

Aoki '03

$$m_\pi^2 = 2Bm' \left[1 + \frac{m'(2B + w_1 a)}{32\pi^2 f^2} \log \frac{2Bm'}{\Lambda^2} + \frac{w_0 a^2}{32\pi^2 f^2} \log \frac{2Bm'}{\Lambda^2} \right]$$

$$+ \text{analytic}$$

- $a \longrightarrow 0$ gives the correct continuum expression
- w_0, w_1 : combinations of low-energy constants W_i
- The coefficient of $m' \ln m'$ is no longer universal at non-zero a
- The lattice spacing generates an $a^2 \ln m'$ contribution

  $\Longrightarrow$ This term dominates [....] for small m'

  But: A resummation of the $(\ln m')^n$ terms can be performed ....

Resummend Pseudo-scalar mass

Aoki '03

$$m_\pi^2 = 2\tilde{B}m' \left[1 + \frac{m'(2B + w_1 a)}{32\pi^2 f^2} \log \frac{2Bm'}{\Lambda^2}\right] \left\{\log \frac{2Bm'}{\tilde{\Lambda}^2}\right\}^{\tilde{w}_0 a^2/32\pi^2 f^2}$$

$$+ \text{analytic}$$

Expansion of $\{\ldots\}^{a^2}$ gives result on previous slide

Assumption: Aoki phase scenario

Similar modifications for f_π and m_{AWI}

Qualitative feature in WChPT

The coefficients of the chiral logs can be altered by O(a) corrections

Applying to numerical data

Two groups analyzed their data using WChPT:

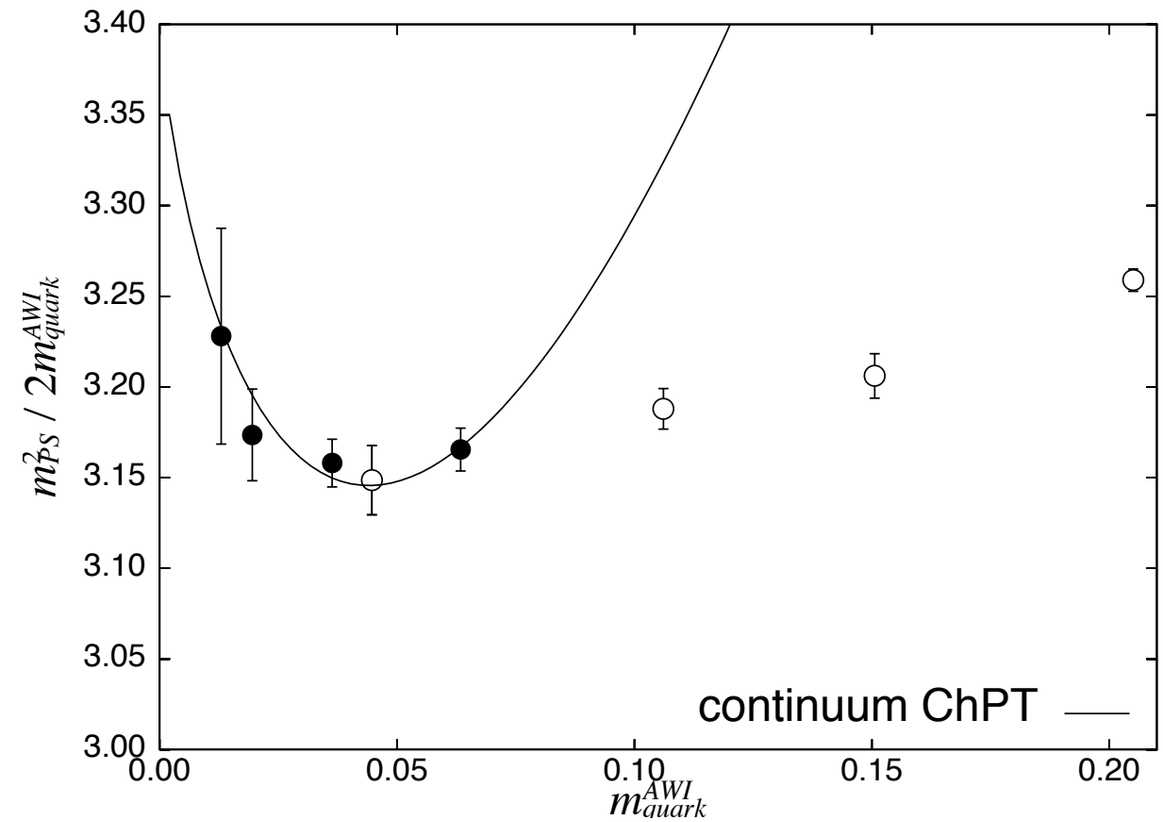
- CP-PACS $\longrightarrow$ talk by Y. Namekawa
  Namekawa et.al '04
- qq+q $\longrightarrow$ talk by E. Scholz
  Farchioni et.al. '03/'04

CP-PACS

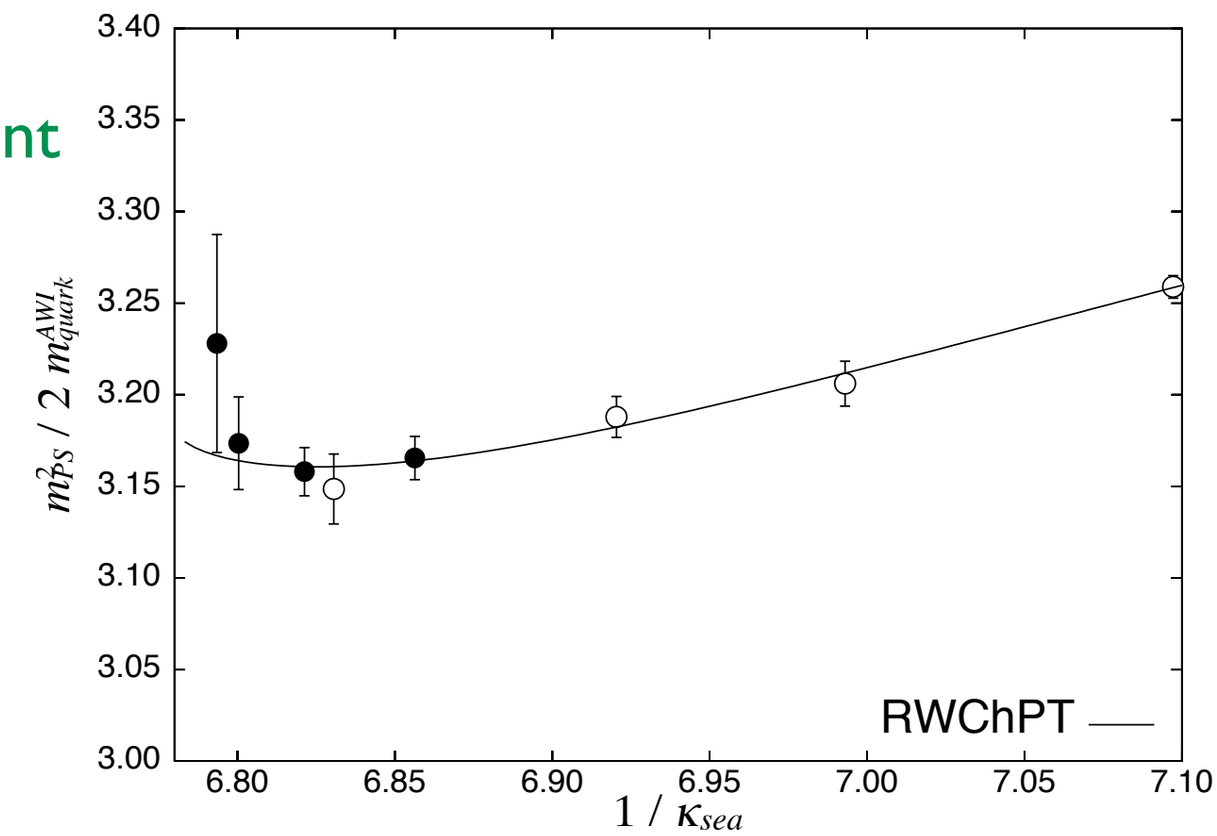
Namekawa et.al '04

- RG improved gauge action
- 2 flavor tadpole improved clover quark action
- One Lattice spacing: $a = 0.2 fm$
- 8 Sea quark masses

$$0.35 \leq \frac{M_\pi}{M_\rho} \leq 0.8$$



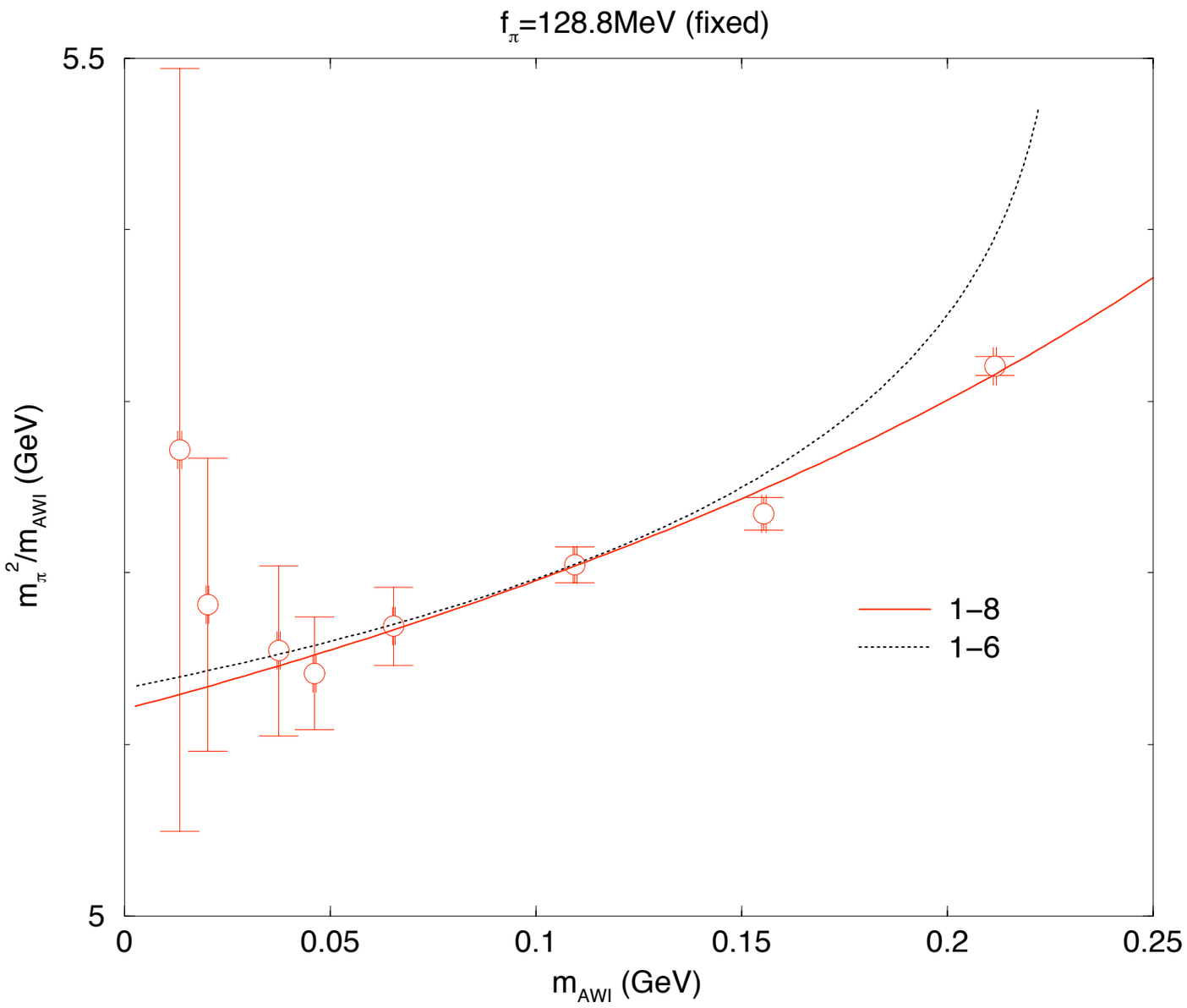
O(a) contribution suppresses the coefficient in front of m log m to ~ 20 % of the continuum ChPT value



A crude fit

Simultaneous fit

- M_π^2
- m_{AWI}



CP-PACS

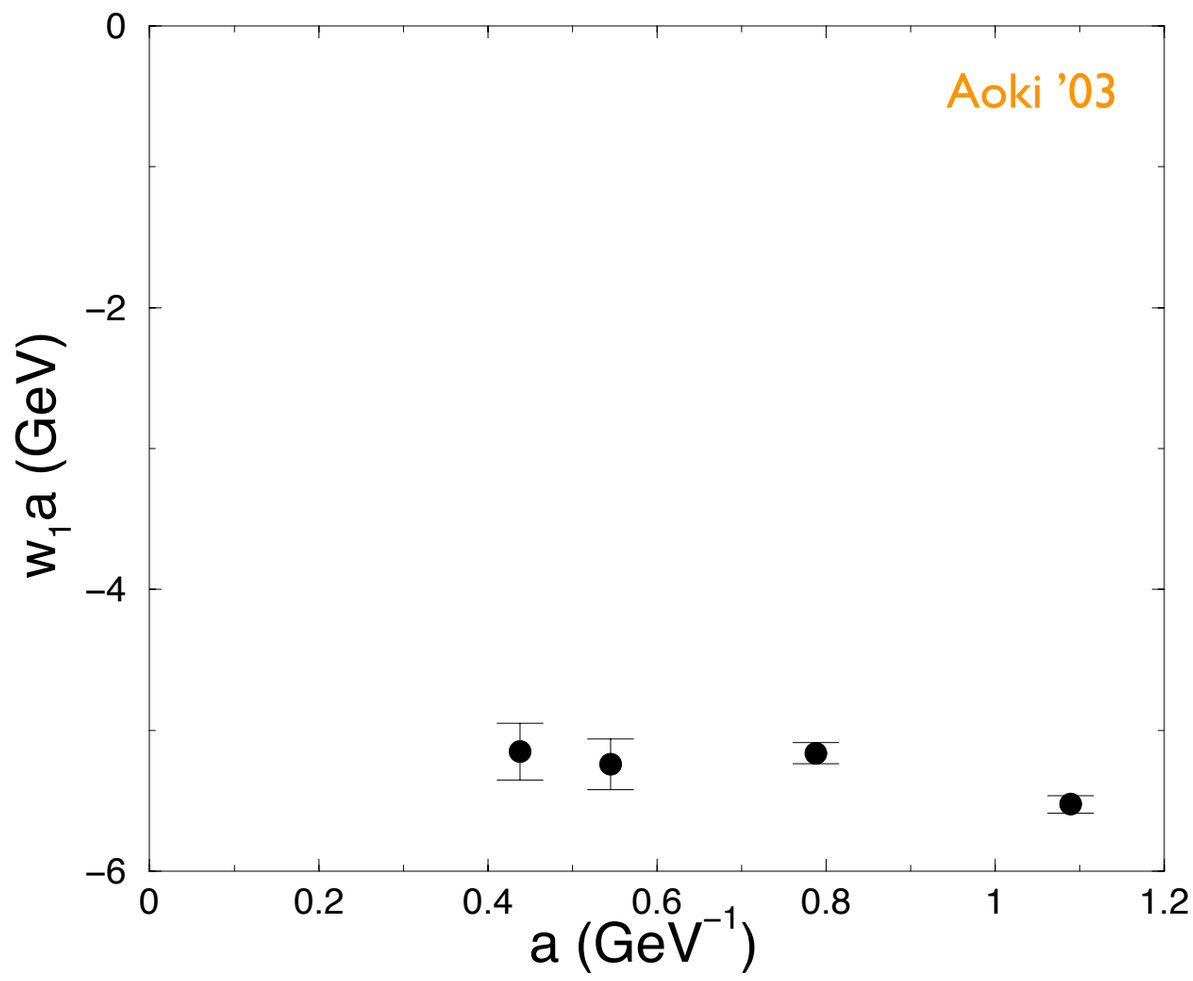
'Success' of WChPT:

Suppression of the log-coefficient

$$\frac{m_\pi^2}{2Bm'} = 1 + \frac{m'(2B + w_1 a)}{32\pi^2 f^2} \log \frac{2Bm'}{\Lambda^2} + \ldots$$

4 quark masses $0.55 \leq \frac{M_\pi}{M_\rho} \leq 0.8$

4 lattice spacings $\approx 0.09 \ \ldots \ 0.2 fm$



Not the expected a dependence

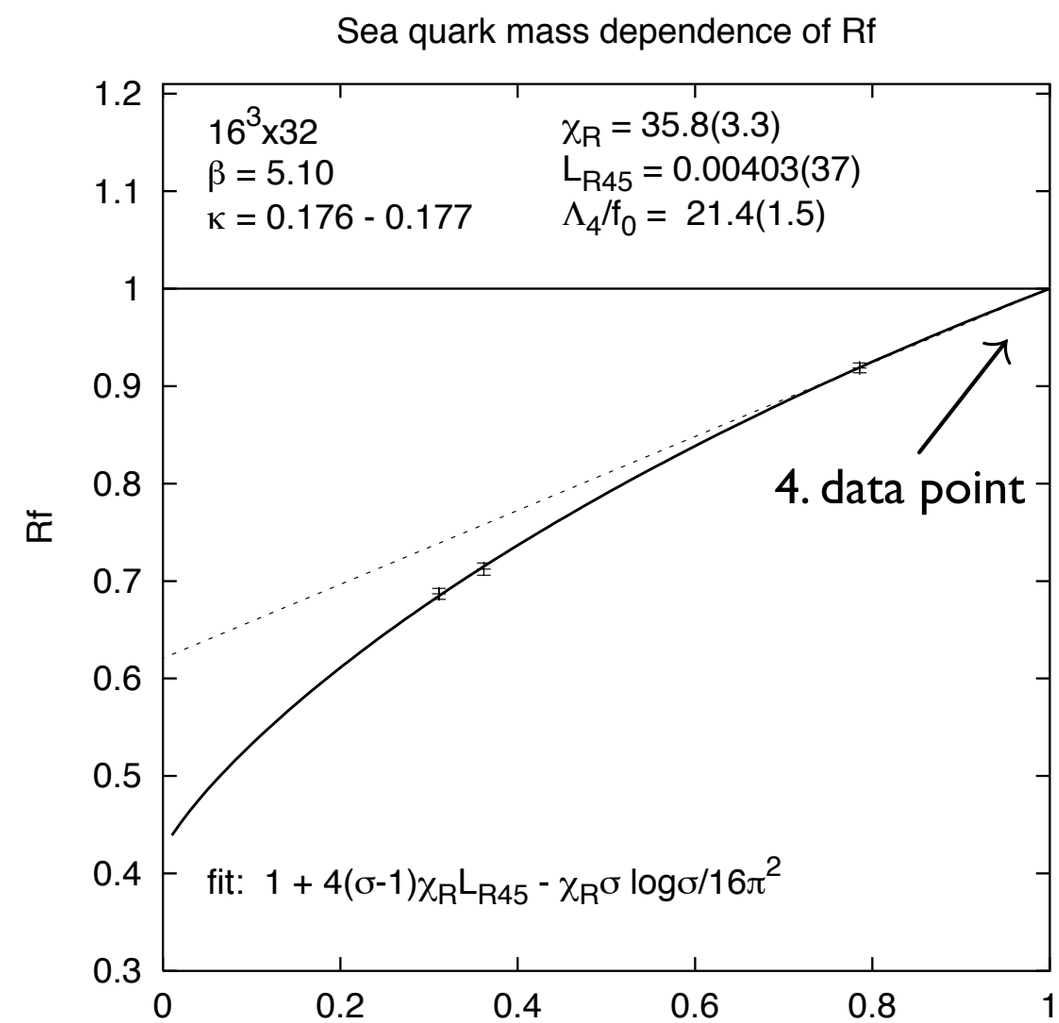
qq+q

Farchioni et.al. '03/'04

- Plaquette gauge action
- 2 flavor unimproved Wilson quarks
- One lattice spacing $a \approx 0.2 fm$
- 4 sea quark masses

$$0.47 \leq \frac{M_\pi}{M_\rho} \leq 0.76$$

$$Rf = \frac{f_\pi(m)}{f_\pi(m_{\text{heaviest}})} \qquad m = m_{\text{AWI}}$$



$$\sigma = m/m_{\text{heaviest}}$$

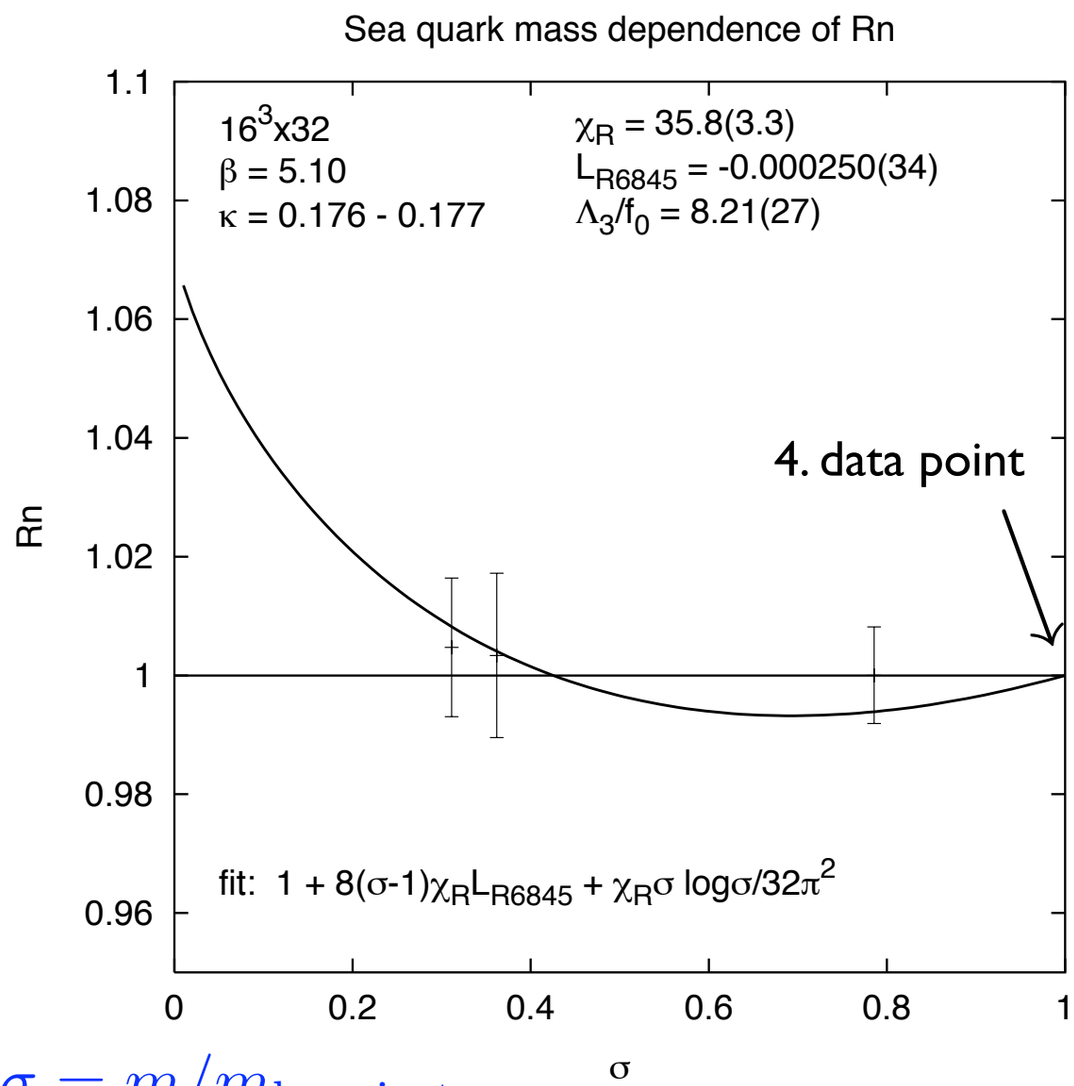
Good fit to Continuum ChPT

Curvature observed in Rf

qq+q

Farchioni et.al. '03/'04

$$Rn = \frac{M_\pi^2(m_{\mathrm{AWI}})}{\sigma M_\pi^2(m_{\mathrm{AWI,heaviest}})}$$



$\sigma = m/m_{\mathrm{heaviest}}$

Consistent to Continuum ChPT

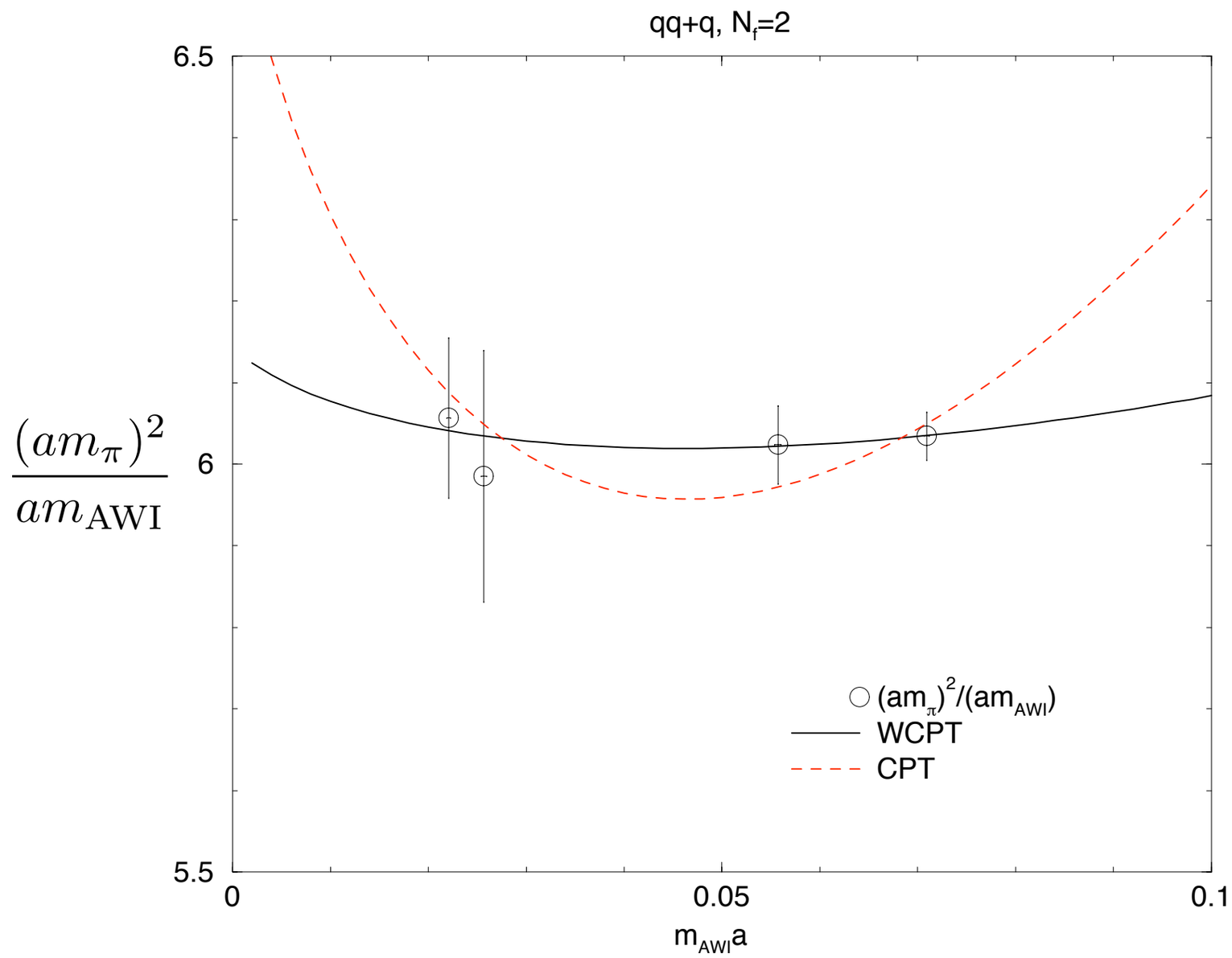
No curvature in Rn

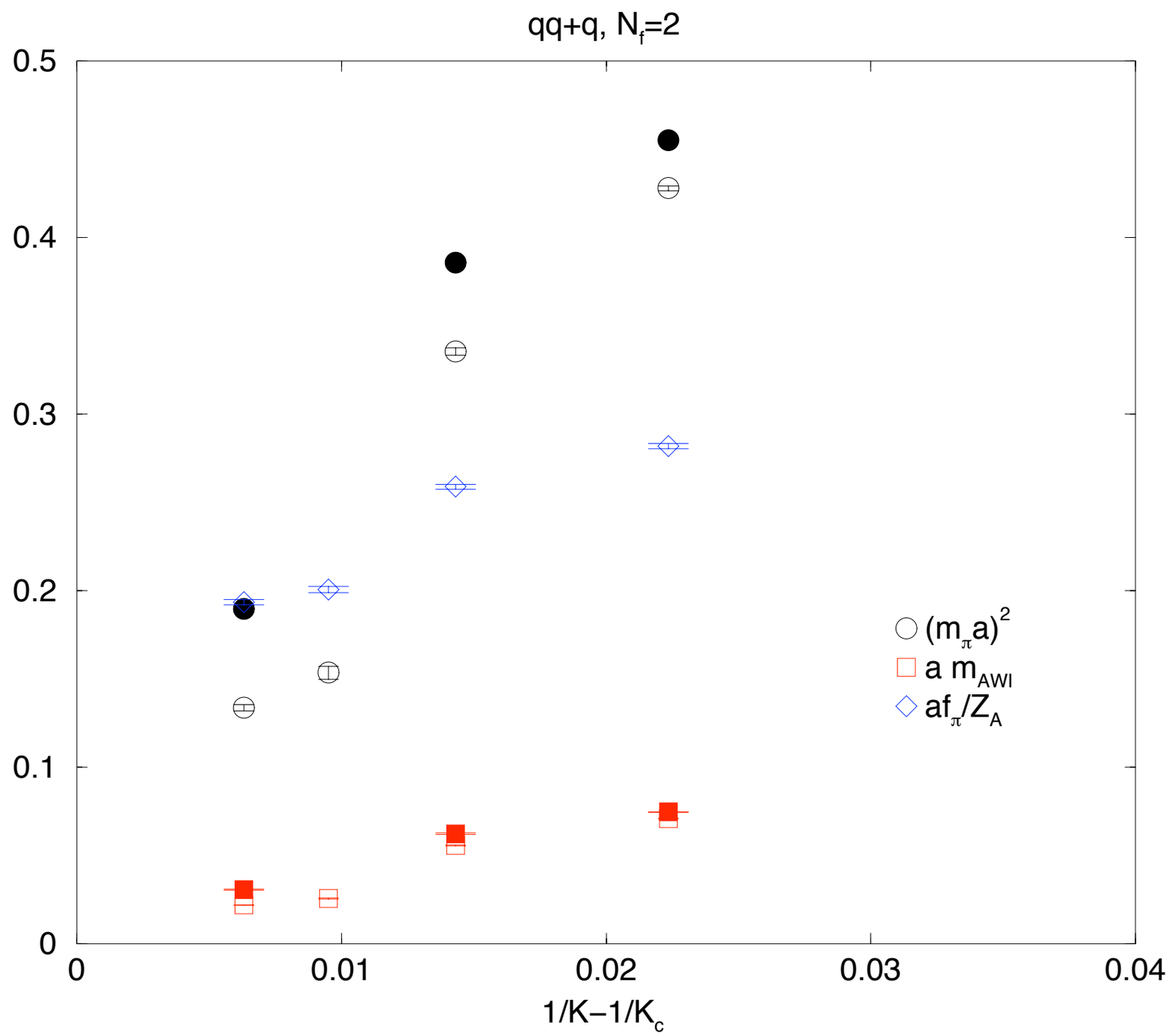
qq+q findings:

- Consistent fits with Continuum ChPT
- Analytic NNLO terms are important
- O(a) terms are small and can be neglected

Conclusion: Small O(a) effects ?

A crude fit




qq+q, N_f=2
0.5
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
0.04
$1/K-1/K_c$
$(m_\pi a)^2$
$a\ m_{AWI}$
af_π/Z_A

Additional theoretical developments

- tmQCD $\longrightarrow$ plenary talk by R. Frezzotti
- "Mixed fermion" theories
- Nucleon properties

tmQCD

Twisted mass term: $$m\, e^{i\omega\gamma_5\tau_3} = m + i\mu\gamma_5\tau_3$$

Why tmQCD:

- No exceptional configurations
- Automatic O(a) improvement at maximal twist

Frezzotti, Rossi '03

Questions:

- tmQCD breaks flavor and parity $\longrightarrow$ Size ?
- How does the O(a) improvement show up in ChPT ?

ChPT for tmQCD

Symanzik action: $S_{eff} = S_{\mathrm{tmQCD}} + a\,\mathrm{Pauli\,term} + \mathcal{O}(a^2)$

$\Longrightarrow$ $$\mathcal{L}_{\mathrm{chiral}}\left[m, \mu, a, a^2\right]$$

a: Muenster, Schmidt '03/'04

a^2: Sharpe, Wu
this conference

$\Longrightarrow$ m_π^2, f_π including the lattice spacing

O(a) improvement at maximal twist is explicit

Flavor symmetry breaking: For $m' \approx \mu \approx a \gg a^2$

W: combination of low-energy constants

$\Longrightarrow$ $$m_{\pi_3}^2 - m_{\pi_{1,2}}^2 = a^2 W \frac{\mu^2}{m'^2 + \mu^2} = \mathcal{O}(a^2)$$

Mixed fermion theories

Q: What are "mixed" theories?

A: Generalization of partially quenched theories:

$$\text{sea sector:} \quad D_S + m_S$$
$$\text{valence sector:} \quad D_V + m_V$$

D_S : Staggered, Wilson

D_V : Overlap, Domain-Wall,...

Naive expectation:

Difference is of $\mathcal{O}(a)$ and vanishes in the Continuum Limit

talks by
Schroers
Tweedie
Fleming
Renner

- Danger: Loss of unitarity
- Analytic control of this $\mathcal{O}(a)$ difference ?

Example: Wilson sea + Overlap valence

Why this combination ?

Rupak, Shoresh, OB '02/03

Exact chiral symmetry in the valence sector

- small quark masses possible
- Expectation: Better determination of (some) GL coefficients

Main results:

- Chiral Lagrangian through $\mathcal{O}(a^2)$
  more low-energy constants than Wilson sea + Wilson valence
- Reduced a dependence in m_π^2
  compared to Wilson sea + Wilson valence

In progress: Staggered sea + Ginsparg-Wilson valence

Bernard, Rupak, Shoresh, OB

Nucleon properties

Symanzik action: $S_{eff} = S_{QCD} + a\,\mathrm{Pauli\,term}$

$\Longrightarrow$ Baryon ChPT + $\mathcal{O}(a)$ Jenkins, Manohar '90

Calculated:

- Nucleon masses, magnetic moments, axial-vector currents, ... Beane, Savage '03
- Electromagnetic properties (charge radii, magnetic moments, ...) Arndt, Tiburzi '04

Staggered fermions

+ Fast to simulate
+ Exact U(1) symmetry for massless quarks
− Fermion doubling: Each flavor comes with 4 tastes

Taste reduction on the lattice: $\det D \to \sqrt[4]{\det D}$ "fourth root trick"

1. The $\sqrt[4]{\det D}$-theory has no local lattice action: Universality ?

2. The $\sqrt[4]{\det D}$-theory has no local Symanzik action: How to include a in ChPT ?

Staggered Chiral Perturbation Theory (SChPT)

Main idea:

Bernard '02
Aubin, Bernard '03

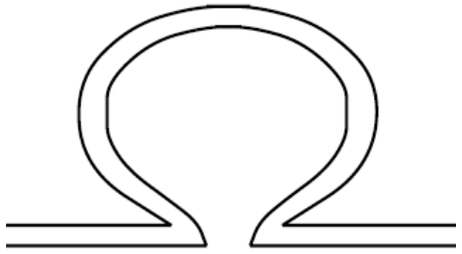
1. Lattice theory with N_f staggered fermions $\longrightarrow$ Symanzik's effective theory with $4N_f$ fermions

2. Symanzik's effective theory $\longrightarrow$ ChPT

3. Compute physical observables like $M_\pi^2, f_\pi, \ldots$

4. Adjust by hand to one taste per flavor
   = include factors of 1/4
   for sea quark loops

No factor $\frac{1}{4}$



One factor $\frac{1}{4}$

"quark flow" diagrams

Symanzik action for staggered fermions

Locality and symmetries of the lattice theory

Luo '97
Lee, Sharpe '99

$$\Rightarrow \qquad S_{eff} = S_4 + a^2 S_6 + \ldots$$

- S_4 : Continuum QCD with $4N_f$ fermions

  N_f flavors, each has a SU(4) taste symmetry

- No $O(a)$ contribution is compatible with the symmetries (also: no $1/a$)
- S_6 : Taste symmetry is broken ("Taste violation")

  SO(4) rotation invariance is also broken

Chiral Lagrangian

Lee, Sharpe '99
Bernard '02
Aubin, Bernard '02/03

$$\mathcal{L}_2 = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass} + a^2 \mathcal{V}$$

$$\mathcal{V} = \sum c_k \mathcal{O}_k$$

- $\mathcal{V}$ contains 8 operators (6 for $N_f = 1$)

  c_k corresponding new low-energy constants

- $\mathcal{V}$ contains no derivatives $\longrightarrow$ SO(4) rotation invariance
- $\mathcal{V}$ has an accidental SO(4) taste symmetry $\longrightarrow$ mass degeneracy at LO

$$m_\pi^2(\xi_i) = 2Bm + a^2 \Delta(\xi_i)$$

ξ_i : Taste label

Mass spitting $\Delta(\xi_i)$ is the same for

$$\xi_4 \leftrightarrow \xi_k$$
$$\xi_{45} \leftrightarrow \xi_{k5} \qquad k, l = 1, 2, 3$$
$$\xi_{4l} \leftrightarrow \xi_{kl}$$

SChPT at NLO

LO: $p^2 \quad m \quad a^2$

NLO: $p^4 \quad p^2 m \quad m^2$

$p^2 a^2 \quad m a^2 \quad a^4$

Sharpe, van de Water
(this conference)

- Accidental Taste SO(4) is broken at NLO by $p^2 a^2$ terms
- The same breaking operators affect decay constants and PGB masses

$\Longrightarrow$ non-trivial predictions

example:

$$\left(\frac{f_k - f_4}{f_4}\right) = -\left(\frac{m^2_{\pi_k} - m^2_{\pi_4}}{m^2_{\pi_4}}\right)$$

No unknown
low-energy constants !

Additional predictions for dispersion relations

Pseudo-scalar mass to 1-loop (3 deg. flavor)

Aubin, Bernard '02/03

In the Continuum limit :

$$\frac{(m_{\pi_5^+})^2}{2Bm} = 1 + \frac{1}{24\pi^2 f^2} m_{\pi_I}^2 \ln \frac{m_{\pi_I}^2}{\Lambda^2} + \text{analytic} \quad \longrightarrow \quad \text{Continuum expression}$$

$$+ \frac{4}{24\pi^2 f^2}\left[m_{\eta'_V}^2 \ln \frac{m_{\eta'_V}^2}{\Lambda^2} - m_{\pi_V}^2 \ln \frac{m_{\pi_V}^2}{\Lambda^2}) \right] \quad \longrightarrow \quad 0$$

$$+ \frac{4}{24\pi^2 f^2}\left[m_{\eta'_A}^2 \ln \frac{m_{\eta'_A}^2}{\Lambda^2} - m_{\pi_A}^2 \ln \frac{m_{\pi_A}^2}{\Lambda^2}) \right] + a^2 C \quad \longrightarrow \quad 0$$

- Reduces to the continuum expression for $a \to 0$ (not easy to see here)
- Non-zero a : Additional log contributions involving other particles

  Continuum log behaviour may be changed significantly !
- Similar modifications in $f_{\pi_5^+}$

Applying to MILC Data

- 1-loop Symanzik imp. gauge action

  with tadpole improvement

- 2+1 flavor Asqtad staggered quark action

- Two lattice spacings: $a = 0.125 fm$
  $a = 0.09 fm$

- Small quark masses:

$$0.3 \leq \frac{m_\pi}{m_\rho}$$

for more results
see the plenary talks

Hadron spectrum
by K. Ishikawa

QCD coupling and quark masses
by P. Rakow

Status of Lattice calculations
for Flavor physics
by M. Wingate

Mass splittings

LO mass:

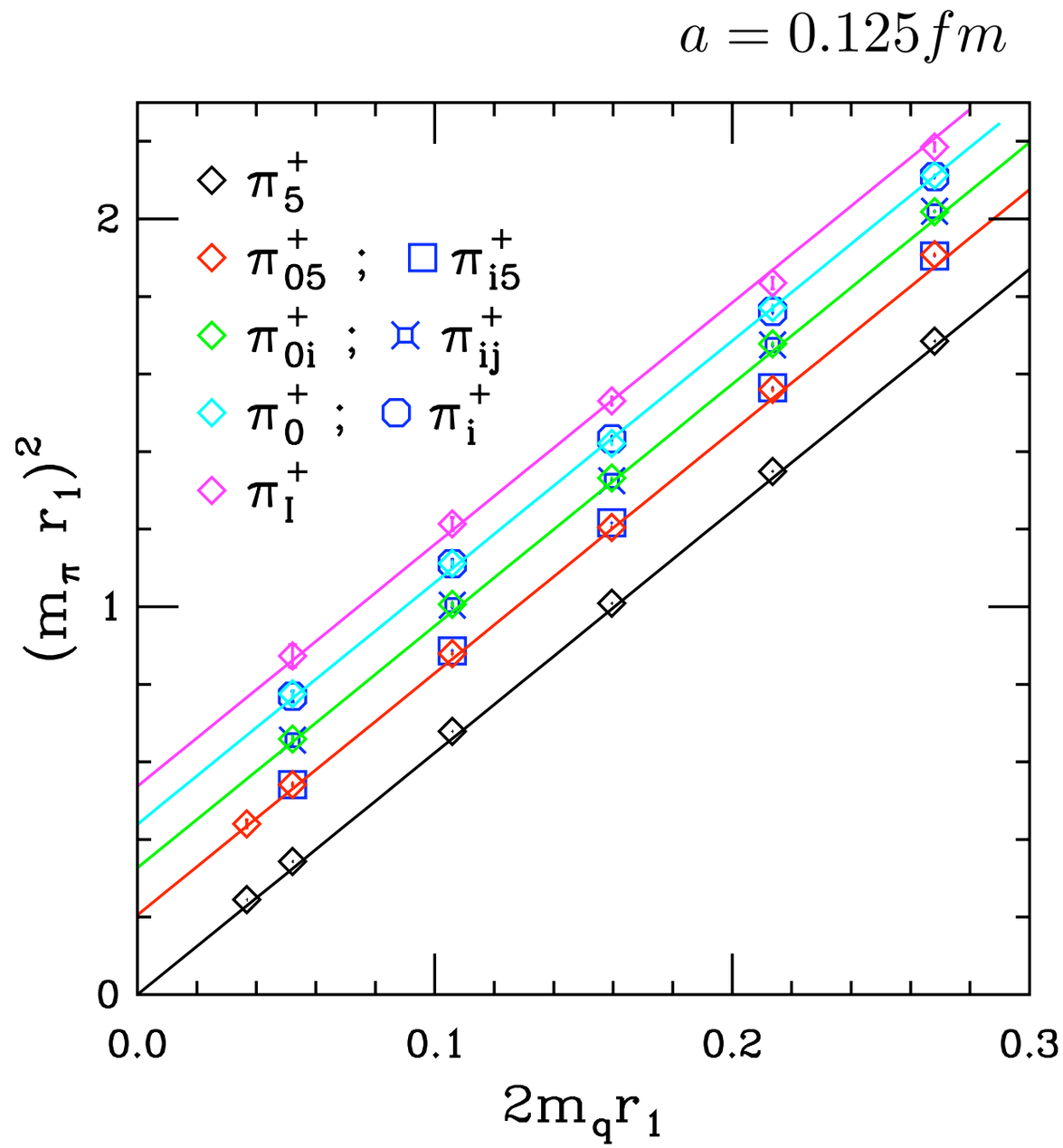
$$m_\pi^2(\xi_i) = 2Bm + a^2\Delta(\xi_i)$$

- SO(4) symmetry is clearly visible

- Splittings are quite large:

$$\Delta \approx (280 - 450MeV)^2$$

proper power counting: $m \approx a^2$



plot by C. Bernard

Pseudo-scalar mass: some details about the fit

- Simultaneous fit of $m^2_{\pi_5^+}/(m_{\mathrm{val\,x}} + m_{\mathrm{val\,y}})$ and $f_{\pi_5^+}$:
- 240 data points, small statistical errors ~ 0.1 - 0.7 %
- Fit forms

$$\frac{m^2_{\pi_5^+}}{(m_{\mathrm{val\,x}} + m_{\mathrm{val\,y}})} = B\big(1 + \mathrm{NLO} + \text{"NNLO"}\big)$$

$$f_{\pi_5^+} = f\big(1 + \mathrm{NLO} + \text{"NNLO"}\big)$$

NLO: 1-loop SChPT

"NNLO": Analytic continuum terms, $\mathcal{O}(m_q^3)$ only ! (NNLO logs not available yet)

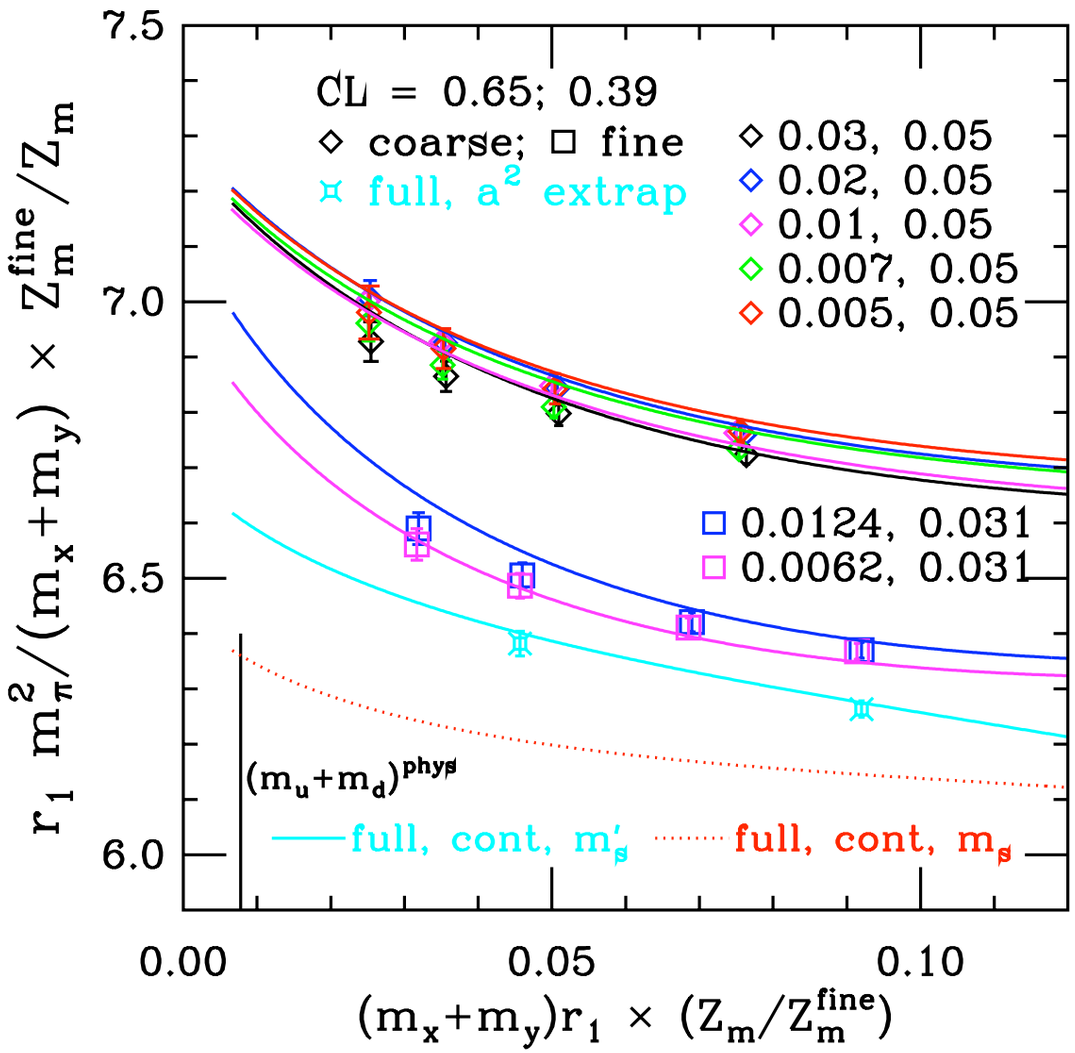
- Total number of fit parameters: 36 + 4 (20 are constrained)

SChPT

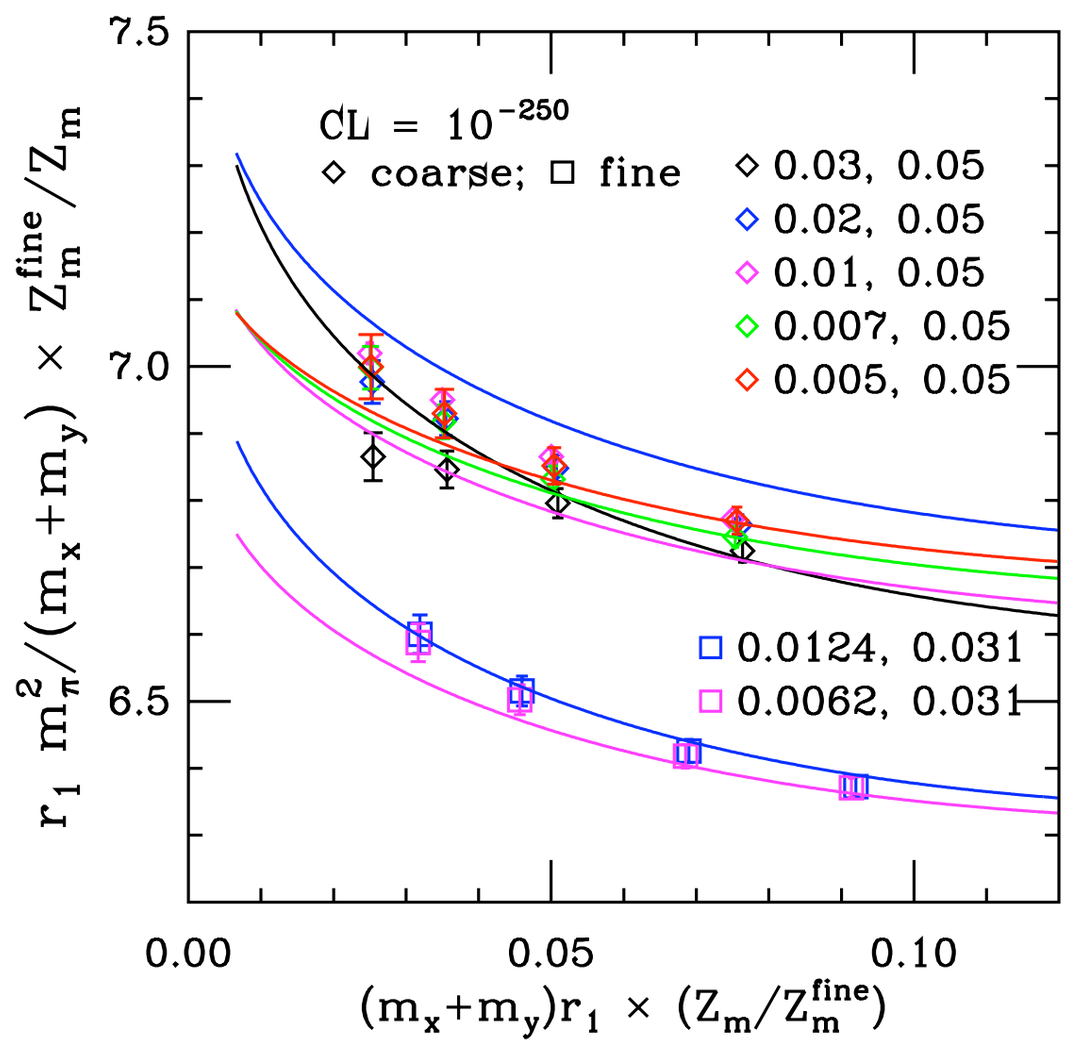
MILC data for PS mass

C. Bernard
this conference

Including a^2 terms



Excluding a^2 terms



- Good fits are not possible without taste-breaking terms in SChPT (CL < 10^{-50})
  (even with 36 free parameters !)

Topological susceptibility

Billeter, deTar, Osborn
this conference

LO continuum ChPT:

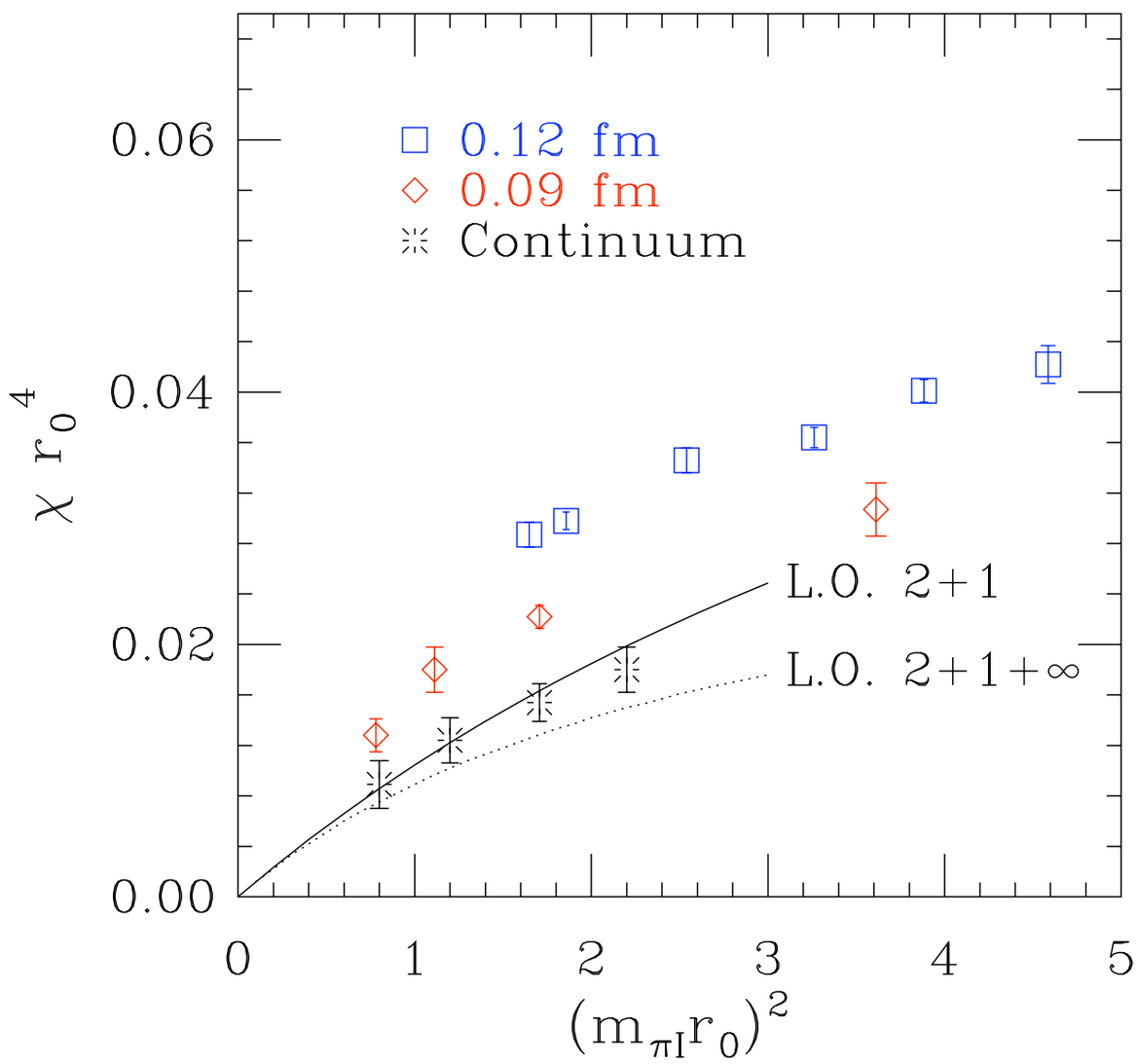
$$\chi = \frac{f_\pi^2 m_\pi^2}{(4N_f)}$$

LO Staggered ChPT:

$$m_\pi \longrightarrow m_{\pi_I}$$

χ does not vanish for $m_{\rm quark} \to 0$

at non-zero lattice spacing !



Data extrapolated to Cont Limit
is consistent with LO ChPT

Heavy-light SChPT

Aubin, Bernard
this conference

Starting point: $$S_{eff} = S_4^{\text{glue,light q}} + S_4^{\text{heavy Q}} + a^2 S_6$$

$$S_4^{\text{heavy Q}} = \int \overline{Q}(iv \cdot D)Q$$ LO HQET

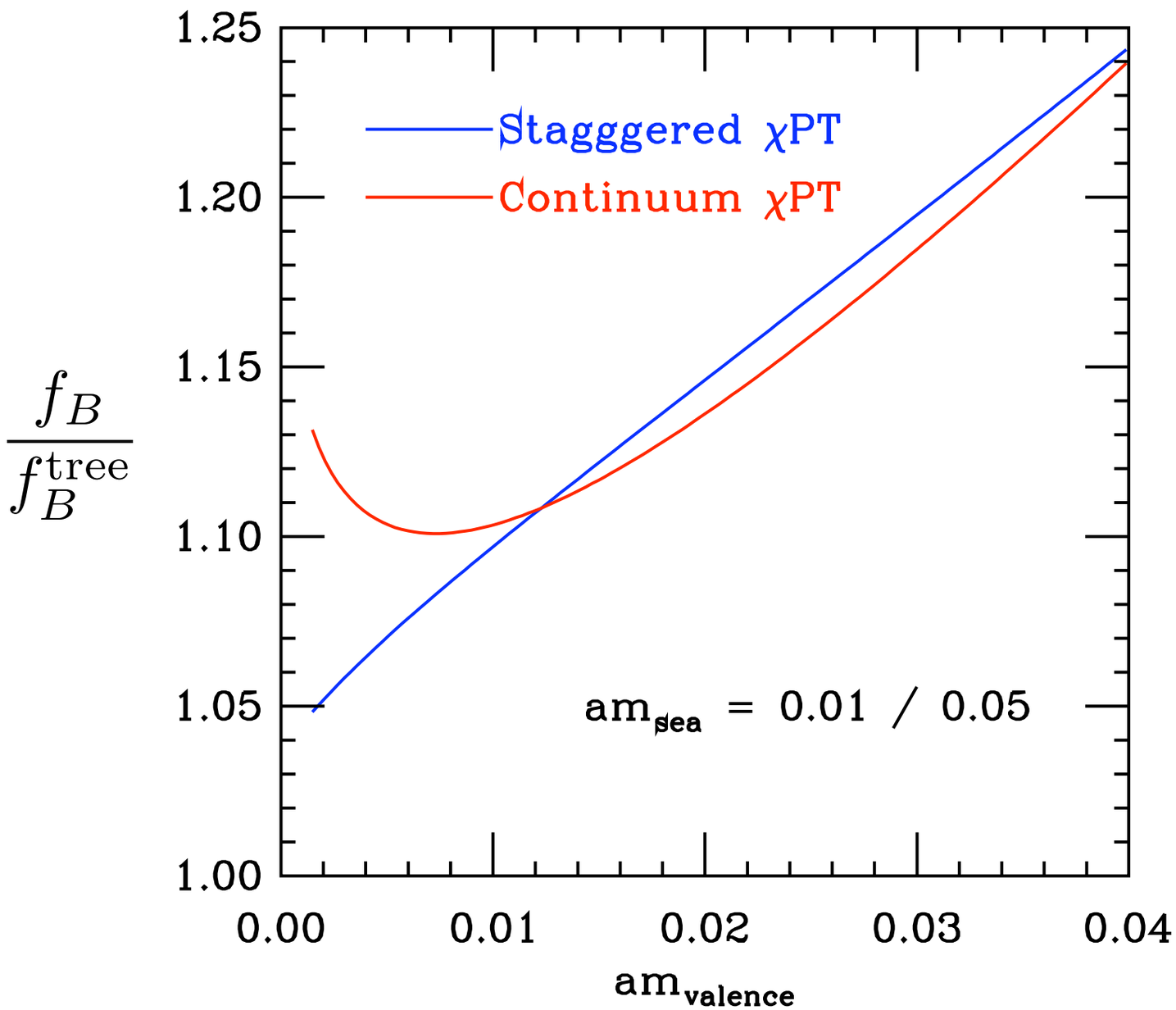
$\Longrightarrow$ Heavy-light ChPT + $\mathcal{O}(a^2)$

Grinstein et.al. '92,
Wise'92

Note: joint expansion in m, a^2, k $\longrightarrow$ residual heavy meson momentum
(here assumed to be small)

$\Longrightarrow$ $f_B, f_{B_s} \ldots$ Additional log contributions $\propto a^2$

Instead of an equation:



plot by Aubin

A problem when taking the continuum limit first ?

Generic form for the pseudo-scalar decay constant:

$$f = f_0 \left[1 + (m + a^2) \ln(m + a^2) + \text{analytic} \right]$$

$m \gg a^2 \rightarrow$ Expand the logarithm and use polynomial in a^2 for continuum extrapolation

$m \approx a^2 \rightarrow$ Cannot expand the logarithm

Uncertainty when assuming polynomial ansatz in a^2 ?

Conclusion

- The two-step matching

$$\text{Lattice} \rightarrow \text{Symanzik's eff. theory} \rightarrow \text{ChPT}$$

allows us to systematically construct a chiral Lagrangian at non-zero a

- ChPT at non-zero a can be very different

Be careful with expectations from Continuum ChPT

- Use the right effective theory for the chiral extrapolation